Asia Pac. J. Math. 2026 **13**:39

MODELLING AND PROJECTING MORTALITY OF THE ELDERLY POPULATION: COMPARISON OF CLASSICAL AND COHERENT MODELS

AHMED LAKEHAL[1,*], FAROUK METIRI[1,*], MOHAMMED RIAD REMITA[2]

[1]LaPS laboratory, Badji Mokhtar University, Box 12, Annaba, 23000, Algeria

[2]National School of Artificial Intelligence, Algiers, Algeria

[*]Corresponding author: ahmed.lakehal@univ-annaba.dz, farouk.metiri@univ-annaba.dz

Received Dec. 1, 2025

Abstract. Mortality projection is essential for population forecasts and actuarial calculations. In most situations, projecting the mortality rates of men and women separately results in some incoherence with regard to the projected progression of male-female mortality. Therefore, coherent mortality forecasting is necessary to prevent any unrealistic convergence or divergence in this regard. In this work, we propose a mortality projection model for the Algerian population aged 50 years and over for men and women during the period 1977–2017 and compare it with the models of Lee-Carter [17], Cairns-Blake-Dowd [3], and Hyndman [16]. The results show that the proposed model is the best in terms of quality of adjustment and coherence between mortality projections for men and women.

2020 Mathematics Subject Classification. 62P05; 91B05; 91G05; 62P25.

Key words and phrases. mortality; product-ratio method; adjustment quality; coherent forecasting; Algerian population.

1. Introduction

For population forecasts and actuarial calculations, mortality projection is essential. Due to the disparity in mortality between males and females, sex-based projections are necessary. Through the use of data analytic techniques, projection mortality models seek to reduce the historical mortality surface into a smaller number of parameters linked to age and period [3,17]. After that, the projection procedure is simplified by using time series models to project the future time components. The coherence of forecasts between males and females can have some flaws when the mortality rates of the two sexes are projected separately. This recent finding may be explained by a crossover in the projected life expectancies for men and women at the projection's horizon, an inflated divergence, or an increase in female mortality at certain ages not shown in the historical data [13].

DOI: 10.28924/APJM/13-39

Some earlier publications have addressed the projection of mortality in Algeria. Using the Lee-Carter model [17], Flici (2016a) [9] has attempted to project mortality rates separately for men and women for the Algerian population sixty years of age and older. He utilised the male and female life tables that the Office of National Statistics (ONS) released between 1977 and 2008 for that reason. In another study, Flici(2016b) [10] compared a variety of models, which are the Cairns-Blake-Dowd generalised models [3,4] and the Lee & Carter generalised models [5,17,21], by projecting mortality independently for men and women for the Algerian population aged 50 and over and based on the data from 1977 to 2013. This resulted in the intersection of life expectancy at the projection horizon for men and women in all models evaluated. In order to enhance the male-female coherence of mortality predictions for the Algerian population, Flici (2021) [13] proposed comparing two coherent models: the model of Li & Lee (2005) [18] and the Hyndman et al.(2013) [16]. Based on all these studies using Algerian data, it can be concluded that the terrorist events of the 1990s had a significant impact on the regularity of Algeria's historical mortality statistics [14]. This apparent irregularity often makes it more difficult to make a robust mortality projection in the future. This meant that a model had to be found that reduced the number of parameters that needed to be estimated, ensured better regularity and robustness, maintains coherence between the projections of men and women, and had a good fit for the population's historical mortality surface in Algeria.

In this study, our primary goal is to suggest a mortality projection model for the Algerian population aged 50 years and older, including men and women, during the period 1977–2017 and to compare it with the models of Lee-Carter (LC) [17], Cairns-Blake-Dowd (CBD) [3], and Hyndman [16], so that the proposed model is the best in terms of quality of adjustment and coherence between mortality projections for men and women.

2. Mortality Projection Models

2.1. **The model of Lee-Carter.** This technique for extrapolating historical trends was first applied to US data and swiftly gained acceptance as the norm in the insurance sector [20]. A mortality model based on three parameters—two linked to age and one related to time—was suggested by Lee & Carter (1992) [17]. By using the following formula, denoted $M1$, the log of the central mortality rate may be modelled [13]:

$$\ln \mu_{x;t} = \alpha_x^{(1)} + \beta_x^{(1)} * k_t^{(1)} + \varepsilon_{x;t}, \tag{1}$$

with:

$\mu_{x;t}$: the age-specific death rate $(ASDR)$ at date t for age x (it is supposed that this rate is piecewise constant),

$\alpha_x^{(1)}$: Age-specific component x represents the time mean of the $\ln \mu_{x;t}$,

$k_t^{(1)}$: Index of the general trend of mortality,

$\beta_x^{(1)}$: Index of the sensitivity of the age-specific death rate in relation to the general trend of mortality, meaning that: $\beta_x^{(1)} = \frac{d\ln\mu_{x;t}}{dk_t^{(1)}}$,

$\varepsilon_{x;t}$: Error term that reflects the peculiarities at age x and at date t that are not captured by the model, with $\varepsilon_{x;t}$ being independent and identically distributed, and each $\varepsilon_{x;t}$ follows the law $N(0,\sigma^2)$.

The authors suggested that in order to estimate the model's parameters, one could first estimate parameter $\alpha_x^{(1)}$ by calculating the central mortality rate's average logarithm over time at age x, as shown as follows:

$$\alpha_x^{(1)} = \frac{1}{n}\sum_{t=t_1}^{t_n} \ln\mu_{x;t} \tag{2}$$

Next, a decomposition of the residual matrix $\ln\mu_{x;t} - \alpha_x^{(1)}$ yields two components, $\beta_x^{(1)}$ and $k_t^{(1)}$. For the purpose of performing the decomposition process, the singular value decomposition approach may be employed to minimise the sum of square errors between the two sides of the equation as follows:

$$\ln\mu_{x;t} - \alpha_x^{(1)} = \beta_x^{(1)} * k_t^{(1)} \tag{3}$$

Furthermore, certain identification constraints need to be put on the estimated parameters in order to ensure the uniqueness of the solution, as shown as follows:

$$\sum_{x=x_1}^{x_m} \beta_x^{(1)} = 1 \tag{4}$$

$$\sum_{t=t_1}^{t_n} k_t^{(1)} = 0 \tag{5}$$

Wilmoth (1993) [23] suggested a one-step decomposition process based on the minimization of square errors by weight sum (WSSE). The quantity to be minimised is:

$$min(WSSE) = W_{x;t} * (\ln\mu_{x;t} - \alpha_x^{(1)} - \beta_x^{(1)} * k_t^{(1)})^2, \tag{6}$$

and the weight $W_{x;t}$ is assumed to be the total number of deaths reported at age x in the year t [11].

2.2. **The CBD Model.** Based on the simplicity of the mortality curve after a particular age, Cairns et al.(2006) [3] suggested a model for projecting mortality. The mortality curves have an exponential shape when the childhood and early-life motor and maternal mortality bumps are taken out. The logit's introduction roughly results in a linear form [11]. The formula suggested by the CBD model is as follows, denoted M5 [22]:

$$logit(q_{x;t}) = \ln(\frac{q_{x;t}}{1-q_{x;t}}) = k_t^{(1)} + k_t^{(2)} * (x - \bar{x}) + \varepsilon_{x;t}, \tag{7}$$

The M5 model may be seen as a linear equation with the following form for each year:

$$logit(q_{x;t}) = \alpha_t + \beta_t * x, \tag{8}$$

using β_t as the slope and α_t as the intercept. The intercept in $M5$ is intended to be the midpoint of the age interval $[x_1; x_n]$, which is the only distinction; therefore, $k_t^{(1)}$ can be approximated by the time evolution series of $logit(q_{x;t})$ corresponding to the middle of the population and $k_t^{(2)}$ as the mortality Logit line's slope for each age [11].

2.3. **The Product-Ratio Method.** For coherent forecasting with a smaller number of parameters, Hyndman et al.(2013) [16] suggested a novel method. This approach is commonly referred to as the Product-Ratio Method (PRM).The authors' major point was to consider the age pattern associated with the mortality-sex ratio (MSR). The male $ASDR$ divided by the female $ASDR$ corresponding to x and t yields the MSR at age x and time t. The model's introduction of such a component must prevent any illogical divergence or cross-over for long-term forecasts. With the usage of the PRM, the historical mortality surfaces of men and women are to be reshaped into two new components, which will be projected into the future: a differential mortality component and a joint mortality function. Male and female mortality surfaces must be combined onto one surface, according to Hyndman et al.(2013) [16]. For this problem, a geometrical average can be applied. The average $ASDR$, denoted $\mu^*_{x;t}$, may be determined as follows if we let $\mu^m_{x;t}$ and $\mu^f_{x;t}$ represent the $ASDR$ for males and females at age x and year t, respectively:

$$\mu^*_{x;t} = \sqrt{\mu^m_{x;t} * \mu^f_{x;t}}, \tag{9}$$

The differential mortality function, the second component, is calculated as follows:

$$R_{x;t} = \sqrt{\frac{\mu^m_{x;t}}{\mu^f_{x;t}}}, \tag{10}$$

Usually, the male death rate is divided by the female death rate to get the MSR. Reconstituting the mortality surfaces of men and women is made easier by introducing the root:

$$\mu^m_{x;t} = \mu^*_{x;t} * R_{x;t}, \tag{11}$$

$$\mu^f_{x;t} = \frac{\mu^*_{x;t}}{R_{x;t}}, \tag{12}$$

In the final, The LC model is used to project $\mu^*_{x;t}$ and $R_{x;t}$ independently. Equation (1), which describes the LC model, may be used to predict the joint mortality surface:

$$\ln \mu^*_{x;t} = \alpha^*_x + \beta^*_x * k^*_t + \varepsilon^*_{x;t}, \tag{13}$$

We utilise the following formula to forecast $R_{x;t}$:

$$R_{x;t} = A_x + B_x * K_t + \zeta_{x;t}, \tag{14}$$

With A_x, B_x and K_t having the same meanings as α^*_x, β^*_x and k^*_t, respectively. $\zeta_{x;t}$ is the error term that reflects the peculiarities at age x and at date t that are not captured by the model, with $\zeta_{x;t}$ being independent and identically distributed, and each $\zeta_{x;t}$ follows the law $N(0, \sigma^2)$.

To guarantee the uniqueness of each solution, two constraints are placed on each model [13].

$$\sum_{x=x_1}^{x_m} \beta_x^* = \sum_{x=x_1}^{x_m} B_x = 1, \tag{15}$$

and

$$\sum_{t=t_1}^{t_n} k_t^* = \sum_{t=t_1}^{t_n} K_t = 0. \tag{16}$$

2.4. **The second product-ratio method.** In our study, we propose another approach for coherent forecasting with a smaller number of parameters, so that we called this method **The second product-ratio method** (**PRM 2**). The model's introduction of such a component must prevent any illogical divergence or cross-over for long-term forecasts. With the usage of $PRM2$, the historical mortality surfaces of men and women are to be reshaped into two new components, both of which are to be projected in the future. According to this, male and female mortality surfaces must be combined onto one surface. If we let $q_{x;t}^m$ and $q_{x;t}^f$ be the mortality rates for males and females at age x and year t, respectively. we have:

$$q'_{x;t}{}^m = \frac{q_{x;t}^m}{1 - q_{x;t}^m}, \tag{17}$$

and

$$q'_{x;t}{}^f = \frac{q_{x;t}^f}{1 - q_{x;t}^f}, \tag{18}$$

The geometrical average of $q'_{x;t}{}^m$ and $q'_{x;t}{}^f$, noted $q_{x;t}{}^*$, can be calculated as:

$$q_{x;t}^* = \sqrt{q'_{x;t}{}^m * q'_{x;t}{}^f}, \tag{19}$$

The second component is calculated by:

$$S_{x;t} = \sqrt{\frac{q'_{x;t}{}^m}{q'_{x;t}{}^f}}, \tag{20}$$

Reconstituting the male and female mortality surfaces is made easier by introducing the root in this way:

$$q_{x;t}^f = \frac{q_{x;t}^*}{q_{x;t}^* + S_{x;t}}, \tag{21}$$

and

$$q_{x;t}^m = \frac{q_{x;t}^*}{q_{x;t}^* + \frac{1}{S_{x;t}}}, \tag{22}$$

In the final, the CBD model is used to project $q_{x;t}^*$ and $S_{x;t}$ independently.

The CBD model, as given in equation (7), may be applied to forecast $q_{x;t}^*$:

$$\ln q_{x;t}^* = k_t^{*(1)} + k_t^{*(2)} * (x - \bar{x}) + \varepsilon_{x;t}^{*(2)}, \tag{23}$$

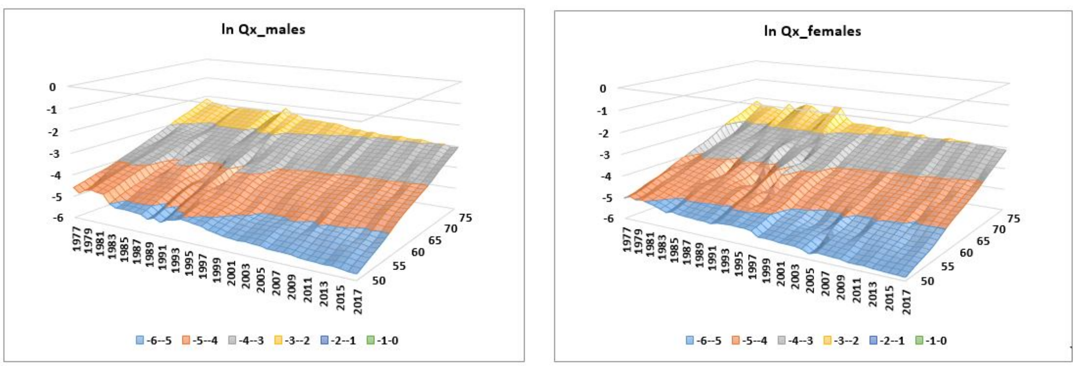
To forecast $S_{x;t}$, we use the following formulation:

$$S_{x;t} = K_t^{(1)} + K_t^{(2)} * (x - \bar{x}) + \zeta_{x;t}^{(2)}, \tag{24}$$

With $K_t^{(1)}$ and $K_t^{(2)}$ having respectively the same interpretations as $k_t^{*(1)}$ and $k_t^{*(2)}$. The specific error of age x and year t is denoted by $\zeta_{x;t}^{(2)}$, which should be normally distributed.

3. Data Description

We obtained an Algerian mortality database (2020) [1] from the ONS (Office National des Statistiques) linked to the mortality of the Algerian population. These data are interpolated [12], including the years in columns (from 1977 to 2017), a detailed age description in rows (50 years to 79 years for men and women), and the corresponding mortality quotients in cells.
Figure 1 shows the surface of mortality by age for men and women, with ages 50-79, and for the period 1977-2017.



(A) male (B) female

Figure 1. Surface of mortality in the period (1977-2017)

4. Estimation of Parameters

In the LC model, the alpha parameter $\alpha_x^{(1)}$ is defined as the average over time of the logarithm of the age-specific death rate at age x, and then we break down the residual matrix into two vectors $\beta_x^{(1)^*}$ and $k_t^{(1)^*}$, such as:

$$\ln \mu_{x;t} - \alpha_x^{(1)} \approx \beta_x^{(1)^*} * k_t^{(1)^*}, \tag{25}$$

respecting the constraints

$$\sum_{x=x_1}^{x_m} \beta_x^{(1)^*} = 1, \tag{26}$$

and

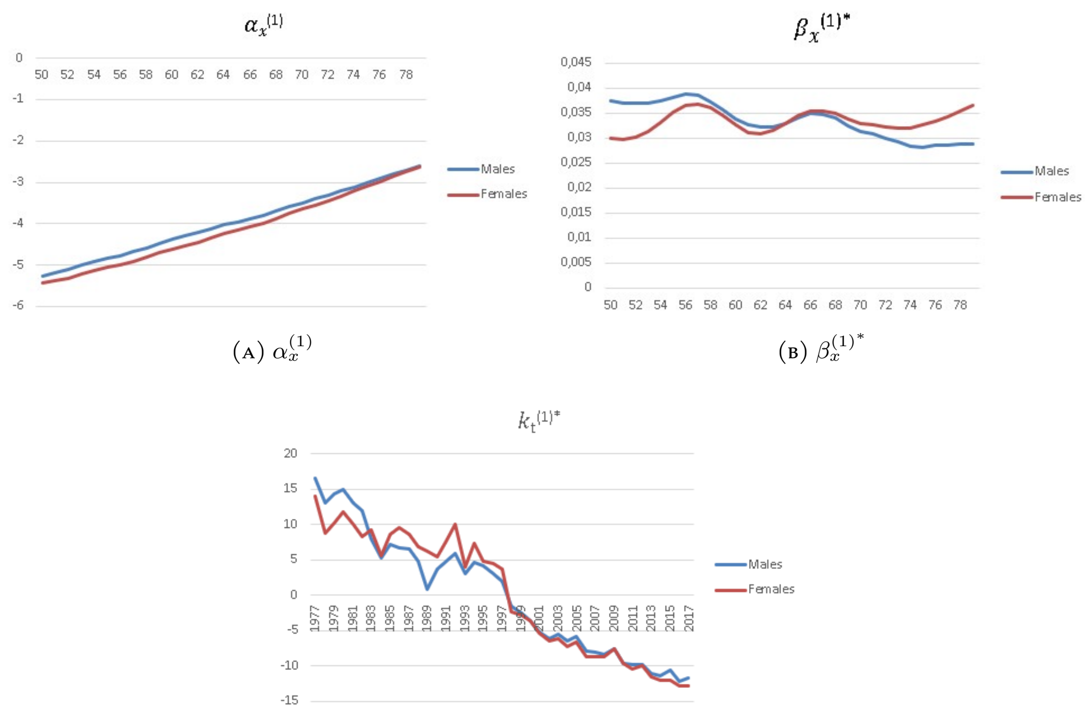
$$\sum_{t=t_1}^{t_n} k_t^{(1)^*} = 0. \tag{27}$$

The quantity to be minimised is the mean of squared errors (MSE) to guarantee the uniqueness of the solution, as shown as follows:

$$\min MSE = \frac{1}{m * n} \sum_{x=x_1}^{x_m} \sum_{t=t_1}^{t_n} (\ln \mu_{x;t} - \alpha_x^{(1)} - \beta_x^{(1)^*} * k_t^{(1)^*})^2, \tag{28}$$

Take note that every application included in this work uses XL-Solver.

The results of M1 are illustrated in figure 2.



(A) $\alpha_x^{(1)}$ (B) $\beta_x^{(1)^*}$

(C) $k_t^{(1)^*}$

FIGURE 2. Estimate of parameters for $M1$ (1977-2017).

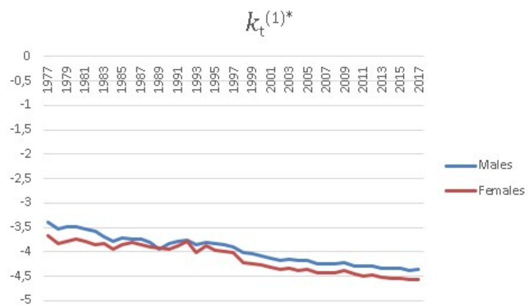
In the CBD model, the method of estimating $k_t^{(1)^*}$ and $k_t^{(2)^*}$, is the same as the previous method, such as:

$$\ln(\frac{q_{x;t}}{1 - q_{x;t}}) \approx k_t^{(1)^*} + k_t^{(2)^*} * (x - \bar{x}), \tag{29}$$

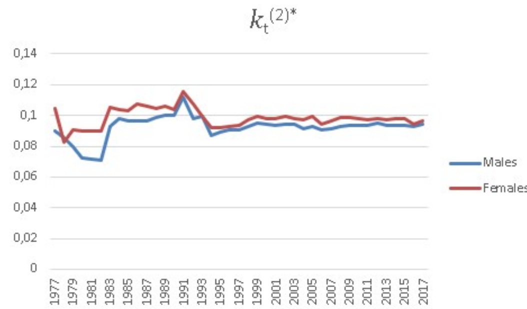
and

$$\min MSE = \frac{1}{m * n} \sum_{x=x_1}^{x_m} \sum_{t=t_1}^{t_n} (\ln(\frac{q_{x;t}}{1 - q_{x;t}}) - k_t^{(1)^*} - k_t^{(2)^*} * (x - \bar{x}))^2, \tag{30}$$

The results of M5 are illustrated in figure 3.



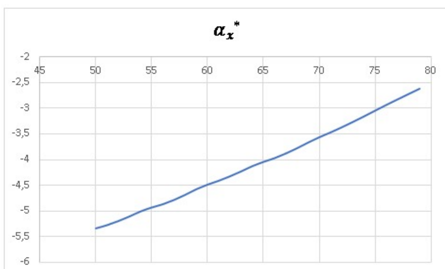
(A) $k_t^{(1)^*}$



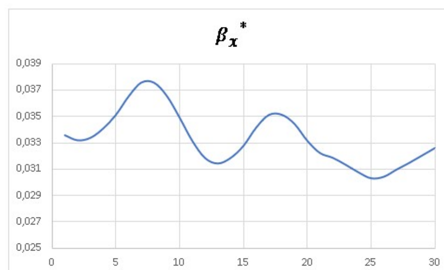
(B) $k_t^{(2)^*}$

FIGURE 3. Estimate of parameters for $M5$ (1977-2017).

In PRM, the method of estimating α_x^*, β_x^*, k_t^*, A_x, B_x and K_t is the same as the previous methods. The results of PRM are illustrated in figure 4.

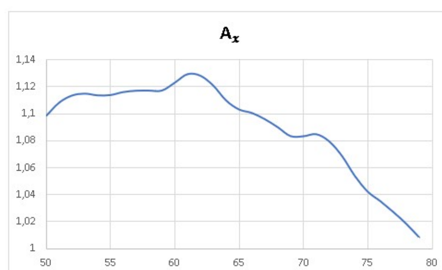


(A) α_x^*

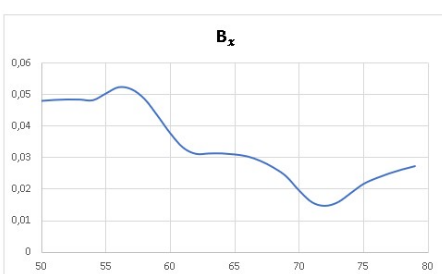


(B) β_x^*

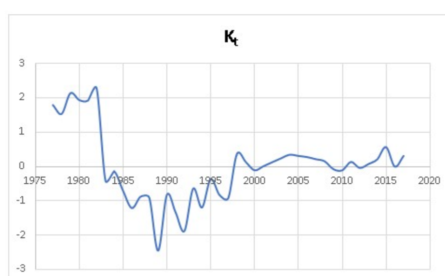
(C) k_t^*



(D) A_x



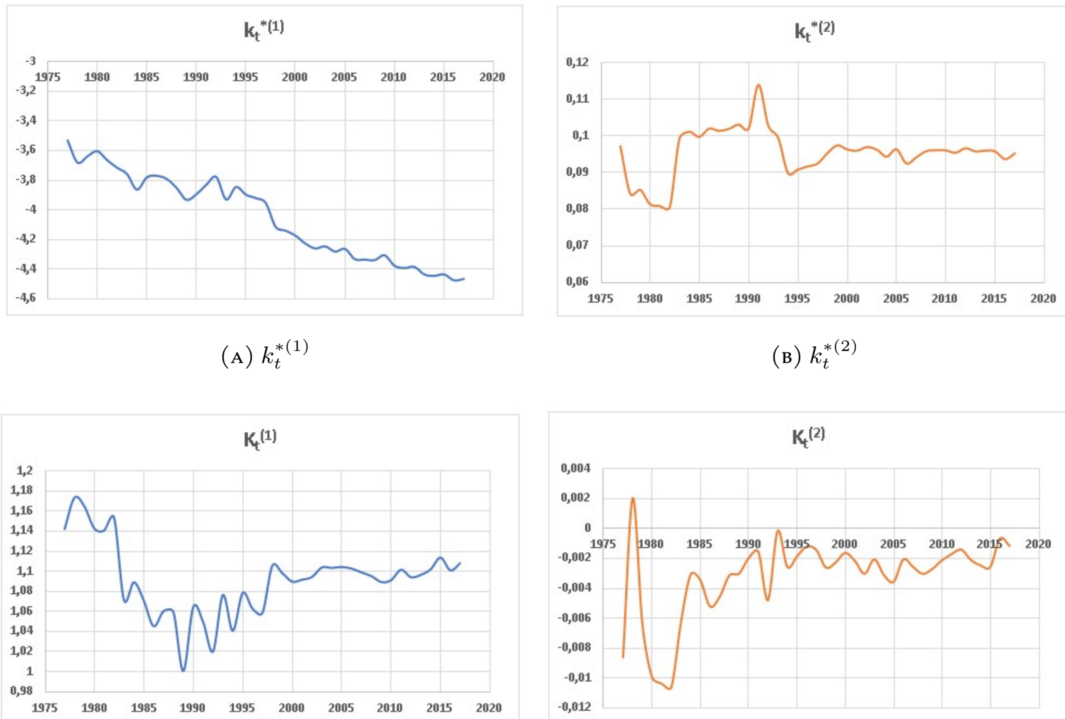
(E) B_x



(F) K_t

FIGURE 4. Estimate of parameters for PRM (1977-2017).

In $PRM2$, the method of estimating $k_t^{*(1)}$, $k_t^{*(2)}$, $K_t^{(1)}$ and $K_t^{(2)}$ is the same as the previous methods. Figure 5 shows the results of $PRM2$.



(A) $k_t^{*(1)}$ (B) $k_t^{*(2)}$

(C) $K_t^{(1)}$ (D) $K_t^{(2)}$

FIGURE 5. Estimate of parameters for $PRM2$ (1977-2017).

5. Adjustment Quality

To assess the quality of adjustment of the different models, we first use the mean of square errors (MSE). In the present study, the age-specific mortality indicators used in all models are different. This is because the $M5$ model is based on $logit(q_{x;t})$, the $M1$ model is based on $\ln \mu_{x;t}$, the PRM approach is based on both $\ln \mu^*_{x;t}$ and $R_{x;t}$, and the $PRM2$ approach is based on both $\ln q^*_{x;t}$ and $S_{x;t}$. For this, a common indication should be used to compare these models, and $q^b_{x;t}$ appears appropriate for all of them, so that: $q^b_{x;t} = q^m_{x;t} * q^f_{x;t}$. Each model is calibrated using the relevant mortality indicator, and the following comparison criteria may be written as:

$$MSE = \frac{1}{m * n} \sum_{x=x_1}^{x_m} \sum_{t=t_1}^{t_n} (q^b_{x;t} - \hat{q}^b_{x;t})^2, \tag{31}$$

$q^b_{x;t}$ and $\hat{q}^b_{x;t}$ are the observed and adjusted mortality indicators by age, respectively, for age x and time t, so that: $\hat{q}^b_{x;t} = \hat{q}^m_{x;t} * \hat{q}^f_{x;t}$. A perfectly good calibration and evaluation criterion might be the mean square error (MSE); however, comparing models with various numbers of parameters is inappropriate. The quality of adjustment usually improves according to the number of parameters in the underlying model.

As a result, comparing models with different numbers of parameters only based on the MSE is not particularly practical; the comparison criteria must consider the difference in parameter counts between the compared models. The number of parameters and observations used for model calibration is taken into consideration when using the Bayesian Information Criteria (BIC) or the Akaike Information Criteria (AIC). The formula that has been modified for the procedure of estimating the least squares is used here. The variance of errors σ_p^2, the number of observations p equal to $m * n$, and the number of parameters noted i found in the model may be used to express the AIC and the BIC, such as [15]:

$$BIC = p * \ln(\sigma_p^2) + i * \ln(p), \tag{32}$$

$$AIC = 2 * i + p * \ln(\sigma_p^2), \tag{33}$$

The variance may be roughly calculated using:

$$\sigma_p^2 = \frac{SSE}{p} = MSE \tag{34}$$

in the case of estimates by least squares.

The MSE, BIC, and AIC obtained using all models are summarised in Table 1.

Table 1: Adjustment quality

	MSE	BIC	AIC
$M1$	$2,561E-07$	-17231	-18265
$M5$	$1,322E-07$	-18315	-19154
PRM	$3,906E-07$	-16712	-17745
$PRM2$	$1,322E-07$	-18315	-19154

The models can be classified as follows based on the adjustment quality they provided: $PRM2$, $M5$, $M1$, and PRM, and we note that both models $PRM2$ and $M5$ provided the same adjustment quality, but in this study we will continue to use the model $PRM2$ because it will give us coherent results on the predictions of both males and females, as the upcoming results will show.

6. Projection of Mortality

To project the mortality with the $PRM2$ model into the future, we only choose the period [1998–2017] as a predictive basis, as the indexes of the trend of mortality $k_t^{*(1)}$, $k_t^{*(2)}$, $K_t^{(1)}$ and $K_t^{(2)}$ have regular trends in this period, as shown in Figure 5.

6.1. Projection of $k_t^{*(1)}$.

We propose to project $k_t^{*(1)}$ using a linear model of time series, so that [19]:

$$k_t^{(1)} = a * t + b + \varepsilon_t, \tag{35}$$

depending on the time t.

The results obtained are illustrated in figure 6.

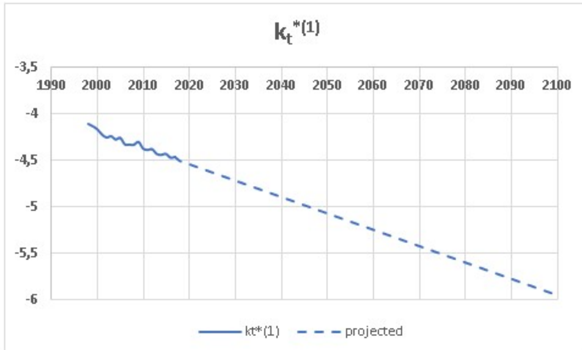


FIGURE 6. Projection of $k_t^{*(1)}$ with the linear model

6.2. **Projection of $k_t^{*(2)}$.** Since the $k_t^{*(2)}$ series is stationary, using an AR(1) time series model with a drift term added, we propose to project it so that [2]:

$$k_t^{*(2)} = \alpha_1 + \alpha_2 * k_{t-1}^{*(2)} + \varepsilon_t, \tag{36}$$

The results are illustrated in figure 7.

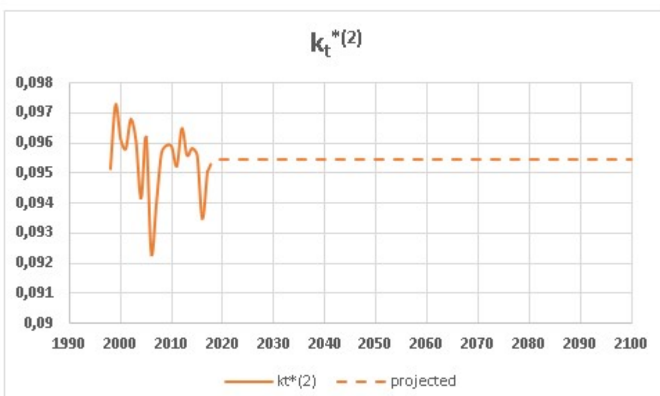


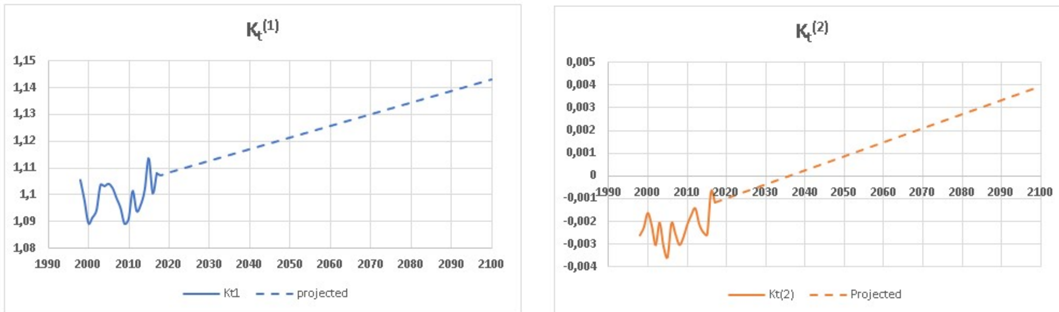
FIGURE 7. Projection of $k_t^{*(2)}$ with AR(1).

6.3. **Projection of $K_t^{(1)}$ & $K_t^{(2)}$.** Since the $K_t^{(1)}$ and $K_t^{(2)}$ series are integrated of order 1, we propose to project them using an ARIMA $(1, 1, 0)$ model of time series for which a drift term is added, so that [2]:

$$\Delta K_t^{(1)} = \alpha_1 + \alpha_2 * \Delta K_{t-1}^{(1)} + \varepsilon_t, \tag{37}$$

$$\Delta K_t^{(2)} = \alpha_1 + \alpha_2 * \Delta K_{t-1}^{(2)} + \varepsilon_t, \tag{38}$$

Figure 8 provides illustrations of the findings.



(A) $K_t^{(1)}$ (B) $K_t^{(2)}$

FIGURE 8. Projection of $K_t^{(1)}$ & $K_t^{(2)}$ with ARIMA(1,1,0).

7. The Dynamic Life Table's Construction

In this portion, we're going to create a dynamic life table with the $PRM2$ model for men and women. The data we have so far enable us to project mortality rates from 50 years of age to 79 years for both men and women. For actuarial applications, it is recommended that life tables be extended to a high age that is in close proximity to the survival age limit. The Denuit and Goderniaux model (2005) [6] extends the projected life tables as follows:

$$\ln(q_x) = a * x^2 + b * x + c + \varepsilon_x, \tag{39}$$

The authors imposed an upper age limit by setting a probability of death equal to 1 at the age of 130: $q_{130} = 1$ and $p_{130} = 0$ [7]. Here, we set a limit on the survival age by establishing a mortality probability equal to 1 at the age of 110: $q_{110} = 1$. The age range $[60, 79]$ was used to calibrate the model.

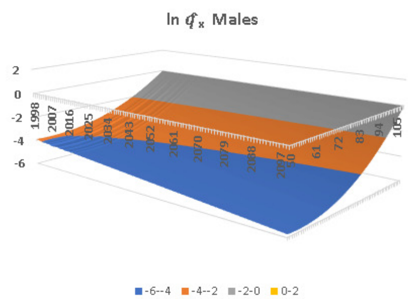
In Figure 9, we show the estimated mortality surface, knowing that it is extrapolated to the age of 110 with the Denuit and Gouderniaux model and projected to the year 2100 with the $PRM2$ model for men and women.

8. Projected Life Expectancy

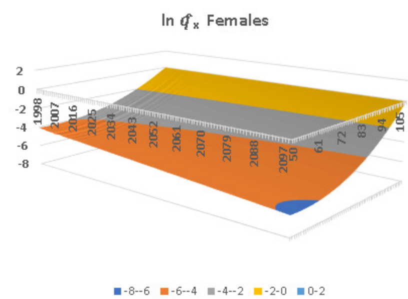
Our objective is to have an idea of the trajectory of life expectancy at 50 years, which will allow us to trace the future longevity of the Algerian population aged 50 and over for men and women.

The expected life expectancy at 50 years for men and women based on the $PRM2$ model is shown in Figure 10.

This model leads to a coherent development of men's and women's life expectancy at age 50.



(A) Male



(B) Female

FIGURE 9. the estimated mortality surface.

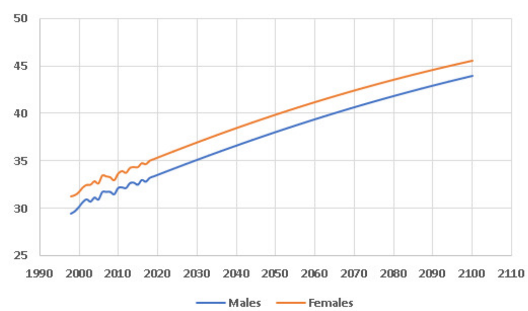


FIGURE 10. Projected life expectancy at age 50 for both sexes.

9. CONCLUSION

A variety of models have been created to assist in mortality forecasting, beginning with the studies of Lee & Carter (1992) [17] and Cairns et al.(2006) [3]. The prospective mortality models allowed for the prediction of mortality rates at various ages with a smaller number of parameters and more ease. The primary concept was to separate the historical mortality surface into components related to age and period. Then, using standard time series models, the forecasting procedure just needs to project the time component into the future.

Furthermore, mortality projections should be assessed in terms of complementary criteria in addition to adjustment quality. It is possible to include male-female coherence in that framework. Mathematical models can occasionally result in an incoherent development of the expected life expectancy by sex. It can be demonstrated by an unrealistic divergence or crossover between men and women.

The goal of this work was to enhance the projections' quality with respect to male-female coherence. For this, a mortality projection model was proposed for the Algerian population aged 50 years and over for men and women during the period 1977–2017 and compared with the models of LC, CBD, and Hyndman according to the quality of adjustment. Based on the results that were obtained, both the proposed model $(PRM2)$ and the CBD model fit the historical surfaces better than the other models according to all criteria, namely, AIC, BIC, and MSE. We chose to use the model $PRM2$ because it gave us coherent results on the predictions and evolution of life expectancy at the age of 50 for both males and females.

Due to $PRM2$'s superior performance, it is recommended to use it in the construction of dynamic life tables for pricing actuarial products such as pensions, thus avoiding underpricing issues. Data quality should be improved by enhancing the collection of accurate data, especially for critical age groups (e.g., youth and elderly), to improve the accuracy of future models. This research could be extended by proposing to study the impact of socio-economic factors on mortality trends, such as improved healthcare or changing lifestyles.

In conclusion, this study was able to provide a practical framework for mortality forecasting in Algeria, emphasising the importance of coherent models to ensuring realistic results. The $PRM2$ model is a promising tool to support decision-makers in developing insurance policies and health planning, with further research needed to generalise the results to additional age groups.

Acknowledgements. The authors appreciate the editor's and referee's comments and advice. Their thoughts and opinions significantly enhanced the article.

Conflicts of Interest. The authors declare that there are no conflicts of interest regarding the publication of this paper.

References

[1] Algerian Mortality Database, Office National des Statistiques, (2020), Retrieved September 4, 2020. https://www.ons.dz/spip.php?rubrique182.

[2] R. Bourbonnais, Économétrie, Dunod, Paris, 2015.

[3] A.J.G. Cairns, D. Blake, K. Dowd, A Two-Factor Model for Stochastic Mortality with Parameter Uncertainty: Theory and Calibration, J. Risk Insur. 73 (2006), 687–718. https://doi.org/10.1111/j.1539-6975.2006.00195.x.

[4] A.J.G. Cairns, D. Blake, K. Dowd, G.D. Coughlan, D. Epstein, et al., A Quantitative Comparison of Stochastic Mortality Models Using Data from England and Wales and the United States, North Am. Actuar. J. 13 (2009), 1–35. https://doi.org/10.1080/10920277.2009.10597538.

[5] I. Currie, Smoothing and Forecasting Mortality Rates With P-Splines, Conference Presentation, Institute of Actuaries, 2006. https://www.actuaries.org.uk/system/files/documents/pdf/currie.pdf.

[6] M. Denuit, A. Goderniaux, Closing and Projecting Life Tables Using Log-Linear Models, Bull. Swiss Assoc. Actuar. 1 (2005), 29–49.

[7] M. Denuit and A. Quashie, Modèles d'Extrapolation de la Mortalité aux Grands Âges, Catholic University of Louvain, Belgium, 2005. https://api.semanticscholar.org/CorpusID:161778462.

[8] F. Flici, Longevity and Life Annuities Reserving in Algeria: The Use of a Multi Components Lee–carter Model, in: 18th East Asian Actuarial Conference (EAAC), Taipei, Taiwan, 2014.

[9] F. Flici, Provisioning of Life Annuities in Algeria Between a Static and a Prospective Approach, Bull. Fr. Actuar. 16 (2016), 5–40.

[10] F. Flici, Longevity and Life Annuities Reserving in Algeria: A Comparison of Mortality Model, in: IAALS Conference, International Actuarial Association, Hong Kong, 2016.

[11] F. Flici, Longevity and Pension Plan Sustainability in Algeria: Taking the Retirees Mortality Experience Into Account, Doctoral Thesis, Ecole Nationale Supérieure de Statistique et d'Economie Appliquée (ENSSEA), Algiers, Algeria, (2017).

[12] F. Flici, Closing-Out the Algerian Life Tables: for More Accuracy and Adequacy at Old-Age, Actuar. Res. Clear. House 2020 (2020), 2.

[13] F. Flici, Coherent Mortality Forecasting for the Algerian Population, Assur. Gest. Risq./Insur. Risk Manage. 87 (2021), 209–231. https://doi.org/10.7202/1076125ar.

[14] F. Flici, N.E. Hammouda, Analysis of Half-Century of Mortality Evolution in Algeria: 1992–2012, in: Middle East Economics Association (MEEA) Conference, Tlemcen, Algeria, 2014.

[15] F. Flici, K. Senouci, and Y. Hannani, Tables de Mortalité d'Expérience Incorporant une Échelle de Projection: Adaptation au Cas des Retraites en Algérie, Bull. Fr. Actuar. 17 (2017), 5–32.

[16] R.J. Hyndman, H. Booth, F. Yasmeen, Coherent Mortality Forecasting: The Product-Ratio Method with Functional Time Series Models, Demography 50 (2012), 261–283. https://doi.org/10.1007/s13524-012-0145-5.

[17] R.D. Lee, L.R. Carter, Modeling and Forecasting U. S. Mortality, J. Am. Stat. Assoc. 87 (1992), 659–671. https://doi.org/10.2307/2290201.

[18] N. Li, R. Lee, Coherent Mortality Forecasts for a Group of Populations: An Extension of the Lee-Carter Method, Demography 42 (2005), 575–594. https://doi.org/10.1353/dem.2005.0021.

[19] M.S. Paolella, Linear Models and Time-Series Analysis: Regression, ANOVA, ARMA and GARCH, John Wiley & Sons Ltd., Chichester, 2019.

[20] F. Planchet, Modèles de Durée, Thesis, Lyon 1 University, France, 2017.

[21] A. Renshaw, S. Haberman, A Cohort-Based Extension to the Lee–Carter Model for Mortality Reduction Factors, Insurance: Math. Econ. 38 (2006), 556–570. https://doi.org/10.1016/j.insmatheco.2005.12.001.

[22] Y.R. Safitri, S. Mardiyati, M. Malik, The Cairns-Blake-Dowd Model to Forecast Indonesian Mortality Rates, AIP Conf. Proc. 2168 (2019), 020039. https://doi.org/10.1063/1.5132466.

[23] J.R. Wilmoth, Computational Methods for Fitting and Extrapolating the Lee–Carter Model of Mortality Change, Technical Report, Department of Demography, University of California, Berkeley, 1993.