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APPROXIMATION OF FIXED POINTS VIA A NEW FOUR-STEP ITERATION METHOD FOR SUZUKI MAPPINGS

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Abstract. This paper provides a new iterative method for estimating fixed points (FPs) for Suzuki mappings (Suz-map) using a Banach space framework. With the necessary assumptions, we obtain some convergence results. Furthermore, numerical computations and graphical representations show that the suggested iterative method has a faster convergence rate than many other current methods.

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Key words and phrases. Banach space; convergent sequence; fixed point; iteration method; Suzuki mapping.

1. Introduction

In nonlinear functional analysis, fixed-point (FP) theory is crucial. Numerous writers have studied FP theory on a wide range of subjects, as it is a potent branch of contemporary mathematics with multiple applications [1–5]. Analytical solutions for many nonlinear situations are challenging. FP approximation approaches are employed to solve these issues.

Mann [6] presents the following iterative technique:

$$g_{n+1} = (1 - r_n)\, g_n + r_n \aleph\, g_n \text{ for all } n \geq 0 \tag{1.1}$$

where $r_n \in (0,1)$ and $g_\circ \in G(G \subseteq H$ where H is Banach space).

Ishikawa [7] developed the following iterative method:

$$\begin{aligned} w_n &= (1 - \mathfrak{R}_n)\, g_n + \mathfrak{R}_n \aleph\, g_n \\ g_{n+1} &= (1 - r_n)\, g_n + r_n \aleph\, w_n \end{aligned} \tag{1.2}$$

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A novel iterative technique was developed by Noor [8] in 2000.

$$\begin{aligned} \nu_n &= (1-\mu_n)\, g_n + \mu_n \aleph\, g_n \\ w_n &= (1-\mathfrak{R}_n)\, g_n + \mathfrak{R}_n \aleph\, \nu_n \\ g_{n+1} &= (1-r_n)\, g_n + r_n \aleph\, w_n \end{aligned} \tag{1.3}$$

for all $n \geq 0$ where $r_n,\ \mathfrak{R}_n, \mu_n \in (0,1)$.

Agarwal et al. [9] described an iterative technique as follows.

$$\begin{aligned} w_n &= (1-\mathfrak{R}_n)\, g_n + \mathfrak{R}_n \aleph\, g_n \\ g_{n+1} &= (1-r_n)\, \aleph g_n + r_n \aleph\, w_n \end{aligned} \tag{1.4}$$

Abbas and Nazir [10] introduced an iterative approach as follows:

$$\begin{aligned} \nu_n &= (1-\mu_n)\, g_n + \mu_n \aleph\, g_n \\ w_n &= (1-\mathfrak{R}_n)\, \aleph g_n + \mathfrak{R}_n \aleph\, \nu_n \\ g_{n+1} &= (1-r_n)\, \aleph\, w_n + r_n \aleph\, \nu_n \end{aligned} \tag{1.5}$$

While Thakur et al. [11] devised an iterative approach as follows.

$$\begin{aligned} \nu_n &= (1-\mu_n)\, g_n + \mu_n \aleph\, g_n \\ w_n &= (1-\mathfrak{R}_n)\, \nu_n + \mathfrak{R}_n \aleph \nu_n \\ g_{n+1} &= (1-r_n)\, \aleph\, \nu_n + r_n \aleph w_n \end{aligned} \tag{1.6}$$

Numerous iterative techniques can be seen in [12–15]. Researchers have proposed new iterative methods for estimating the FPs of various mapping types. For example, the FP of nonexpansive mappings is estimated in research publications [16–20], and the approximation of FPs for generalized nonexpansive mappings is studied in [21–25].

This paper presents a new four-step iterative method for approximating FPs under Suz-maps. Furthermore, we show that for Suz-maps, the proposed technique converges quicker than a number of other contemporary iterative schemes. Furthermore, a number of convergence findings are shown for our unique iterative technique. In addition, to demonstrate the effectiveness of our new iterative technique, we perform a numerical experiment.

2. PRELIMINARIES

We examine certain notation and concepts in this section that can aid in explaining our results. In this work, $\mathfrak{I}(\aleph)$ refers to the set of all FP of $\aleph$, where H is a Banach space(BN-space)

Definition 2.1. [26] *A BN- space H is termed as uniformly convex (UCB)if for every $\wp \in (0, 2]$, there is $\tau > 0$ such that $\forall p, \mathfrak{z} \in H$*

$$\left.\begin{aligned} \|p\| \leq 1 \\ \|\mathfrak{z}\| \leq 1 \\ \|p - \mathfrak{z}\| > \wp \end{aligned}\right\} \textit{implies} \frac{\|p + z\|}{2} \leq \tau \tag{2.1}$$

Definition 2.2. [27] *A map $\aleph : G \to G$ is termed Suzuki mapping$(Suz\text{-}map)$ if the following is hold:*

$$\frac{1}{2}\|p - \aleph p\| \leq \|p - \mathfrak{z}\| \Rightarrow \|\aleph p - \aleph \mathfrak{z}\| \leq \|p - \mathfrak{z}\|, \tag{2.2}$$

for every two elements $p, \mathfrak{z} \in G$.

Definition 2.3. [28] *Consider a bounded sequence $\{g_n\}$ in H. If $\emptyset \neq G \subseteq H$ (where G is convex and closed), then the asymptotic radius of $\{g_n\}$ is determined by*

$$\mathfrak{N}(G, g_n) = \inf\left\{\lim_{n\to\infty} \sup \|g_n - p\| : p \in\ G\right\}.$$

Similarly, the asymptotic center of $\{g_n\}$, which corresponds to G, is determined and shown using the formula:

$$\mathcal{A}(G, g_n) = \left\{p \in G : \lim_{n\to\infty} \sup \|g_n - p\| = \mathfrak{N}(G, \{g_n\})\right\}$$

Lemma 2.4. [27] *Let $\emptyset \neq G \subseteq H$, and $\aleph : G \longrightarrow G$. If $\aleph$ is a Suz-map, then the property $\|\aleph p - \aleph \beth\| \leq \|p - \beth\|$ holds for each $p \in G$ and each $\beth \in \mathfrak{I}(\aleph)$.*

Lemma 2.5. [27] *Let $\emptyset \neq G \subseteq H$, and $\aleph : G \to G$. If $\aleph$ is a Suz-map, then the property*

$$\|p - \aleph \mathfrak{z}\| \leq 3\|p - \aleph p\| + \|p - \mathfrak{z}\|. \tag{2.3}$$

holds for each $p, \mathfrak{z} \in G$.

Lemma 2.6. [29] *Consider H be a UCB- space and assume $0 < \wp \leq g_n \leq \sigma < 1$. If there is real number $\lambda \geq 0$ with $\{d_n\}$ and $\{b_n\}$ in H fulfill $\lim_{n\to\infty} \sup \|d_n\| \leq \lambda, \lim_{n\to\infty} \sup \|b_n\| \leq \lambda$ and $\lim_{n\to\infty} \sup \|g_n d_n + (1 - g_n)b_n\| = \lambda$, then $\lim_{n\to\infty} \sup \|d_n - b_n\| = 0$.*

3. MAIN RESULTS

In this part, a novel iteration method is introduced, and some convergence findings are obtained. The new iteration method is defined as follows:

$$\begin{aligned} \nu_n &= \frac{\varrho g_n + \aleph g_n}{\varrho + 1} \\ w_n &= \aleph\left(\frac{\varrho \nu_n + \aleph \nu_n}{\varrho + 1}\right) \\ s_n &= \aleph(w_n) \\ g_{n+1} &= \aleph(\aleph w_n) \end{aligned} \tag{3.1}$$

where $\varrho \geq 0$.

The FP for the Suz-map was approximated in this part using the iterative method (3.1). In Banach space, we prove various convergence results. We provide numerical examples to contrast the rapidity of the proposed method with that of some of the most prominent iterative methods.

Lemma 3.1. *Let $G \subseteq H$ (G is closed convex subset) and $\aleph : G \to G$ is a Suz-map with $\Im(\aleph) \neq \emptyset$. Suppose $\{g_n\}$ is given in (3.1). Then $\lim_{n\to\infty} \|g_n - \gimel\|$ exists $\forall \gimel \in \Im(\aleph)$.*

Proof. Consider $\gimel \in ((\aleph)$. Based on Lemma 2.4, one gets

$$\begin{aligned}
\|\nu_n - \gimel\| &= \left\| \frac{\varrho g_n + \aleph g_n}{\varrho + 1} - \gimel \right\| \\
&= \left\| \frac{\varrho}{\varrho+1} (g_n - \gimel) + \frac{1}{\varrho+1} (\aleph g_n - \gimel) \right\| \\
&\leq \frac{\varrho}{\varrho+1} \|g_n - \gimel\| + \frac{1}{\varrho+1} \aleph \|g_n - \gimel\| \\
&\leq \frac{\varrho}{\varrho+1} \|g_n - \gimel\| + \frac{1}{\varrho+1} \|g_n - \gimel\| \\
&= \|g_n - \gimel\| \qquad (3.2)
\end{aligned}$$

Moreover,

$$\begin{aligned}
\|u_n - \gimel\| &= \left\| \aleph\left(\frac{\varrho \nu_n + \aleph \nu_n}{\varrho + 1}\right) - \gimel \right\| \\
&\leq \left\| \frac{\varrho \nu_n + \aleph \nu_n}{\varrho + 1} - \gimel \right\| \\
&\leq \left\| \frac{\varrho}{\varrho+1} (\nu_n - \gimel) + \frac{1}{\varrho+1} (\aleph \nu_n - \gimel) \right\|
\end{aligned}$$

$$\begin{aligned}
&\leq \frac{\varrho}{\varrho+1} \|\nu_n - \gimel\| + \frac{1}{\varrho+1} \|\aleph \nu_n - \gimel\| \\
&\leq \frac{\varrho}{\varrho+1} \|\nu_n - \gimel\| + \frac{1}{\varrho+1} \|\nu_n - \gimel\| \\
&= \|\nu_n - \gimel\| \qquad (3.3)
\end{aligned}$$

Also we have

$$\begin{aligned}
\|s_n - \gimel\| &= \|\aleph(u_n) - \gimel\| \\
&\leq \|u_n - \gimel\| \\
&\leq \|\nu_n - \gimel\| \\
&= \|g_n - \gimel\| \qquad (3.4)
\end{aligned}$$

$$\begin{aligned}\|g_{n+1} - \gimel\| &= \|\aleph(\aleph u_n) - \gimel\| \\ &\leq \|\aleph(u_n) - \gimel\| \\ &\leq \|u_n - \gimel\| \\ &\leq \|\nu_n - \gimel\| = \|g_n - \gimel\| \end{aligned} \tag{3.5}$$

Thus $\lim_{n\to\infty} \|g_n - \gimel\|$ exists, $\forall \gimel \in \Im(\aleph)$. □

Theorem 3.2. *Let $G \subseteq H$ (G is a closed convex subset) and $\aleph : G \to G$ is a Suz-map. Suppose g_n is given in (3.1). Then, $\Im(\aleph) \neq \emptyset$ if and only if $\{g_n\}$ is bounded and $\lim_{n\to\infty} \|\aleph g_n - g_n\| = 0$.*

Proof. First, consider $\{g_n\}$ is bounded and $\lim_{n\to\infty} \|\aleph g_n - g_n\| = 0$. We shall prove that $\Im(\aleph) \neq 0$. For this, let $\gimel \in \mathcal{A}(G, \{g_n\})$. Employing Lemma 2.5, we have

$$\begin{aligned}\mathcal{A}(\aleph\gimel, \{g_n\}) &= \lim_{n\to\infty} \sup\|g_n - \aleph\gimel\| \\ &\leq 3 \lim_{n\to\infty} \sup\|g_n - \aleph g_n\| + \lim_{n\to\infty} \sup\|g_n - \gimel\|, \\ &= \lim_{n\to\infty} \sup\|g_n - \gimel\|, \\ &= \mathcal{A}(\gimel, \{g_n\}). \end{aligned} \tag{3.6}$$

It follows that $\aleph\gimel \in \mathcal{A}(G, \{g_n\})$. In UCB-space, asymptotic centers are singletons; this means $\aleph\gimel = \gimel$. Therefore, FP is nonempty.

In contrast, we presume that $\Im(\aleph) \neq \emptyset$. Lemma 3.1 indicates that $\{g_n\}$ is bounded and $\lim_{n\to\infty} \|g_n - \gimel\|$ exists. Now, if

$$\lim_{n\to\infty} \|g_n - \gimel\| = \ell, \tag{3.7}$$

Then, using Lemma 3.1's proof and (16) as a guide,

$$\lim_{n\to\infty} \sup \|\nu_n - \gimel\| \leq \lim_{n\to\infty} \sup \|g_n - \gimel\| = \ell. \tag{3.8}$$

Applying Lemma 2.4,

$$\lim_{n\to\infty} \sup\|\aleph g_n - \gimel\| \leq \lim_{n\to\infty} \sup \|g_n - \gimel\| = \ell, \tag{3.9}$$

Additionally, by looking at Lemma 3.1's proof, we can see

$$\|g_{n+1} - \gimel\| \leq \|\nu_n - \gimel\|. \tag{3.10}$$

It gives, together with (3.7),

$$\ell \leq \lim_{n\to\infty} \inf\|\nu_n - \gimel\|. \tag{3.11}$$

From (3.8) and (3.11), we obtain

$$\ell = \lim_{n\to\infty} \|\nu_n - \gimel\|. \tag{3.12}$$

From (3.12), we have

$$\begin{aligned}
\eth &= \lim_{n\to\infty} \|\nu_n - \gimel\|, \\
&= \lim_{n\to\infty} \|\frac{\varrho g_n + \aleph g_n}{\varrho + 1} - \gimel\|, \\
&= \lim_{n\to\infty} \|\frac{\varrho}{\varrho + 1}(g_n - \gimel) + \frac{1}{\varrho + 1}(\aleph g_n - \gimel)\|. \qquad (3.13)
\end{aligned}$$

Hence,

$$\eth = \lim_{n\to\infty} \|\frac{\varrho}{\varrho + 1}(g_n - \gimel) + \frac{1}{\varrho + 1}(\aleph g_n - \gimel)\| \qquad (3.14)$$

Using (3.7), (3.9), and (3.14) along with Lemma 2.6, we get

$$\lim_{n\to\infty} \|\aleph g_n - g_n\| = 0. \qquad (3.15)$$

□

4. NUMERICAL RESULTS

This section provides certain examples to support the convergence result discussed in the previous section. The iterative graph displayed in Eq. (3.1) converges to Fp faster than other iterative methods, as evidenced by the numerical and graphical analysis of these instances.

Example 4.1. *Let* $H = \mathbb{R}$ *and consider* $G = [0, 1]$. *Set a map* $\aleph : G \to G$ *as follows:*

$$\aleph(\mathfrak{z}) = \begin{cases} 1 - \mathfrak{z}, & if\ \mathfrak{z} < \frac{1}{6} \\ \frac{\mathfrak{z}+4}{5}, & if\ \mathfrak{z} \geq \frac{1}{6} \end{cases}$$

for all $\mathfrak{z} \in G$.

For $\mathfrak{z}_\circ = 0.9, \varrho = 2$ *and* $r_n, \mathfrak{R}_n, \mu_n = 0.8$.

Initially, to demonstrate that $\aleph$ *on* G *possesses the Suzuki property. The partition of the proof is as follows: Let* $\mathfrak{z}_1, \mathfrak{z}_2 \in G$ *and assume that*

$$\frac{1}{2}\|\mathfrak{z}_1 - \aleph\mathfrak{z}_1\| \leq \|\mathfrak{z}_1 - \mathfrak{z}_2\|$$

Case 1: Choose $\mathfrak{z}_1 < \frac{1}{6}$ and, $\mathfrak{z}_2 < \frac{1}{6}$ then $\aleph(\mathfrak{z}_1) = 1 - \mathfrak{z}_1$ and $\aleph(\mathfrak{z}_2) = 1 - \mathfrak{z}_2$. Hence

$$\|\aleph\mathfrak{z}_1 - \aleph\mathfrak{z}_2\| = |1 - \mathfrak{z}_1 - (1 - \mathfrak{z}_2)| = |\mathfrak{z}_1 - \mathfrak{z}_2|$$

Hence,

$$\frac{1}{2}\|\mathfrak{z}_1 - \aleph\mathfrak{z}_1\| \leq \|\mathfrak{z}_1 - \mathfrak{z}_2\| \Rightarrow \|\aleph\mathfrak{z}_1 - \aleph\mathfrak{z}_2\| \leq \|\mathfrak{z}_1 - \mathfrak{z}_2\|.$$

Case 2: Choose Select Choose $\mathfrak{z}_1 \geq \frac{1}{6}$ and, $\mathfrak{z}_2 \geq \frac{1}{6}$ then $\aleph(\mathfrak{z}_1) = \frac{\mathfrak{z}_1+4}{5}$ and $\aleph(\mathfrak{z}_2) = \frac{\mathfrak{z}_2+4}{5}$. Hence

$$\|\aleph\,\mathfrak{z}_1 - \aleph\mathfrak{z}_2\| = \left|\frac{\mathfrak{z}_1 + 4}{5} - (\frac{\mathfrak{z}_2 + 4}{5})\right| = \frac{1}{5}|\mathfrak{z}_1 - \mathfrak{z}_2| \leq |\mathfrak{z}_1 - \mathfrak{z}_2|$$

Hence,

$$\frac{1}{2}\|\mathfrak{z}_1-\aleph\mathfrak{z}_1\| \le \|\mathfrak{z}_1-\ \mathfrak{z}_2\| \Rightarrow \|\aleph\mathfrak{z}_1-\aleph\mathfrak{z}_2\| \le \|\mathfrak{z}_1-\mathfrak{z}_2\|.$$

Case 3: Select Choose $\mathfrak{z}_1<\frac{1}{6}$ and, $\mathfrak{z}_2\ge\frac{1}{6}$ then $\aleph\left(\mathfrak{z}_1\right)=1-\mathfrak{z}_1$ and $\aleph\left(\mathfrak{z}_2\right)=\frac{\mathfrak{z}_2+4}{5}$. Since$\mathfrak{z}_1<\frac{1}{6}$ then we have

$$\|\mathfrak{z}_1-\aleph\mathfrak{z}_1\|=|1-\mathfrak{z}_1-(\mathfrak{z}_1)|=|1-2\mathfrak{z}_1|\Rightarrow\frac{1}{2}\left|\mathfrak{z}_1-\aleph\mathfrak{z}_1\right|\le\ \left|\mathfrak{z}_1-\ \mathfrak{z}_2\right|$$

Thus $\frac{1}{2}\left|1-2\mathfrak{z}_1\right|\le\left|\mathfrak{z}_1-\mathfrak{z}_2\right|$ that is mean $\left|\frac{1}{2}-\mathfrak{z}_1\right|\le\left|\mathfrak{z}_1-\mathfrak{z}_2\right|\Rightarrow,\mathfrak{z}_2\ge\frac{1}{2}$ Now,

$$\|\aleph\mathfrak{z}_1-\aleph\mathfrak{z}_2\|=\left|1-\mathfrak{z}_1-\left(\frac{\mathfrak{z}_2+4}{5}\right)\right|=\frac{1}{5}\left|1-5\mathfrak{z}_1-\mathfrak{z}_2\right|$$

and since $\mathfrak{z}_1<\frac{1}{6}$ and $\mathfrak{z}_2\ge\frac{1}{2}$ then we have

$$\|\aleph\ \mathfrak{z}_1-\aleph\mathfrak{z}_2\|=\frac{1}{5}\left|1-5u_1-u_2\right|\le\left|u_1-u_2\right|$$

Case 4: Select Choose $\mathfrak{z}_1\ge\frac{1}{6}$ and, $\mathfrak{z}_2<\frac{1}{6}$. This case in a similar way to the third case, thus obtaining

$$\frac{1}{2}\|\mathfrak{z}_1-\aleph\mathfrak{z}_1\|\le\|\mathfrak{z}_1-\mathfrak{z}_2\|\Rightarrow\|\aleph\mathfrak{z}_1-\aleph\mathfrak{z}_2\|\le\|\mathfrak{z}_1-\mathfrak{z}_2\|$$

Thus, in all cases, $\aleph$ satisfies Suzuki property on G.

TABLE 1. Comparison of convergence rates for various iteration approaches

step	Mann	Ishikawa	Noor	Agarwal	Abbas	Thakur	New iteration
	0.9	0.9	0.9	0.9	0.9	0.9	0.9
1	0.96400000	0.97424000	0.97587840	0.99024000	0.99384960	0.99648640	0.99956978
2	0.98704000	0.99336422	0.99418148	0.99904742	0.99962173	0.99987655	0.99999815
3	0.99533440	0.99829062	0.99859648	0.99990703	0.99997673	0.99999566	0.99999999
4	0.99832038	0.99955966	0.99966145	0.99999093	0.99999857	0.99999985	1.00000000
5	0.99939534	0.99988657	0.99991834	0.99999911	0.99999991	0.99999999	
6	0.99978232	0.99997078	0.99998030	0.99999991	0.99999999	1.00000000	
7	0.99992164	0.99999247	0.99999525	0.99999999	1.00000000		
8	0.99997179	0.99999806	0.99999885	1.00000000			
9	0.99998984	0.99999950	0.99999972				
10	0.99999634	0.99999987	0.99999993				
11	0.99999868	0.99999997	0.99999998				
12	0.99999953	0.99999999	1.00000000				
13	0.99999983	1.00000000					
14	0.99999994						
15	0.99999998						
16	0.99999999						
17	1.00000000						

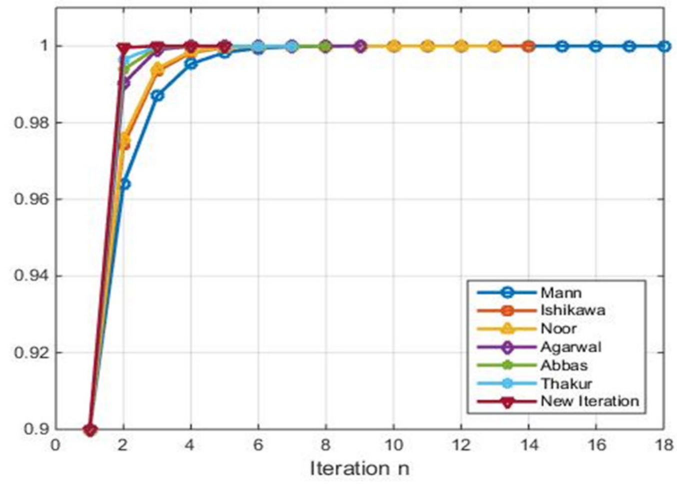

FIGURE 1. Graphical illustration of the convergence of iterative techniques.

Example 4.2. *Let* $H = \mathbb{R}$ *and consider* $G = [0, 1]$. *Set a map* $\aleph : G \to G$ *as follows:*

$$\aleph(\mathfrak{z}) = \frac{\mathfrak{z} + 1}{4}$$

for all $\mathfrak{z} \in G$.

For $\mathfrak{z}_\circ = 0, \varrho = 2$ *and* $r_n, \mathfrak{R}_n, \mu_n = 0.8, n = 1, 2, \dots$.

TABLE 2. Comparison of convergence rates for various iteration approaches

step	Mann	Ishikawa	Noor	Agarwal	Abbas	Thakur	New iteration
	0	0	0	0	0	0	0
1	0.20000000	0.24000000	0.24800000	0.29000000	0.30450000	0.31600000	0.33040365
2	0.28000000	0.30720000	0.31148800	0.32770000	0.33083925	0.33243200	0.33330758
3	0.31200000	0.32601600	0.32774093	0.33260100	0.33311760	0.33328646	0.33333311
4	0.32480000	0.33128448	0.33190168	0.33323813	0.33331467	0.33333090	0.33333333
5	0.32992000	0.33275965	0.33296683	0.33332096	0.33333172	0.33333321	
6	0.33196800	0.33317270	0.33323951	0.33333172	0.33333319	0.33333333	
7	0.33278720	0.33328836	0.33330931	0.33333312	0.33333332		
8	0.33311488	0.33332074	0.33332718	0.33333331	0.33333333		
9	0.33324595	0.33332981	0.33333176	0.33333333			
10	0.33329838	0.33333235	0.33333293				
11	0.33331935	0.33333306	0.33333323				
12	0.33332774	0.33333326	0.33333331				
13	0.33333110	0.33333331	0.33333333				
14	0.33333244	0.33333333					
15	0.33333298						
16	0.33333319						
17	0.33333328						
18	0.33333328						
19	0.33333332						
20	0.33333333						

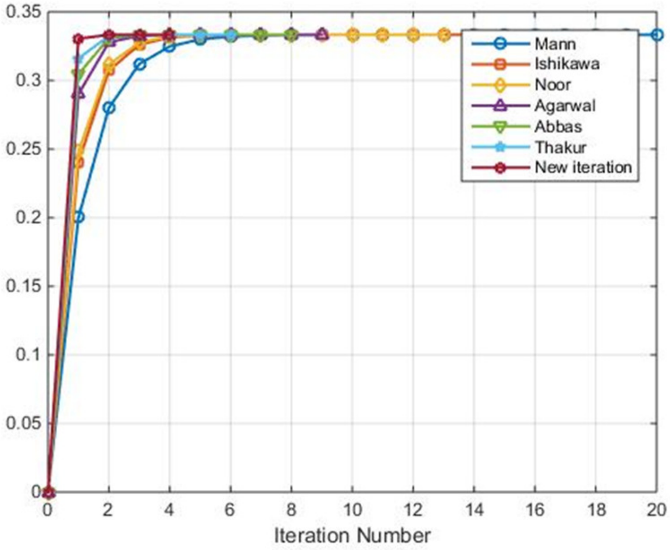

FIGURE 2. Graphical illustration of the convergence of iterative techniques.

The data in Tables 1 and 2, and the graphs in Figures 1 and 2, for each of the above examples, clearly show that the proposed new method reaches the FP more quickly than different approaches.

5. CONCLUSION

In this study, we offer an efficient new iterative method for approximating the fixed point of Suzuki mappings. The proposed iterative approach yields significant convergence results in a Banach space. some examples demonstrate the theoretical outcome.

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