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LATTICE-VALUED FUZZY SUBSTRUCTURES IN SHEFFER STROKE UP-ALGEBRAS

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Abstract. This paper introduces the notions of $\mathscr{L}$-fuzzy SUP-subalgebras and $\mathscr{L}$-fuzzy SUP-ideals within the framework of Sheffer stroke UP-algebras, alongside the corresponding level subsets of such fuzzy structures. We establish a bidirectional correspondence between these $\mathscr{L}$-fuzzy substructures and their level sets: namely, the upper level set of an $\mathscr{L}$-fuzzy SUP-subalgebra (respectively, SUP-ideal) is an SUP-subalgebra (respectively, SUP-ideal), and conversely, every SUP-subalgebra (ideal) determines an $\mathscr{L}$-fuzzy SUP-subalgebra (ideal) via its characteristic function. These findings deepen the understanding of lattice-valued fuzzy logic in Sheffer stroke UP-algebras and extend the classical correspondence between fuzzy structures and their crisp counterparts in an algebraic context.

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1. Introduction

The Sheffer operation, also known as the Sheffer stroke or NAND operator, was introduced by Sheffer [13] and holds a central place in logic due to its functional completeness: any classical logical operation can be expressed using only this operation. This universality confers structural simplicity and expressive power, enabling the reformulation of logical systems using a single connective. Furthermore, the axioms of Boolean algebras—the algebraic counterpart of classical propositional logic—can be derived using only the Sheffer operation, underscoring its fundamental role in the synthesis of logic and algebra.

Building upon this foundational idea, Sheffer stroke UP-algebras were proposed as an algebraic framework that integrates Sheffer stroke logic with algebraic structure, leading to what Iampan [3]

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termed UP-algebras. These structures provide a flexible and general context for analyzing logical systems with enriched algebraic semantics. The UP-algebraic framework opens new perspectives in decision theory, computational logic, and algebraic logic by bridging logical operations with algebraic formality.

Sheffer stroke UP-algebras [6] are prenormed algebraic systems where the Sheffer stroke serves as the primary operation, together with a distinguished constant acting as a multiplicative identity. They generalize Boolean algebras and provide an extended environment for modeling logical systems, with potential applications in functional analysis, operator theory, and fuzzy logic. Recent developments have shown that the Sheffer stroke framework accommodates a wide range of fuzzy algebraic structures. For instance, Chunsee et al. [1] employed fuzzy set theory to extend ideal theory to Sheffer stroke BE-algebras, while Iampan et al. [4] studied fuzzy subalgebras under triangular conorms in the UP-algebra context. Oner et al. [7] further explored both fuzzy and hesitant fuzzy sets over Sheffer stroke UP-algebras, highlighting their expressive richness. A series of studies by Rajesh and collaborators [10–12] has also contributed significantly to the field by investigating length-based and mean-based fuzzy ideals and subalgebras in Sheffer stroke Hilbert algebras. These results collectively demonstrate the versatility of Sheffer stroke algebras as a unifying algebraic setting for diverse fuzzy generalizations.

In this paper, we develop a theory of $\mathscr{L}$-fuzzy SUP-subalgebras and $\mathscr{L}$-fuzzy SUP-ideals in the context of Sheffer stroke UP-algebras, where $\mathscr{L}$ is a complete lattice. In particular, we investigate their corresponding level subsets and establish a precise connection between fuzzy substructures and their crisp counterparts. Specifically, we prove that an upper level set of an $\mathscr{L}$-fuzzy SUP-subalgebra (or SUP-ideal) forms an SUP-subalgebra (or SUP-ideal), and conversely, every SUP-subalgebra (or SUP-ideal) induces an $\mathscr{L}$-fuzzy structure through its characteristic function. This bidirectional correspondence enhances the understanding of lattice-valued fuzzy logic within this algebraic setting and reinforces the role of Sheffer stroke UP-algebras as a rich platform for algebraic fuzzification.

2. PRELIMINARIES

To explore the structure and behavior of $\mathscr{L}$-fuzzy sets on Sheffer stroke UP-algebras, it is essential to first recall the algebraic foundations on which these concepts are built. This section reviews key definitions and properties related to the Sheffer stroke operation and UP-algebras, which serve as the algebraic setting for our fuzzy generalizations.

The Sheffer stroke operation, denoted by $|$, is a binary operation characterized by its role in constructing logical systems from a single connective. When this operation is equipped with specific axioms, it gives rise to an algebraic system known as a Sheffer stroke UP-algebra, or more succinctly, an SUP-algebra.

SUP-algebras are algebraic structures of type $(2, 0)$, consisting of a nonempty set equipped with the Sheffer stroke operation and a constant element, typically denoted 0. They satisfy a set of axioms that encode the behavior of the operation in a way analogous to logical negation and implication. These axioms induce a partial order and enable the definition of substructures, such as SUP-subalgebras and SUP-ideals, that mirror classical notions of closure and absorption.

To set the stage for the fuzzy extensions introduced later, this section also recalls the notions of fuzzy sets, particularly in the sense of lattice-valued membership functions, and their compatibility with SUP-algebraic operations. These preliminaries provide the structural and logical groundwork needed for the development of $\mathscr{L}$-fuzzy SUP-subalgebras and SUP-ideals in subsequent sections.

Definition 2.1. [13] *Let $\langle X, |\rangle$ be a groupoid. The operation $|$ is said to be a Sheffer stroke operation if it satisfies the following conditions: for all $x, y, z \in X$,*

(S1) $x|y = y|x$

(S2) $(x|x)|(x|y) = x$

(S3) $x|((y|z)|(y|z)) = ((x|y)|(x|y))|z$

(S4) $(x|((x|x)|(y|y)))|(x|((x|x)|(y|y))) = x.$

Definition 2.2. [6] *A Sheffer stroke UP-algebra (briefly, SUP-algebra) is a structure $\langle X, |, 0\rangle$ of type $(2, 0)$ such that 0 is the fixed element in X and the following conditions are satisfied for all $x, y, z \in X$,*

(SUP-1) $$\begin{aligned}&(((z|(x|x))|(z|(x|x)))|(((y|(x|x))|(z|(y|y)))|((y|(x|x))|\\ &\quad(z|(y|y)))))|(((z|(x|x))|(z|(x|x)))|(((y|(x|x))|(z|(y|y)))|\\ &\quad((y|(x|x))|(z|(y|y))))) = 0\end{aligned}$$

(SUP-2) $x|x = x|(0|0)$

(SUP-3) $(x|(y|y))|(x|(y|y)) = 0$ *and* $(y|(x|x))|(y|(x|x)) = 0 \Rightarrow x = y.$

Proposition 2.1. [6] *Let $\langle X, |, 0\rangle$ be an SUP-algebra. Then the binary relation $x \leq y$ if and only if $(y|(x|x))|(y|(x|x)) = 0$ is a partial order on X.*

Lemma 2.1. [6] *Let $\langle X, |, 0\rangle$ be an SUP-algebra. Then for all $x, y, z \in X$, we have*

(1) $x \leq y \Rightarrow y|(z|z) \leq x|(z|z)$ *and* $z|(x|x) \leq z|(y|y)$

(2) $x \leq y \Leftrightarrow y|y \leq x|x$

(3) $y|(x|x) \leq x$

(4) $y \leq (y|(x|x))|(y|(x|x))$

(5) $x \leq y \Rightarrow x \leq (y|(z|z))|(y|(z|z))$

(6) $z|(y|y) \leq z|(y|(x|x))$

(7) $((z|(y|y))|(z|(y|y)))|(x|x) \leq z|(y|(x|x))$

(8) $x|((y|(z|z))|(y|(z|z))) \leq (x|(y|y))|((x|(z|z))|(x|(z|z))).$

Definition 2.3. [6] *A nonempty subset G of an SUP-algebra $\langle X, |, 0\rangle$ is called an SUP-subalgebra of X if $(x|(y|y))|(x|(y|y)) \in G$ for all $x, y \in G$.*

Definition 2.4. [5,6] *A nonempty subset G of an SUP-algebra $\langle X, |, 0\rangle$ is called an SUP-ideal of X if for all $x, y \in X$,*

(1) $y \in G \Rightarrow (y|(x|x))|(y|(x|x)) \in G$

(2) $(y|(x|x))|(y|(x|x)) \in G$ *and* $x \in G \Rightarrow y \in G$.

A fuzzy set [14] in a nonempty set X is defined to be a function $\mu : X \to [0, 1]$, where $[0, 1]$ is the unit closed interval of real numbers.

Definition 2.5. [8] *A fuzzy set μ in an SUP-algebra $\langle X, |, 0\rangle$ is called a fuzzy SUP-subalgebra of X if it satisfies the following:*

$$(\forall x, y \in X)(\mu((x|(y|y))|(x|(y|y))) \geq \min\{\mu(x), \mu(y)\}).$$

Definition 2.6. [9] *A fuzzy set μ in an SUP-algebra $\langle X, |, 0\rangle$ is called a fuzzy SUP-ideal of X if it satisfies the following:*

(1) $(\forall x, y \in X)(\mu((x|(y|y))|(x|(y|y))) \geq \mu(x))$

(2) $(\forall x, y \in X)(\mu(x) \geq \min\{\mu((x|(y|y))|(x|(y|y))), \mu(y)\})$

Lemma 2.2. [2] *Let $\mathscr{L} = (\mathscr{L}, \leq, \wedge, \vee, ', 0_{\mathscr{L}}, 1_{\mathscr{L}})$ be a Boolean lattice. Then the following properties hold:*

(1) $(\forall u, v \in \mathscr{L})((u \vee v)' = u' \wedge v')$

(2) $(\forall u, v \in \mathscr{L})((u \wedge v)' = u' \vee v')$

(3) $(\forall u, v \in \mathscr{L})(u \leq v \Leftrightarrow u' \geq v')$

(4) $(\forall u, v \in \mathscr{L})(u = v \Leftrightarrow u' = v')$

(5) $(\forall u, v \in \mathscr{L})(u < v \Leftrightarrow u' > v')$

3. LATTICE-VALUED FUZZY SETS IN SHEFFER STROKE UP-ALGEBRAS

In this section, we develop the theory of $\mathscr{L}$-fuzzy subsets on Sheffer stroke UP-algebras, with a particular focus on $\mathscr{L}$-fuzzy SUP-subalgebras and $\mathscr{L}$-fuzzy SUP-ideals. Here, $\langle L, |\rangle$ denotes a Sheffer stroke UP-algebra unless otherwise specified, and we adopt the simplified notation x^y to represent the expression $x|(y|y)$ for all $x, y \in L$. We also let $\mathscr{L} = (\mathscr{L}, \leq, \wedge, \vee)$ denote a complete lattice, serving as the range of truth values for fuzzy sets.

The aim of this section is twofold: first, to formally define $\mathscr{L}$-fuzzy SUP-subalgebras and SUP-ideals using lattice-valued membership functions; and second, to establish a bidirectional correspondence between these fuzzy substructures and their level subsets. Specifically, we prove that:

- the upper level set of an $\mathscr{L}$-fuzzy SUP-subalgebra (respectively, SUP-ideal) is an SUP-subalgebra (respectively, SUP-ideal), and

- conversely, every SUP-subalgebra (or SUP-ideal) yields an $\mathscr{L}$-fuzzy SUP-subalgebra (or SUP-ideal) via its characteristic function.

This lattice-valued generalization not only extends classical fuzzy algebraic theory but also demonstrates the expressive power of Sheffer stroke UP-algebras in capturing fuzzy logic through an algebraic lens. The results presented herein serve as the core contribution of this work.

Definition 3.1. *An $\mathscr{L}$-fuzzy set $\mathcal{L}$ in L is called an $\mathscr{L}$-fuzzy SUP-subalgebra of L if*

$$(\forall x, y \in L)\left(\mathcal{L}_\mu(x^y|x^y) \geq \mathcal{L}_\mu(x) \wedge \mathcal{L}_\mu(y) \right). \tag{3.1}$$

Lemma 3.1. *If $\mathcal{L}$ is an $\mathscr{L}$-fuzzy SUP-subalgebra of L, then*

$$(\forall x \in L)\left(\mathcal{L}_\mu(0) \geq \mathcal{L}_\mu(x) \right). \tag{3.2}$$

Proof. Let $\mathcal{L}$ be an $\mathscr{L}$-fuzzy SUP-subalgebra of L. Then

$$\mathcal{L}_\mu(0) = \mathcal{L}_\mu((x|(x|x))|(x|(x|x))) \geq \mathcal{L}_\mu(x) \wedge \mathcal{L}_\mu(x) = \mathcal{L}_\mu(x)$$

for all $x \in L$. □

From now on, let $\mathscr{L}$ denote a complete lattice, written as $\mathscr{L} = (\mathscr{L}, \leq, \wedge, \vee, 0_{\mathscr{L}}, 1_{\mathscr{L}})$.

Let A be a subset of L. Then the characteristic function χ_A of L is a function of L into $\{0_{\mathscr{L}}, 1_{\mathscr{L}}\}$ defined as follows:

$$\chi_A(x) = \begin{cases} 1_{\mathscr{L}} & \text{if } x \in A \\ 0_{\mathscr{L}} & \text{otherwise.} \end{cases}$$

By the definition of characteristic function, χ_A is a function of L into $\{0_{\mathscr{L}}, 1_{\mathscr{L}}\} \subset \mathscr{L}$. We denote the $\mathscr{L}$-fuzzy set $\mathcal{L}_A$ in L is described by its membership function χ_A, is called the characteristic $\mathscr{L}$-fuzzy set of A in L.

Lemma 3.2. *Let the constant 0 of L be in A. Then $\chi_A(0) \geq \chi_A(a)$ for all $a \in L$.*

Proof. For all $a \in L, \chi_A(0) = 1_{\mathscr{L}} \geq \chi_A(a)$. □

Lemma 3.3. *Let A be a nonempty subset of L. If $\chi_A(0) \geq \chi_A(a)$ for all $a \in L$, then the constant 0 of L is in A.*

Proof. Assume that $\chi_A(0) \geq \chi_A(a)$ for all $a \in L$. Since A is a nonempty subset of L, we have an element u in A, that is, $\chi_A(u) = 1_{\mathscr{L}}$. Thus, $1_{\mathscr{L}} \geq \chi_A(0) \geq \chi_A(u) = 1_{\mathscr{L}}$. So $\chi_A(0) = 1_{\mathscr{L}}$, that is, $0 \in A$. □

Theorem 3.1. *A nonempty subset A of L is an SUP-subalgebra of L if and only if the characteristic $\mathscr{L}$-fuzzy set $\mathcal{L}_A$ is an $\mathscr{L}$-fuzzy SUP-subalgebra of L.*

Proof. Assume that A is an SUP-subalgebra of L. Let $a, b \in L$.

Case 1: $a, b \in A$. Then $\chi_A(a) = 1_{\mathscr{L}} = \chi_A(b)$, so $\chi_A(a) \wedge (b) = 1_{\mathscr{L}}$. Since A is an SUP-subalgebra of L, $a^b|a^b \in A$ and so $\chi_A(a^b|a^b) = 1_{\mathscr{L}}$. Therefore, $\chi_A(a^b|a^b) = 1_{\mathscr{L}} \geq 1_{\mathscr{L}} = \chi_A(a) \wedge \chi_A(b)$.

Case 2: $a \notin A$ or $b \notin A$. Then $\chi_A(a) = 0_{\mathscr{L}}$ or $\chi_A(b) = 0_{\mathscr{L}}$, so $\chi_A(a) \wedge \chi_A(b) = 0_{\mathscr{L}}$. Therefore, $\chi_A(a^b|a^b) \geq 0_{\mathscr{L}} = \chi_A(a) \wedge \chi_A(b)$.

Hence, $\mathcal{L}_A$ is an $\mathscr{L}$-fuzzy SUP-subalgebra of L.

Conversely, assume that $\mathcal{L}_A$ is an $\mathscr{L}$-fuzzy SUP-subalgebra of L. Let $a, b \in A$. Then $\chi_A(a) = 1_{\mathscr{L}} = \chi_A(b)$, so $\chi_A(a) \wedge \chi_A(b) = 1_{\mathscr{L}}$. Since $\mathcal{L}_A$ is an $\mathscr{L}$-fuzzy SUP-subalgebra of L, we have $1_{\mathscr{L}} \geq \chi_A(a^b|a^b) \geq \chi_A(a) \wedge \chi_A(b) = 1_{\mathscr{L}}$. By anti-symmetry, we have $\chi_A(a^b|a^b) = 1_{\mathscr{L}}$, that is, $(a|(b|b))|(a|(b|b)) \in A$. Hence, A is an SUP-subalgebra of L. □

Definition 3.2. *Let $\mathcal{L}$ be an $\mathscr{L}$-fuzzy set in L with the membership function $\mathcal{L}_\mu$. For any $t \in \mathscr{L}$, the sets*

$$\mathtt{U}(\mathcal{L}_\mu, t) = \{x \in L : \mathcal{L}_\mu(x) \geq t\}$$

$$\mathtt{L}(\mathcal{L}_\mu, t) = \{x \in L : \mathcal{L}_\mu(x) \leq t\}$$

are referred to as an upper t-level subset, a lower t-level subset of $\mathcal{L}$, respectively.

Theorem 3.2. *An $\mathscr{L}$-fuzzy set $\mathcal{L}$ is an $\mathscr{L}$-fuzzy SUP-subalgebra of L if and only if $\mathtt{U}(\mathcal{L}_\mu, t)$ is, if it is nonempty, an SUP-subalgebra of L for every $t \in \mathscr{L}$.*

Proof. Assume $\mathcal{L}$ is an $\mathscr{L}$-fuzzy SUP-subalgebra of L. Let $t \in \mathscr{L}$ be such that $\mathtt{U}(\mathcal{L}_\mu, t) \neq \emptyset$. Let $x, y \in L$. Then

$$\begin{aligned} x, y \in \mathtt{U}(\mathcal{L}_\mu, t) &\Rightarrow \mathcal{L}_\mu(x) \geq t, \mathcal{L}_\mu(y) \geq t \\ &\Rightarrow \mathcal{L}_\mu(x) \wedge \mathcal{L}_\mu(y) \geq t \\ &\Rightarrow \mathcal{L}_\mu(x^y|x^y) \geq \mathcal{L}_\mu(x) \wedge \mathcal{L}_\mu(y) \geq t \\ &\Rightarrow \mathcal{L}_\mu(x^y|x^y) \geq t \\ &\Rightarrow x^y|x^y \in \mathtt{U}(\mathcal{L}_\mu, t). \end{aligned}$$

Hence, $\mathtt{U}(\mathcal{L}_\mu, t)$ is an SUP-subalgebra of L.

Conversely, assume for all $t \in \mathscr{L}, \mathtt{U}(\mathcal{L}_\mu, t)$ is an SUP-subalgebra of L if it is nonempty. Let $x, y \in L$. Choose $t = \mathcal{L}_\mu(x) \wedge \mathcal{L}_\mu(y) \in \mathscr{L}$. Then $\mathcal{L}_\mu(x) \geq t$ and $\mathcal{L}_\mu(y) \geq t$. Thus, $x, y \in \mathtt{U}(\mathcal{L}_\mu, t) \neq \emptyset$. As the hypothesis, we get $\mathtt{U}(\mathcal{L}_\mu, t)$ is an SUP-subalgebra of L and so $x^y|x^y \in \mathtt{U}(\mathcal{L}_\mu, t)$. Thus, $\mathcal{L}_\mu(x^y|x^y) \geq t = \mathcal{L}_\mu(x) \wedge \mathcal{L}_\mu(y)$. Hence, $\mathcal{L}$ is an $\mathscr{L}$-fuzzy SUP-subalgebra of L. □

Definition 3.3. *Let $\mathscr{L} = (\mathscr{L}, \leq, \wedge, \vee, ', 0_{\mathscr{L}}, 1_{\mathscr{L}})$ be a Boolean lattice. Let $\mathcal{L}$ be an $\mathscr{L}$-fuzzy set in L. The $\mathscr{L}$-fuzzy set $\mathcal{L}'$ defined by*

$$(\forall a \in L)(\mathcal{L}'_\mu(a) = (\mathcal{L}_\mu(a))' = \mathcal{L}_\mu(a)')$$

is called the complement of $\mathcal{L}$ in L.

Theorem 3.3. *Let $\mathscr{L} = (\mathscr{L}, \leq, \wedge, \vee, ', 0_{\mathscr{L}}, 1_{\mathscr{L}})$ be a Boolean lattice. Then $\mathcal{L}'$ is an $\mathscr{L}$-fuzzy SUP-subalgebra of L if and only if $\mathtt{L}(\mathcal{L}_\mu, t)$ is, if it is nonempty, an SUP-subalgebra of L for every $t \in \mathscr{L}$.*

Proof. Assume $\mathcal{L}'$ is an $\mathscr{L}$-fuzzy SUP-subalgebra of L. Let $t \in \mathscr{L}$ be such that $\mathtt{L}(\mathcal{L}_\mu, t) \neq \emptyset$. Let $x, y \in L$. Then

$$\begin{aligned}
x, y \in \mathtt{L}(\mathcal{L}_\mu, t) &\Rightarrow \mathcal{L}_\mu(x) \leq t, \mathcal{L}_\mu(y) \leq t \\
&\Rightarrow \mathcal{L}_\mu(x) \vee \mathcal{L}_\mu(y) \leq t \\
&\Rightarrow (\mathcal{L}_\mu(x) \vee \mathcal{L}_\mu(y))' \geq t' \\
&\Rightarrow \mathcal{L}_\mu(x)' \wedge \mathcal{L}_\mu(y)' \geq t' \\
&\Rightarrow \mathcal{L}_\mu(x^y|x^y)' \geq \mathcal{L}_\mu(x)' \wedge \mathcal{L}_\mu(y)' \geq t' \\
&\Rightarrow \mathcal{L}_\mu(x^y|x^y)' \geq t' \\
&\Rightarrow \mathcal{L}_\mu(x^y|x^y) \leq t \\
&\Rightarrow x^y|x^y \in \mathtt{L}(\mathcal{L}_\mu, t).
\end{aligned}$$

Hence, $\mathtt{L}(\mathcal{L}_\mu, t)$ is an SUP-subalgebra of L.

Conversely, assume for all $t \in \mathscr{L}, \mathtt{L}(\mathcal{L}_\mu, t)$ is an SUP-subalgebra of L if it is nonempty. Let $x, y \in L$. Choose $t = \mathcal{L}_\mu(x) \vee \mathcal{L}_\mu(y) \in \mathscr{L}$. Then $\mathcal{L}_\mu(x) \leq t$ and $\mathcal{L}_\mu(y) \leq t$. Thus, $x, y \in \mathtt{U}(\mathcal{L}_\mu, t) \neq \emptyset$. As the hypothesis, we get $\mathtt{L}(\mathcal{L}_\mu, t)$ is an SUP-subalgebra of L and so $x^y|x^y \in \mathtt{L}(\mathcal{L}_\mu, t)$. Thus, $\mathcal{L}_\mu(x^y|x^y) \leq t = \mathcal{L}_\mu(x) \vee \mathcal{L}_\mu(y)$. By Lemma 2.2 (1), we have $\mathcal{L}_\mu(x^y|x^y)' \geq t = \mathcal{L}_\mu(x)' \wedge \mathcal{L}'_\mu(y)$. Hence, $\mathcal{L}'$ is an $\mathscr{L}$-fuzzy SUP-subalgebra of L. □

Definition 3.4. *An $\mathscr{L}$-fuzzy set $\mathcal{L}$ in L is called an $\mathscr{L}$-fuzzy SUP-ideal of L if*

$$(\forall x, y \in L) \left(\begin{array}{l} \mathcal{L}_\mu(x^y|x^y) \geq \mathcal{L}_\mu(x) \\ \mathcal{L}_\mu(x) \geq \mathcal{L}_\mu(x^y|x^y) \wedge \mathcal{L}_\mu(y) \end{array} \right). \tag{3.3}$$

Theorem 3.4. *Every $\mathscr{L}$-fuzzy SUP-ideal of L is an $\mathscr{L}$-fuzzy SUP-subalgebra of L.*

Proof. The theorem follows directly from (3.3). □

Theorem 3.5. *A nonempty subset A of L is an SUP-ideal of L if and only if the characteristic $\mathscr{L}$-fuzzy set $\mathcal{L}_A$ is an $\mathscr{L}$-fuzzy SUP-ideal of L.*

Proof. Assume that A is an SUP-ideal of L. Let $x, y \in L$.

Case 1: $x \in A$. Then $\chi_A(x) = 1_{\mathscr{L}}$. Since A is an SUP-ideal of L, $x^y|x^y \in A$ and so $\chi_A(x^y|x^y) = 1_{\mathscr{L}}$. Therefore, $\chi_A(x^y|x^y) = 1_{\mathscr{L}} \geq 1_{\mathscr{L}} = \chi_A(x)$.

Case 2: $x \notin A$. Then $\chi_A(x) = 0_{\mathscr{L}}$. Therefore, $\chi_A(x^y|x^y) \geq 0_{\mathscr{L}} = \chi_A(x)$.

Let $x, y \in L$.

Case 1: $x^y|x^y, y \in A$. Then $\chi_A(x^y|x^y) = 1_{\mathscr{L}} = \chi_A(y)$, so $\chi_A(x^y|x^y) \wedge \chi_A(y) = 1_{\mathscr{L}}$. Since A is an SUP-ideal of L, $x \in A$ and so $\chi_A(x) = 1_{\mathscr{L}}$. Therefore, $\chi_A(x) = 1_{\mathscr{L}} \geq 1_{\mathscr{L}} = \chi_A(x^y|x^y) \wedge \chi_A(y)$.

Case 2: $x^y|x^y \notin A$ or $y \notin A$. Then $\chi_A(x^y|x^y) = 0_{\mathscr{L}}$ or $\chi_A(y) = 0_{\mathscr{L}}$, so $\chi_A(x^y|x^y) \wedge \chi_A(y) = 0_{\mathscr{L}}$. Therefore, $\chi_A(x) \geq 0_{\mathscr{L}} = \chi_A(x^y|x^y) \wedge \chi_A(y)$.

Hence, $\mathcal{L}_A$ is an $\mathscr{L}$-fuzzy SUP-ideal of L.

Conversely, assume that $\mathcal{L}_A$ is an $\mathscr{L}$-fuzzy SUP-ideal of L. Let $x, y \in L$ be such that $x \in A$. Then $\chi_A(x) = 1_{\mathscr{L}}$. Since $\mathcal{L}_A$ is an $\mathscr{L}$-fuzzy SUP-ideal of L, $1_{\mathscr{L}} \geq \chi_A(x^y|x^y) \geq \chi_A(x) = 1_{\mathscr{L}}$. By anti-symmetry, we have $\chi_A(x^y|x^y) = 1_{\mathscr{L}}$, that is, $x^y|x^y \in A$. Let $x, y \in L$ be such that $x^y|x^y, y \in A$. Then $\chi_A(x^y|x^y) = 1_{\mathscr{L}} = \chi_A(y)$, so $\chi_A(x^y|x^y) \wedge \chi_A(y) = 1_{\mathscr{L}}$. Since $\mathcal{L}_A$ is an $\mathscr{L}$-fuzzy SUP-ideal of L, $1_{\mathscr{L}} \geq \chi_A(x) \geq \chi_A(x^y|x^y) \wedge \chi_A(y) = 1_{\mathscr{L}}$. By anti-symmetry, we have $\chi_A(x) = 1_{\mathscr{L}}$, that is, $x \in A$. Hence, A is an SUP-ideal of L. □

Theorem 3.6. *An $\mathscr{L}$-fuzzy set $\mathcal{L}$ is an $\mathscr{L}$-fuzzy SUP-ideal of L if and only if $\mathtt{U}(\mathcal{L}_\mu, t)$ is, if it is nonempty, an SUP-ideal of L for every $t \in \mathscr{L}$.*

Proof. Assume $\mathcal{L}$ is an $\mathscr{L}$-fuzzy SUP-ideal of L. Let $t \in \mathscr{L}$ be such that $\mathtt{U}(\mathcal{L}_\mu, t) \neq \emptyset$. Let $x, y \in L$. Then

$$\begin{aligned} x \in \mathtt{U}(\mathcal{L}_\mu, t) &\Rightarrow \mathcal{L}_\mu(x) \geq t \\ &\Rightarrow \mathcal{L}_\mu(x^y|x^y) \geq \mathcal{L}_\mu(x) \geq t \\ &\Rightarrow \mathcal{L}_\mu(x^y|x^y) \geq t \\ &\Rightarrow x^y|x^y \in \mathtt{U}(\mathcal{L}_\mu, t). \end{aligned}$$

Let $x, y \in L$. Then

$$\begin{aligned} x^y|x^y, y \in \mathtt{U}(\mathcal{L}_\mu, t) &\Rightarrow \mathcal{L}_\mu(x^y|x^y) \geq t, \mathcal{L}_\mu(y) \geq t \\ &\Rightarrow \mathcal{L}_\mu(x) \geq \mathcal{L}_\mu(x^y|x^y) \wedge \mathcal{L}_\mu(y) \geq t \\ &\Rightarrow \mathcal{L}_\mu(x) \geq t \\ &\Rightarrow x \in \mathtt{U}(\mathcal{L}_\mu, t). \end{aligned}$$

Hence, $\mathtt{U}(\mathcal{L}_\mu, t)$ is an SUP-ideal of L.

Conversely, assume for all $t \in \mathscr{L}, \mathtt{U}(\mathcal{L}_\mu, t)$ is an SUP-ideal of L if it is nonempty. Let $x, y \in L$. Choose $t = \mathcal{L}_\mu(x) \in \mathscr{L}$. Then $\mathcal{L}_\mu(x) \geq t$. Thus, $x \in \mathtt{U}(\mathcal{L}_\mu, t) \neq \emptyset$. As the hypothesis, we get $\mathtt{U}(\mathcal{L}_\mu, t)$ is an SUP-ideal of L and so $x^y|x^y \in \mathtt{U}(\mathcal{L}_\mu, t)$. Thus, $\mathcal{L}_\mu(x^y|x^y) \geq t = \mathcal{L}_\mu(x)$. Let $x, y \in L$. Choose $t = \mathcal{L}_\mu(x^y|x^y) \wedge \mathcal{L}_\mu(y) \in \mathscr{L}$. Then $\mathcal{L}_\mu(x^y|x^y) \geq t$ and $\mathcal{L}_\mu(y) \geq t$. Thus, $x^y|x^y, y \in \mathtt{U}(\mathcal{L}_\mu, t) \neq \emptyset$. As the hypothesis, we get $\mathtt{U}(\mathcal{L}_\mu, t)$ is an SUP-ideal of L and so $x^y|x^y, y \in \mathtt{U}(\mathcal{L}_\mu, t)$. Thus, $\mathcal{L}_\mu(x) \geq t = \mathcal{L}_\mu(x^y|x^y) \wedge \mathcal{L}_\mu(y)$. Hence, $\mathcal{L}$ is an $\mathscr{L}$-fuzzy SUP-ideal of L. □

Theorem 3.7. *Let $\mathscr{L} = (\mathscr{L}, \leq, \wedge, \vee, ', 0_{\mathscr{L}}, 1_{\mathscr{L}})$ be a Boolean lattice. Then $\mathcal{L}'$ is an $\mathscr{L}$-fuzzy SUP-ideal of L if and only if $\mathtt{L}(\mathcal{L}_\mu, t)$ is, if it is nonempty, an SUP-ideal of L for every $t \in \mathscr{L}$.*

Proof. Assume $\mathcal{L}'$ is an $\mathscr{L}$-fuzzy SUP-ideal of L. Let $t \in \mathscr{L}$ be such that $\mathtt{L}(\mathcal{L}_\mu, t) \neq \emptyset$. Let $x, y \in L$. Then

$$\begin{aligned} x \in \mathtt{L}(\mathcal{L}_\mu, t) &\Rightarrow \mathcal{L}_\mu(x) \leq t \\ &\Rightarrow \mathcal{L}_\mu(x)' \geq t' \\ &\Rightarrow \mathcal{L}_\mu(x^y|x^y)' \geq \mathcal{L}_\mu(x)' \geq t' \\ &\Rightarrow \mathcal{L}_\mu(x^y|x^y)' \geq t' \\ &\Rightarrow \mathcal{L}_\mu(x^y|x^y) \leq t \\ &\Rightarrow x^y|x^y \in \mathtt{L}(\mathcal{L}_\mu, t). \end{aligned}$$

Let $x, y \in L$. Then

$$\begin{aligned} x^y|x^y, y \in \mathtt{L}(\mathcal{L}_\mu, t) &\Rightarrow \mathcal{L}_\mu(x^y|x^y) \leq t, \mathcal{L}_\mu(y) \leq t \\ &\Rightarrow \mathcal{L}_\mu(x^y|x^y) \vee \mathcal{L}_\mu(y) \leq t \\ &\Rightarrow (\mathcal{L}_\mu(x^y|x^y) \vee \mathcal{L}_\mu(y))' \geq t' \\ &\Rightarrow \mathcal{L}_\mu(x^y|x^y)' \wedge \mathcal{L}_\mu(y)' = (\mathcal{L}_\mu(x) \vee \mathcal{L}_\mu(y))' \geq t' \\ &\Rightarrow \mathcal{L}_\mu(x)' \geq \mathcal{L}_\mu(x)' \wedge \mathcal{L}_\mu(y)' \geq t' \\ &\Rightarrow \mathcal{L}_\mu(x)' \geq t' \\ &\Rightarrow \mathcal{L}_\mu(x) \leq t \\ &\Rightarrow x \in \mathtt{L}(\mathcal{L}_\mu, t). \end{aligned}$$

Hence, $\mathtt{L}(\mathcal{L}_\mu, t)$ is an SUP-ideal of L.

Conversely, assume for all $t \in \mathscr{L}, \mathtt{L}(\mathcal{L}_\mu, t)$ is an SUP-ideal of L if it is nonempty. Let $x, y \in L$. Choose $t = \mathcal{L}_\mu(x) \in \mathscr{L}$. Then $\mathcal{L}_\mu(x) \leq t$. Thus, $x \in \mathtt{L}(\mathcal{L}_\mu, t) \neq \emptyset$. As the hypothesis, we get $\mathtt{L}(\mathcal{L}_\mu, t)$ is an SUP-ideal of L and so $x^y|x^y \in \mathtt{L}(\mathcal{L}_\mu, t)$. Thus, $\mathcal{L}_\mu(x^y|x^y) \leq t = \mathcal{L}_\mu(x)$. Let $x, y \in L$. Choose $t = \mathcal{L}_\mu(x^y|x^y) \vee \mathcal{L}_\mu(y) \in \mathscr{L}$. Then $\mathcal{L}_\mu(x^y|x^y) \leq t$ and $\mathcal{L}_\mu(y) \leq t$. Thus, $x^y|x^y, y \in \mathtt{L}(\mathcal{L}_\mu, t) \neq \emptyset$. As the hypothesis, we get $\mathtt{L}(\mathcal{L}_\mu, t)$ is an SUP-ideal of L and so $x^y|x^y, y \in \mathtt{L}(\mathcal{L}_\mu, t)$. Thus, $\mathcal{L}_\mu(x) \leq t = \mathcal{L}_\mu(x^y|x^y) \vee \mathcal{L}_\mu(y)$. By Lemma 2.2 (1), we have $\mathcal{L}_\mu(x)' \geq \mathcal{L}_\mu(x^y|x^y)' \wedge \mathcal{L}_\mu(y)'$. Hence, $\mathcal{L}'$ is an $\mathscr{L}$-fuzzy SUP-ideal of L. □

Theorem 3.8. *Given a nonempty subset F of L, let $\mathcal{L}_F = (L, \mathcal{L}_\mu)$ be an $\mathscr{L}$-fuzzy set in L defined as follows:* $\mathcal{L}_\mu(x) = \begin{cases} \alpha_0 & \text{if } x \in F \\ \alpha_1 & \text{otherwise} \end{cases}$ *for all $x \in L$. Then $\mathcal{L}$ is an $\mathscr{L}$-fuzzy SUP-ideal of L if and only if F is an SUP-ideal of L.*

Proof. Assume that $\mathcal{L}_F = (L, \mathcal{L}_\mu)$ is an $\mathscr{L}$-fuzzy SUP-ideal of L. Let $x, y \in L$ be such that $x^y|x^y, x \in F$. Then $\mathcal{L}_\mu(x^y|x^y) \geq \mathcal{L}_\mu(x) = \alpha_0$, and so $\mathcal{L}_\mu(x^y|x^y) = \alpha_0$. This shows that $x^y|x^y \in F$. Also,

$$\mathcal{L}_\mu(x) \geq \mathcal{L}_\mu(x^y|x^y) \wedge \mathcal{L}_\mu(y) = \alpha_0,$$

and so $\mathcal{L}_\mu(x) = \alpha_0$. This shows that $x \in F$. Therefore, F is an SUP-ideal of L.

Conversely, let F be an SUP-ideal of L. For every $x, y \in L$, if $x \in F$, then $x^y|x^y \in F$ which implies that $\mathcal{L}_\mu(x^y|x^y) = \alpha_0 = \mathcal{L}_\mu(x)$. If $x^y|x^y \notin F$, then $\mathcal{L}_\mu(x^y|x^y) = \alpha_1 < \mathcal{L}_\mu(x)$. For every $x, y \in L$, if $x^y|x^y, y \in F$, then $x \in F$ which implies that $\mathcal{L}_\mu(x) = \alpha_0 = \mathcal{L}_\mu(x^y|x^y) \wedge \mathcal{L}_\mu(y)$. If $x^y|x^y \notin F$ or $y \notin F$, then $\mathcal{L}_\mu(x) \geq \alpha_1 = \mathcal{L}_\mu(x^y|x^y) \wedge \mathcal{L}_\mu(y)$. Therefore, $\mathcal{L}_F = (L, \mathcal{L}_\mu)$ is an $\mathscr{L}$-fuzzy SUP-ideal of L. □

Definition 3.5. [6] *Let $\langle A, |_A, 0_A\rangle$ and $\langle B, |_B, 0_B\rangle$ be SUP-algebras. Then a mapping $f : A \to B$ is called a homomorphism if $f(x|_A y) = f(x)|_B f(y)$ for all $x, y \in L$ and $f(0_A) = 0_B$.*

Theorem 3.9. *Let $\langle A, |_A, 0_A\rangle$ and $\langle B, |_B, 0_B\rangle$ be SUP-algebras, $f : A \to B$ be a surjective homomorphism and $\mathcal{B}$ be an $\mathscr{L}$-fuzzy set in B. Then $\mathcal{B} = (B, \mathcal{L}_\mu)$ is an $\mathscr{L}$-fuzzy SUP-subalgebra of B if and only if $\mathcal{B}^f$ is an $\mathscr{L}$-fuzzy SUP-subalgebra of A, where $\mathcal{L}_\mu^f : A \to [0,1]$ is defined by $\mathcal{L}_\mu^f(x) = \mathcal{L}_\mu(f(x))$ for all $x \in A$.*

Proof. Let $f : A \to B$ be a surjective homomorphism and $\mathcal{B}$ be an $\mathscr{L}$-fuzzy SUP-subalgebra of B. Let $x_1, x_2 \in A$. Then

$$\begin{aligned}
&\mathcal{L}_\mu^f((x_1|_A(x_2|_A x_2))|_A(x_1|_A(x_2|_A x_2))) \\
&= \mathcal{L}_\mu(f((x_1|_A(x_2|_A x_2))|_A(x_1|_A(x_2|_A x_2)))) \\
&= \mathcal{L}_\mu(f(x_1)|_B(f(x_2)|_B f(x_2))|_B(f(x_1)|_B(f(x_2)|_B f(x_2)))) \\
&\geq \mathcal{L}_\mu(f(x_1)) \wedge \mathcal{L}_\mu(f(x_2)) \\
&= \mathcal{L}_\mu^f(x_1) \wedge \mathcal{L}_\mu^f(x_2).
\end{aligned}$$

Hence, $\mathcal{B}^f$ is an $\mathscr{L}$-fuzzy SUP-subalgebra of A.

Conversely, let $\mathcal{B}^f$ be an $\mathscr{L}$-fuzzy SUP-subalgebra of A. Let $x|(y|y), x \in B$ be such that $f(x_1) = x|(y|y)$ and $f(x_2) = x$ for $x_1, x_2 \in A$. Then

$$\begin{aligned}
&\mathcal{L}_\mu((x|(y|y)|_B(x|_B x))|_B(x|(y|y)|_B(x|_B x))) \\
&= \mathcal{L}_\mu((f(x_1)|_B(f(x_2)|_B f(x_2)))|_B(f(x_1)|_B(f(x_2)|_B f(x_2)))) \\
&= \mathcal{L}_\mu(f((x_1|_A(x_2|_A x_2))|_A(x_1|_A(x_2|_A x_2))))
\end{aligned}$$

$$
\begin{aligned}
&= \mathcal{L}^f_\mu((x_1|_A(x_2|_Ax_2))|_A(x_1|_A(x_2|_Ax_2))) \\
&\geq \mathcal{L}^f_\mu(x_1) \wedge \mathcal{L}^f_\mu(x_2) \\
&= \mathcal{L}_\mu(f(x_1)) \wedge \mathcal{L}_\mu(f(x_2)) \\
&= \mathcal{L}_\mu(x|(y|y)) \wedge \mathcal{L}_\mu(x).
\end{aligned}
$$

Hence, $\mathcal{B}$ is an $\mathscr{L}$-fuzzy SUP-subalgebra of B. □

Theorem 3.10. *Let $\langle A, |_A, 0_A\rangle$ and $\langle B, |_B, 0_B\rangle$ be SUP-algebras, $f : A \to B$ be a surjective homomorphism, and $\mathcal{B}$ be an $\mathscr{L}$-fuzzy set in B. Then $\mathcal{B}$ is an $\mathscr{L}$-fuzzy SUP-ideal of B if and only if $\mathcal{B}^f$ is an $\mathscr{L}$-fuzzy SUP-ideal of A.*

Proof. Let $f : A \to B$ be a surjective homomorphism and $\mathcal{B}$ be an $\mathscr{L}$-fuzzy SUP-ideal of B. Let $x_1, x_2 \in A$. Then

$$
\begin{aligned}
\mathcal{L}^f_\mu(x_1) &= \mathcal{L}_\mu(f(x_1)) \\
&\leq \mathcal{L}_\mu(f((x_1|_A(x_2|_Ax_2))|_A(x_1|_A(x_2|_Ax_2)))) \\
&= \mathcal{L}^f_\mu((x_1|_A(x_2|_Ax_2))|_A(x_1|_A(x_2|_Ax_2))),
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}^f_\mu(x_1) &= \mathcal{L}_\mu(f(x_1)) \\
&\geq \mathcal{L}_\mu(f((x_1|_A(x_2|_Ax_2))|_A(x_1|_A(x_2|_Ax_2)))) \wedge \mathcal{L}_\mu(f(x_2)) \\
&= \mathcal{L}^f_\mu((x_1|_A(x_2|_Ax_2))|_A(x_1|_A(x_2|_Ax_2))) \wedge \mathcal{L}^f_\mu(x_2).
\end{aligned}
$$

Hence, $\mathcal{B}^f$ is an $\mathscr{L}$-fuzzy SUP-ideal of A.

Conversely, let $\mathcal{B}^f$ be an $\mathscr{L}$-fuzzy SUP-ideal of A. Let $x, y \in B$ be such that $f(x_1) = x$ and $f(x_2) = y$ for $x_1, x_2 \in A$. Then

$$
\begin{aligned}
\mathcal{L}_\mu(x) &= \mathcal{L}_\mu(f(x_1)) \\
&= \mathcal{L}^f_\mu(x_1) \\
&\leq \mathcal{L}^f_\mu((x_1|_A(x_2|_Ax_2))|_A(x_1|_A(x_2|_Ax_2))) \\
&= \mathcal{L}_\mu(f((x_1|_A(x_2|_Ax_2))|_A(x_1|_A(x_2|_Ax_2)))) \\
&= \mathcal{L}_\mu(x^y|x^y),
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}_\mu(x) &= \mathcal{L}_\mu(f(x_1)) \\
&= \mathcal{L}^f_\mu(x_1) \\
&\geq \mathcal{L}^f_\mu((x_1|_A(x_2|_Ax_2))|_A(x_1|_A(x_2|_Ax_2))) \wedge \mathcal{L}^f_\mu(x_2) \\
&= \mathcal{L}_\mu(f((x_1|_A(x_2|_Ax_2))|_A(x_1|_A(x_2|_Ax_2)))) \wedge \mathcal{L}_\mu(f(x_2))
\end{aligned}
$$

$$= \mathcal{L}_{\mu}(x^y|x^y) \wedge \mathcal{L}_{\mu}(y).$$

Hence, $\mathcal{B}$ is an $\mathscr{L}$-fuzzy SUP-ideal of B. □

The proof of the following theorem is similar to the proof of Theorem 3.10.

Theorem 3.11. *Let $\langle A, |_A, 0_A\rangle$ and $\langle B, |_B, 0_B\rangle$ be SUP-algebras, $f : A \to B$ be a surjective homomorphism, and $\mathcal{B}$ be an $\mathscr{L}$-fuzzy set in B. Then $\mathcal{B}$ is an $\mathscr{L}$-fuzzy SUP-subalgebra of B if and only if $\mathcal{B}^f$ is an $\mathscr{L}$-fuzzy SUP-subalgebra of A.*

4. CONCLUSION

In this paper, we introduced the notions of $\mathscr{L}$-fuzzy SUP-subalgebras and $\mathscr{L}$-fuzzy SUP-ideals in the setting of Sheffer stroke UP-algebras and explored their structural properties through the lens of lattice-valued fuzzy logic. By establishing a bidirectional correspondence between these fuzzy substructures and their level subsets, we demonstrated that the algebraic properties of crisp SUP-subalgebras and SUP-ideals are preserved under $\mathscr{L}$-fuzzification via characteristic functions—and conversely, that every $\mathscr{L}$-fuzzy SUP-subalgebra/ideal determines a family of crisp substructures through its level sets.

These findings not only generalize earlier results in fuzzy algebra but also highlight the expressive capacity of Sheffer stroke UP-algebras as a host framework for modeling lattice-valued uncertainty. Furthermore, they provide a foundation for future investigations into morphisms, categorical frameworks, and potential applications in areas such as logic programming, fuzzy inference systems, and computational algebra.

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REFERENCES

[1] N. Chunsee, P. Julatha, A. Iampan, Fuzzy Set Approach to Ideal Theory on Sheffer Stroke BE-Algebras, J. Math. Comput. Sci. 34 (2024), 283–294. https://doi.org/10.22436/jmcs.034.03.07.

[2] B.A. Davey, H.A. Priestley, Introduction to Lattices and Order, Cambridge University Press, 2002. https://doi.org/10.1017/CBO9780511809088.

[3] A. Iampan, A New Branch of the Logical Algebra: UP-Algebras, J. Algebr. Relat. Top. 5 (2017), 35–54. https://doi.org/10.22124/JART.2017.2403.

[4] A. Iampan, T. Oner, N. Rajesh, Fuzzy Subalgebras of Sheffer Stroke UP-Algebras with respect to a Triangular Conorm, ICIC Express Lett. 19 (2025), 1201–1209. https://doi.org/10.24507/icicel.19.11.1201.

[5] T. Oner, T. Katican, On Ideals of Sheffer Stroke UP-Algebras, J. Discret. Math. Sci. Cryptogr. 26 (2023), 2257–2271. https://doi.org/10.47974/JDMSC-1568.

[6] A.B. Saeid, T. Katican, T. Oner, On Sheffer Stroke UP-Algebras, Discuss. Math. - Gen. Algebr. Appl. 41 (2021), 381–394. https://doi.org/10.7151/dmgaa.1368.

[7] T. Oner, T. Katican, A.B. Saeid, (Hesitant) Fuzzy Sets on Sheffer Stroke UP-Algebras, J. Interdiscip. Math. 25 (2022), 1221–1236. https://doi.org/10.1080/09720502.2021.1959999.

[8] T. Oner, N. Rajesh, A. Iampan, I. Senturk, Semidetached SUP-Subalgebras of Sheffer Stroke UP-Algebras, Eur. J. Pure Appl. Math. 18 (2025), 5876. https://doi.org/10.29020/nybg.ejpam.v18i2.5876

[9] N. Rajesh, T. Oner, A. Iampan, I. Senturk, Intuitionistic Fuzzy Structures on Sheffer Stroke UP-Algebras, Eur. J. Pure Appl. Math. 18 (2025), 5627. https://doi.org/10.29020/nybg.ejpam.v18i1.5627

[10] N. Rajesh, T. Oner, A. Iampan, I. Senturk, On Length and Mean Fuzzy Ideals of Sheffer Stroke Hilbert Algebras, Eur. J. Pure Appl. Math. 18 (2025), 5779. https://doi.org/10.29020/nybg.ejpam.v18i1.5779.

[11] N. Rajesh, T. Oner, A. Iampan, A. Rezaei, Investigating Length and Mean-Fuzzy Subalgebras in Sheffer Stroke Hilbert Algebras, Eur. J. Pure Appl. Math. 18 (2025), 5914. https://doi.org/10.29020/nybg.ejpam.v18i2.5914.

[12] N. Rajesh, A. Iampan, T. Oner, A.B. Saeid, Generalized Fuzzy Subalgebras of Sheffer Stroke Hilbert Algebras, Eur. J. Pure Appl. Math. 18 (2025), 6500. https://doi.org/10.29020/nybg.ejpam.v18i3.6500.

[13] H.M. Sheffer, A Set of Five Independent Postulates for Boolean Algebras, with Application to Logical Constants, Trans. Am. Math. Soc. 14 (1913), 481–488. https://doi.org/10.2307/1988701.

[14] L. Zadeh, Fuzzy Sets, Inf. Control. 8 (1965), 338–353. https://doi.org/10.1016/S0019-9958(65)90241-X.