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EXTENSION OF THE MACBEV METHOD FOR SOLVING DECISION-MAKING PROBLEMS WITH INTERVAL WEIGHTS AND DATA(E-MACBEVID)

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Abstract. The Collective Aggregation Model based on the hybridization of the EVAMIX method and the VMAVA+ voting method(MACBEV) is a recently developed multi-criteria group decision-making tool designed to efficiently solve certain decision-making problems. This method exhibits good properties and facilitates optimal action ranking. However, it has some limitations in handling problems with interval-based data. This study aims to propose an extension of this method adapted to solving these types of problems with interval-based data and weightings. The proposed new method, called E-MACBEVID, is primarily based on determining the uncertainty of each weight interval and adapting the MACBEV method to interval-based judgments. To justify the relevance and accuracy of the E-MACBEVID method's methodological approach, we conducted a complexity analysis, a sensitivity analysis based on that of the MACBEV method, and a numerical application to a decision-making problem with data entirely in interval form. This allowed us to identify a better compromise.

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1. Introduction

Many state and non-governmental structures, as well as human organizations, face significant decision-making problems. These decisions, when made, often lead to disagreements among stakeholders. Therefore, several decision-making methods have been developed to resolve these problems by providing consensus-based results [2,14,15].. While these various existing methods facilitate the ranking of actions, they sometimes encounter difficulties that occasionally lead to unexpected rankings [3]. To address this, some group decision-making methods, such as LON-ZO, MAC-AHP, MWPM-GDS, and MACUQ, have been implemented, but these are only applicable to discrete-value problems [9,11], and others provide only approximate results for the ideal solution [10]. Further reflections have

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also led to the development of the TOPSIS method, the WPM method and their extension to group decision-making, and the VMAVA+ voting method [5,6], which is a tool of social choice theory [16]. All these methods have limitations in handling problems with interval-based data. Consequently, a recent method called the MACBEV Method (Collective Aggregation Model based on the hybridization of the EVAMIX method and the VMAVA+ voting method) has been developed, which effectively solves problems, especially those with mixed data [7]. Despite the good properties of the MACBEV method, it cannot rank actions when the data and weights are interval-based or fuzzy. The objective of this work is to broaden the scope of the MACBEV method by extending it to solve problems with entirely interval-based data. To do this, we first use the interval uncertainty formula to estimate the discrete weight of each criterion, and then we proceed with the adaptation by meticulously following the steps of the MACBEV method. Finally, a numerical application example is presented to illustrate the new method.

2. Preliminary

2.1. **Interval arithmetic.** This section is inspired by [12].

Before describing the MACBEV method adapted to problems with interval data, we briefly review intervals and then explain how algebraic operations work on intervals.

Definition 2.1. *The interval is a bounded subset of real numbers. Formally:*

$$([a;b] \quad is \quad an \quad interval) \Longleftrightarrow (X \quad x \in \mathbb{R} \mid a \leq x \leq b) \tag{1}$$

where $a, b \in \mathbb{R}$ (set of all real numbers); in particular a, b, or even both of them may be infinite. Graphically, an interval is defined as a portion of a real line. The set of all intervals is denoted $\mathbb{R}$. The infimum and upper bounds of an interval X are denoted $\underline{X}$ and $\overline{X}$, respectively. The three real-valued functions defined on intervals are:

$$Width : \mathrm{W}(X) = |\overline{X} - \underline{X}| \tag{2}$$

$$Center : \mathrm{mid}(X) = \frac{1}{2}(\underline{X} + \overline{X}) \tag{3}$$

$$Absolute \quad value : |X| = max\{|\underline{X}|, |\overline{X}|\} \tag{4}$$

Let X and Y be two intervals. The result of X$\diamond$Y is a new interval with the following properties:

$$X \diamond Y = Z = \{z = x \diamond y | x \in X, y \in Y\} \tag{5}$$

where $\diamond$ is an element of the set $\{+,-,\div,\times\}$. Arithmetic operations on intervals are performed in the same way as ordinary operations on the endpoints of intervals. They are as follows:

$$X + Y = [\underline{X} + \underline{Y}, \overline{X} + \overline{Y}] \tag{6}$$

$$X - Y = [\underline{X} - \overline{Y}, \overline{X} - \underline{Y}] \tag{7}$$

$$X \times Y = [min(\underline{X} \times \underline{Y}, \underline{X} \times \overline{Y}, \overline{X} \times \underline{Y}, \overline{X} \times \overline{Y}), max(\underline{X} \times \underline{Y}, \underline{X} \times \overline{Y}, \overline{X} \times \underline{Y}, \overline{X} \times \overline{Y})] \tag{8}$$

$$X \div Y = [min(\underline{X} \div \underline{Y}, \underline{X} \div \overline{Y}, \overline{X} \div \underline{Y}, \overline{X} \div \overline{Y}), max(\underline{X} \div \underline{Y}, \underline{X} \div \overline{Y}, \overline{X} \div \underline{Y}, \overline{X} \div \overline{Y})] \tag{9}$$

With the condition $0 \notin Y$ for division.

Also, for comparing any two intervals, it proceeds as follows:

$$[\underline{a}, \overline{a}] \leq [\underline{b}, \overline{b}] \Longleftrightarrow \underline{a} \leq \underline{b}, \overline{a} \leq \overline{b} \tag{10}$$

The definitions presented in this section were used to perform the algebraic operations in this article, especially the definition 10 to rank the actions from best to worst.

2.2. **Description of the EVAMIX method.** According to [8, 13], the EVAMIX (EVAluation of data MIXte) method is a multi-criteria decision-making tool that efficiently aggregates mixed data (ordinal and cardinal). It follows these steps:

Step 1: Calculation of the superiority rate α_{ij} and β_{ij} of the stocks, respectively, for the qualitative (O) and quantitative (C) attributes.

$$\begin{cases} \alpha_{ij} = \left[\sum_{k \in O} \{W_k \times sgn(e_{ik} - e_{jk})\}^c\right]^{1/c} \\ \beta_{ij} = \left[\sum_{k \in C} \{W_k \times (e_{ik} - e_{jk})\}^c\right]^{1/c} \\ i, j \in \{1, ..., m\}, k \in \{1, ..., n\} \end{cases} \quad with \quad sgn(e_{ik} - e_{jk}) = \begin{cases} -1 & for \quad e_{ik} < e_{jk} \\ 0 & for \quad e_{ik} \simeq e_{jk} \\ 1 & for \quad e_{ik} > e_{jk} \end{cases} \tag{11}$$

With any real number c.

Step 2: Determining the normalized superiority rate:

$$\begin{cases} \delta_{ij} = \frac{(\alpha_{ij} - \alpha^-)}{(\alpha^+ - \alpha^-)} \\ d_{ij} = \frac{(\beta_{ij} - \beta^-)}{(\beta^+ - \beta^-)} \\ i, j \in \{1, ..., m\} \end{cases} \tag{12}$$

where α^+ and β^+ are the maximum superiority rates of the different actions for the qualitative and quantitative attributes, respectively, and α^- and β^- are the minimum superiority rates of the actions for the qualitative and quantitative attributes, respectively.

Step 3: Calculating the total dominance of any two actions

$$\begin{cases} D_{ij} = W_O \delta_{ij} + W_C d_{ij} & i,j \in \{1,...,m\} \\ W_O = \sum_{k \in O} W_k \quad et \quad W_C = \sum_{k \in C} W_k \end{cases} \tag{13}$$

Step 4: Determining the final score for each action:

$$S(a_i) = S_i = \left[\sum_j \frac{D_{ji}}{D_{ij}} \right]^{-1} \qquad i,j \in \{1,...,m\} \tag{14}$$

$S(a_i) > S(a_j)$ This means that action a_i is better than action a_j.

The EVAMIX method is widely used in the literature for certain group decision problems. It is easy to apply, has good cleanliness, and its main advantage is its rigorous aggregation of both ordinal and cardinal data. This is not a given for existing methods. It has its limitations when the data is in interval form.

2.3. **Description of the VMAVA+ voting method: (VOTING METHOD BASED ON APPROVAL VOTING AND ARITHMETIC MEAN)+.** This section is inspired by [6].

Step 1: Calculation and modeling of individual preferences

Suppose a set E of n candidates in an election (n > 2) and a set A of k voters with k > 2.

Each voter groups the candidates into subsets Sets of $E = \{E_1, E_2, E_3, E_p\}$ that are elements of $\mathbb{P}(E)$ and that form a partition of E.

$E_{(i;l)}$ is the set of candidates for the i^{th} choice for voter l, with $i = \{1;2;...;p\}$ and $l = \{1;2;...;k\}$. Let u_{lj} be the score assigned to candidate j by voter l. We have:

$u_{lj} = 4 \iff c_j \in E_1$;

$u_{lj} = 3 \iff c_j \in E_2$;

$u_{lj} = 2 \iff c_j \in E_3$;

$u_{lj} = 1 \iff c_j \in E_4$.

Thus, we obtain the following matrix of individual judgments in table 1:

TABLE 1. Individual judgments of voters

Groups →	E_1	E_2	E_3	E_4
Notes →	4	3	2	1
v_1	$E_{1.1}$	$E_{2.1}$	$E_{3.1}$	$E_{4.1}$
v_2	$E_{1.2}$	$E_{2.2}$	$E_{3.2}$	$E_{4.2}$
⋮	⋮	⋮	⋮	⋮
v_k	$E_{1.k}$	$E_{2.k}$	$E_{3.k}$	$E_{4.k}$

We then deduce the following matrix of voters' judgments in table 2:

TABLE 2. Individual judgments of voters

	c_1	c_2	...	c_m
v_1	$u_{1.1}$	$u_{1.2}$	...	$u_{1.n}$
v_2	$u_{2.1}$	$u_{2.2}$	...	$u_{2.n}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
v_k	$u_{k.1}$	$u_{k.2}$	...	$u_{k.n}$

Step 2: Partial Aggregation of Individual Judgments

Let g_A^+ be the aggregation function defining the geometric mean. Thus, we have:

$m_{v_l} = g_A^+(u_{l.1}, u_{l.2}, ..., u_{l.n}) = \sqrt[n]{\prod_{j=1}^{n} u_{l.j}}$

$m_{c_j} = g_A^+(u_{1.j}, u_{2.j}, ..., u_{k.j}) = \sqrt[k]{\prod_{l=1}^{k} u_{l.j}}$

This is summarised in table 3

TABLE 3. Partial aggregation of individual judgments

	c_1	c_2	...	c_m	m_{v_k}
v_1	$u_{1.1}$	$u_{1.2}$	...	$u_{1.n}$	m_{v_1}
v_2	$u_{2.1}$	$u_{2.2}$	...	$u_{2.n}$	m_{v_2}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
v_k	$u_{k.1}$	$u_{k.2}$	...	$u_{k.n}$	m_{v_k}
m_{c_j}	m_{c_1}	m_{c_2}	...	m_{c_n}	-

Step 3: Construction of the sets G_{sup}, G_{avg} and G_{inf}.

through the following procedure:

$u_{lj} > m_{v_l} \iff c_j \in G_{sup,v_l}$

$u_{lj} = m_{v_l} \iff c_j \in G_{moy,v_l}$

$u_{lj} < m_{v_l} \iff c_j \in G_{inf,v_l}$

The results are thus arranged in the following tables(see table 4):

TABLE 4. Sets G_{sup} , G_{avg}, G_{inf}

	G_{sup}	G_{avg}	G_{inf}
v_1	G_{sup,v_1}	G_{moy,v_1}	G_{inf,v_1}
v_2	G_{sup,v_2}	G_{moy,v_2}	G_{inf,v_2}
$\vdots$	$\vdots$	$\vdots$	$\vdots$
v_k	G_{sup,v_k}	G_{moy,v_k}	G_{inf,v_k}

Step 4: Determining the Winner

We know that: $G_{sup} = \bigcup_{l=1}^{k} G_{sup,v_l}$.

Let $\bigvee \subseteq G_{sup}$ be such that $\bigvee = \bigcap_{l=1}^{k} G_{sup,v_l}$ and c a candidate such that $c \in G_{sup}$.

We obtain the following cases:

(1) card $\bigvee$ = $|\bigvee|$=1

(2) 1< $|\bigvee| \leq n$

(3) $|\bigvee| = 0 \iff \bigvee = \{\}$. Consider a family of voters $\nu = \{v_i\}_{i\in\{1;...;k\}}$ and c a candidate such that $\bigcap_{i\in\{1;...;k\}} G_{sup,i} = \{c\}$.

(a) If k is even and $|\nu| \geq \frac{k}{2} + 1$ then c is the winner.

(b) If k is odd and $|\nu| > E(\frac{k}{2})$ then c is the winner.

The VMAVA+ voting method is an indispensable tool in social choice theory. It has long been a significant contributor to selecting the best candidate in voting problems and the best course of action in multi-criteria decision problems. However, it is only usable on discrete data and does not adequately address mixed-evaluation problems or those involving intervals. Given its importance in solving decision problems, it has been hybridized with the EVAMIX method, resulting in the MACBEV method.

2.4. **Description of the MACBEV Method.** This section is inspired by [7].

The method consists of three fundamental steps:

Step 1: Aggregation of each decision-maker's judgment matrix using EVAMIX.

EVAMIX yields the following score matrix(see table 5):

TABLE 5. Matrix of action scores according to decision-makers' judgments

	a_1	a_2	...	a_n
a_1	$S_1(a_1)$	$S_1(a_2)$	...	$S_1(a_n)$
a_2	$S_2(a_1)$	$S_2(a_2)$	...	$S_2(a_n)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
a_k	$S_k(a_1)$	$S_k(a_2)$	...	$S_k(a_n)$

Step 2: Aggregation of the overall action score matrix using VMAVA+

This is achieved by following the processes of the VMAVA+ method. We calculate:

$Md[k] = (prod_{i=1}^{N} S^k[a_i])^{1/N}$: The performance of decision-makers d_k by geometric mean.

$Ma[a_i] = (prod_{k=1}^{K} S^k[a_i])^{1/K}$: The performance of actions a_i by geometric mean. According to each decision-maker, actions are classified into three groups: G_{sup} (upper group), G_{moy} (middle group), and G_{inf} (lower group).

- If $S^k[a_i] > Md[k]$ then $a_i \in G_{sup}(d_k)$
- If $S^k[a_i] = Md[k]$ then $a_i \in G_{moy}(d_k)$
- If $S^k[a_i] < Md[k]$ then $a_i \in G_{inf}(d_k)$.

Step 3: Determining the Winner. The best action is obtained by meticulously following the steps for determining the winner in VMAVA+.

The MACBEV method is a very recent method developed to address the shortcomings of some existing methods. Many methods in the literature struggle to effectively solve mixed-judgment decision problems. The MACBEV method has helped overcome these limitations by efficiently solving these types of problems, and these results lead to greater consensus. Like all other methods, the MACBEV method, in turn, has its own shortcomings, especially when it comes to managing or solving decision-making problems with interval-based data and weights. Hence our motivation to propose this new method, which will take these types of problems into account. It is simply an extension of the MACBEV method to problems with interval-based data and weights.

3. Main Results

3.1. **Description of the E-MACBEVID Method.** The proposed new method follows these steps:

Step 1: Calculation of the uncertain value of the weight of each criterion

This is obtained using the following formula:

$$\widetilde{W_r^k} = \frac{W_r^l(d_k) + W_r^u(d_k)}{2} \quad k = 1, ..., K \quad r = 1, ..., m \tag{15}$$

where $W_r^l(d_k)$ represents the lower end of the interval and $W_r^u(d_k)$ the upper end of the interval.

Step 2: Determining the action dominance rate interval: α_{ij}^k and β_{ij}^k for the ordinal (O) and cardinal (C) criteria, respectively.

$$\begin{cases} \alpha_{ij}^k = \left[\left[\sum_{r\in O}\left\{\widetilde{W_r^k} \times sgn(e_{ir}^l(d_k) - e_{jr}^l(d_k))\right\}^c\right]^{1/c}, \left[\sum_{r\in O}\left\{\widetilde{W_r^k} \times sgn(e_{ir}^u(d_k) - e_{jr}^u(d_k))\right\}^c\right]^{1/c}\right] \\ \beta_{ij}^k = \left[\left[\sum_{k\in C}\left\{\widetilde{W_r^k} \times (e_{ir}^l(d_k) - e_{jr}^l(d_k))\right\}^c\right]^{1/c}, \left[\sum_{k\in C}\left\{\widetilde{W_r^k} \times (e_{ir}^u(d_k) - e_{jr}^u(d_k))\right\}^c\right]^{1/c}\right] \\ i, j \in \{1, ..., n\}, k \in \{1, ..., K\} \end{cases} \tag{16}$$

$$sgn(e^l_{ir}(d_k) - e^l_{jr}(d_k)) = \begin{cases} -1 & for \quad e^l_{ir}(d_k) < e^l_{jr}(d_k) \\ 0 & for \quad e^l_{ir}(d_k) \simeq e^l_{jr}(d_k) \\ 1 & for \quad e^l_{ir}(d_k) > e^l_{jr}(d_k) \end{cases}$$

$$sgn(e^u_{ir}(d_k) - e^u_{jr}(d_k)) = \begin{cases} -1 & for \quad e^u_{ir}(d_k) < e^u_{jr}(d_k) \\ 0 & for \quad e^u_{ir}(d_k) \simeq e^u_{jr}(d_k) \\ 1 & for \quad e^u_{ir}(d_k) > e^u_{jr}(d_k) \end{cases}$$

With any real number c. In our cas, we take c=1to perform the operations.

Step 3: Determining the interval of the normalized dominance index matrix:

$$\begin{cases} \delta^k_{ij} = \left[\delta^l_{ij}(d_k), \delta^u_{ij}(d_k)\right] = \left[\frac{(\alpha^l_{ij}(d_k) - \min(\alpha^k_{ij}))}{(\max(\alpha^k_{ij}) - \min(\alpha^k_{ij}))}, \frac{(\alpha^u_{ij}(d_k) - \min(\alpha^k_{ij}))}{(\max(\alpha^k_{ij}) - \min(\alpha^k_{ij}))}\right] \\ d^k_{ij} = \left[\beta^l_{ij}(d_k), \beta^u_{ij}(d_k)\right] = \left[\frac{\beta^l_{ij}(d_k) - \min(\beta^k_{ij})}{\max(\beta^k_{ij}) - \min(\beta^k_{ij})}, \frac{\beta^u_{ij}(d_k) - \min(\beta^k_{ij})}{\max(\beta^k_{ij}) - \min(\beta^k_{ij})}\right] \\ i, j \in \{1, ..., n\} \end{cases} \tag{17}$$

Step 4: Determining the interval of the total dominance matrix of action a_i with respect to action a_j

$$\begin{cases} D^k_{ij} = \left[D^l_{ij}, D^l_{ij}\right] = \left[W^l_O \delta^l_{ij} + W^l_C d^l_{ij}, W^u_O \delta^u_{ij} + W^u_C d^u_{ij}\right] \qquad i, j \in \{1, ..., m\} \\ W^k_O = \sum_{r \in O} \widetilde{W^k_r} \quad et \quad W^k_C = \sum_{r \in C} \widetilde{W^k_r} \end{cases} \tag{18}$$

Step 5: Calculation of the overall score per action according to the decision-maker D_k:

$$S^k(a_i) = \left[S^l_k(a_i), S^u_k(a_i)\right] = \left[\left[\sum_j \frac{D^l_{ji}}{D^l_{ij}}\right]^{-1}, \left[\sum_j \frac{D^u_{ji}}{D^u_{ij}}\right]^{-1}\right] \qquad i, j \in \{1, ..., m\} \tag{19}$$

Step 6 Aggregation of the overall action score matrix

• We calculate:

$Md\,[k] = \left[\left(\prod_{i=1}^N S^l_k(a_i)\right)^{1/N}, \left(\prod_{i=1}^N S^u_k(a_i)\right)^{1/N}\right]$: Calculation of the performance interval for each decision-maker d_k using the geometric mean.

$Ma\,[k] = \left[\left(\prod_{k=1}^K S^l_k(a_i)\right)^{1/K}, \left(\prod_{k=1}^K S^u_k(a_i)\right)^{1/K}\right]$: Calculation of the performance interval of each action a_i using the geometric mean.

• According to each decision-maker d_k, the actions are ranked into three groups such as G_{sup} (upper group), G_{moy} (middle group), and G_{inf} (lower group)

-If $\left[S^l_k(a_i), S^u_k(a_i)\right] > \left[\left(\prod_{i=1}^N S^l_k(a_i)\right)^{1/N}, \left(\prod_{i=1}^N S^u_k(a_i)\right)^{1/N}\right]$ then $a_i \in G_{sup}(d_k)$

-If $\left[S^l_k(a_i), S^u_k(a_i)\right] = \left[\left(\prod_{i=1}^N S^l_k(a_i)\right)^{1/N}, \left(\prod_{i=1}^N S^u_k(a_i)\right)^{1/N}\right]$ then $a_i \in G_{avg}(d_k)$

-If $\left[S^l_k(a_i), S^u_k(a_i)\right] < \left[\left(\prod_{i=1}^N S^l_k(a_i)\right)^{1/N}, \left(\prod_{i=1}^N S^u_k(a_i)\right)^{1/N}\right]$ then $a_i \in G_{inf}(d_k)$

Step 7: Determining the Winner

We know that: $G_{sup} = \bigcup_{l=1}^{k} G_{sup(d_k)}$. Let $\bigvee \subseteq G_{sup}$ be such that $\bigvee = \bigcap_{l=1}^{k} G_{sup(d_k)}$ and c be a candidate such that $c \in G_{sup}$.

We can distinguish the following cases:

(1) card $\bigvee$ = $|\bigvee|$=1

(2) 1 < $|\bigvee| \leq n$

(3) $|\bigvee| = 0 \iff \bigvee = \{\}$. Consider a family of voters $\nu = \{d_i\}_{i \in \{1;...;k\}}$ and c a candidate such that $\bigcap_{i \in \{1;...;k\}} G_{sup,i} = \{c\}$.

a. If k is even and $|\nu| \geq \frac{k}{2} + 1$ then c is the winner.

b. If k is odd and $|\nu| > E(\frac{k}{2})$ then c is the winner.

3.2. **Study of the E-MACBEVID Complexity.** Complexity analysis, particularly in terms of time, is an important element in evaluating a method's algorithm. Indeed, it facilitates comparing the quality of one method to another and understanding how execution time increases with data size.

Step 1: Element Operation

▷ $\frac{W_r^l(d_k)+W_r^u(d_k)}{2}$: Calculation of the uncertain value of the weight of each criterion c_r at the level of the decision-maker $d_k \to \mathbb{O}(m \times K)$

▷ $\alpha_{ij}^k = \left[\alpha_{ij}^l(d_k); \alpha_{ij}^u(d_k)\right]$: Calculation of the interval of the dominance matrix of any two stocks a_i and a_j according to the decision-maker d_k with regard to the ordinal criteria $c_r \to \mathbb{O}(n^2 \times m' \times k)$

▷ $\beta_{ij}^k = \left[\beta_{ij}^l(d_k), \beta_{ij}^u(d_k)\right]$: Calculation of the interval of the dominance matrix of any two stocks a_i and a_j according to the decision-maker d_k with regard to the cardinal criteria $c_r \to \mathbb{O}(n^2 \times m'' \times k)$

▷ $\delta_{ij}^k = \left[\delta_{ij}^l(d_k), \delta_{ij}^u(d_k)\right]$: The interval of the matrix of the normalized dominance index of a_i with respect to aj of decision-maker d_k with respect to the ordinal criteria $c_r \to \mathbb{O}(n^2 \times m' \times k)$

▷ $d_{ij}^k = \left[\beta_{ij}^l(d_k), \beta_{ij}^u(d_k)\right]$: The interval of the matrix of the normalized dominance index of a_i with respect to aj of decision-maker d_k with respect to the cardinal criteria $c_r \to \mathbb{O}(n^2 \times m'' \times k)$

▷ $D_{ij}^k = \left[D_{ij}^l, D_{ij}^l\right]$: the interval of the total dominance matrix of a_i with respect to aj of decision-maker $d_k \to \mathbb{O}(n^2 \times k)$

$S^k(a_i) = \left[S_k^l(a_i), S_k^u(a_i)\right]$: The interval of the overall score matrix of action a_i according to decision-maker $d_k \to \mathbb{O}(n \times K)$.

▷ $\left[\left(\prod_{i=1}^{n} S_k^l(a_i)\right)^{1/n}, \left(\prod_{i=1}^{n} S_k^u(a_i)\right)\right] \to \mathbb{O}(n \times K)$

▷ $Ma\,[k] = \left[\left(\prod_{k=1}^{K} S_k^l(a_i)\right)^{1/K}, \left(\prod_{k=1}^{K} S_k^u(a_i)\right)^{1/K}\right] \to \mathbb{O}(n \times K)$

▷ $\bigvee = \bigcap_{l=1}^{k} G_{sup(d_k)} \to \mathbb{O}(K)$

Step 2: Total Complexity

From the above, the overall complexity of the proposed method is $\mathbb{O}(\max(n^2 \times m' \times k, n^2 \times m'' \times k))$. The following table 6 presents the complexity of several decision-making methods.

TABLE 6. Multiple criteria methods and their complexities [1]

Method	Reference	Complexity	Multiple Decision-maker
TOPSIS	Hwang and Yoon (1981)	$\mathbb{O}(m.n)$	no
ELECTRE(I,II,III)	Roy (1968, 1978a, 1978b)	$\mathbb{O}(m.n^2)$	no
PROMETHEE(I,II III)	Brans and Vincke (1985) Brans and Mareschal(1986;1994)	$\mathbb{O}(m.n^2)$	no
AHP	Saaty (1980)	$\mathbb{O}(m^3 + m.n^3)$	no
Group AHP	Ossadnik et al. (2016)	$\mathbb{O}(l.m.n^2)$	yes
DS/AHP	7Beynon et al. (2000)	$\mathbb{O}(l.m.n^2)$	yes
AHP/VAHP	Soltanifar and Kamyabi (2024).	$\mathbb{O}(l.m.n^2)$	yes
AHP-SWARA	Zolfani and Saparauskas (2013)	$\mathbb{O}(m.n + m^2)$	no
SWARA-VAHP	Keršulienė et al. (2010)	$\mathbb{O}(l.n^2.m)$	yes
AHP-BWM	Rezaei (2015)	$\mathbb{O}(m.n + m^2)$	no
BWM-VAHP	Mi et al. (2019)	$\mathbb{O}(l.m.n + m^2)$	yes
WM-AHP	Dong et al. (2010)	$\mathbb{O}(l.m.n^2)$	yes
BM-AHP	Ishizaka and Labib (2011)	$\mathbb{O}(m.n^2)$	no
CAHP	Ngoie et al. (2022)	$\mathbb{O}(\max(n.m.l, m.n^3))$	yes

3.3. **Numerical Application of E-MACBEVID to a Choice Problem.** In this section, we will perform a numerical application to illustrate the E-MACBEVID method.

To do this, we will compare three companies and select the best company to carry out a drilling project based on four criteria as follows:

c_1: Technical competence (very good (9), good (7), fair (5), passable (3))

c_2: Cost of drilling

c_3: Quality of service provided (very good (9), good (7), fair (5), passable (3))

c_4: Time required to complete the work

Three experts or decision-makers are asked to provide their judgments in order to reach a consensus. These judgments are presented in matrix form in table 7, 8 and 9.

TABLE 7. decision maker matrix D_1

Criteria →	c_1	c_2	c_3	c_4
Minimum scale →	0	0	0	0
Maximum scale →	10	10	10	10
Actions↓ \ Weight →	$[2;4]$	$[1;6]$	$[3;8]$	$[5;6]$
a_1	$[2;3]$	$[3;5]$	$[3;5]$	$[5;9]$
a_2	$[1;2]$	$[5;7]$	$[1;3]$	$[3;7]$
a_3	$[6;8]$	$[3;7]$	$[8;9]$	$[5;9]$

TABLE 8. decision maker matrix D_2

Criteria →	c_1	c_2	c_3	c_4
Minimum scale →	0	0	0	0
Maximum scale →	10	10	10	10
Actions↓ \ Weight →	$[3;5]$	$[2;7]$	$[5;8]$	$[4;6]$
a_1	$[4;6]$	$[5;7]$	$[5;9]$	$[5;9]$
a_2	$[3;4]$	$[3;7]$	$[2;4]$	$[5;7]$
a_3	$[7;8]$	$[7;9]$	$[7;8]$	$[3;5]$

TABLE 9. decision maker matrix D_3

Criteria →	c_1	c_2	c_3	c_4
Minimum scale →	0	0	0	0
Maximum scale →	10	10	10	10
Actions↓ \ Weight →	$[6;7]$	$[3;8]$	$[7;8]$	$[2;5]$
a_1	$[1;4]$	$[3;7]$	$[2;7]$	$[5;7]$
a_2	$[2;4]$	$[5;7]$	$[7;9]$	$[3;5]$
a_3	$[7;8]$	$[7;9]$	$[7;8]$	$[5;9]$

Step 1: Determining the approximate weight of each criterion.

This is obtained using relation 15. Table 10 shows the weight of each criterion calculated.

TABLE 10. weight matrix of criteria according to each decision-maker

Decision-makers↓ Criteria →	c_1	c_2	c_3	c_4
d_1	3	3.5	8.5	5.5
d_2	8	9	13	10
d_3	13	11	15	7

Step 2: Calculate α_{ij}^{k} and β_{ij}^{k} using relation 16.

Table 11, 12 shows the dominance rates of any two actions of decision-maker D_1 in the cardinal criteria and ordinal criteria respectively.

TABLE 11. Dominance interval according to decision maker D_1 at the level of the cardinal criteria

	a_1	a_2	a_3
a_1	$[0; 0]$	$[20; 20]$	$[-54.5; -49]$
a_2	$[-20; -20]$	$[0; 0]$	$[-74.5; -69]$
a_3	$[49; 54.5]$	$[69; 74.5]$	$[0; 0]$

TABLE 12. Dominance interval according to decision maker D_1 at the level of ordinal criteria

	a_1	a_2	a_3
a_1	$[0; 0]$	$[2; 9]$	$[2; 9]$
a_2	$[-9; -2]$	$[0; 0]$	$[-9; -5.5]$
a_3	$[-9; -2]$	$[2; 5.5]$	$[0; 0]$

Table 13, 14 shows the dominance rates of any two actions of decision-maker D_2 in the cardinal criteria and ordinal criteria respectively.

TABLE 13. Dominance interval according to decision maker D_2 at the level of the cardinal criteria

	a_1	a_2	a_3
a_1	$[0; 0]$	$[47; 89]$	$[-50; 5]$
a_2	$[-89; -47]$	$[0; 0]$	$[-97; -84]$
a_3	$[-5; 50]$	$[84; 97]$	$[0; 0]$

TABLE 14. Dominance interval according to the decision maker D_2 at the level of the ordinal criteria

	a_1	a_2	a_3
a_1	$[0; 0]$	$[9; 10]$	$[1; 1]$
a_2	$[-10; -9]$	$[0; 0]$	$[1; 1]$
a_3	$[-1; -1]$	$[-1; -1]$	$[0; 0]$

Table 15, 16 shows the dominance rates of any two actions of decision-maker D_3 in the cardinal criteria and ordinal criteria respectively.

TABLE 15. Dominance interval according to decision maker D_3 at the level of the cardinal criteria

	a_1	a_2	a_3
a_1	$[0; 0]$	$[-88; -30]$	$[-153; -67]$
a_2	$[30; 88]$	$[0; 0]$	$[-65; -37]$
a_3	$[67; 153]$	$[37; 65]$	$[0; 0]$

TABLE 16. Dominance interval according to decision maker D_3 at the level of ordinal criteria

	a_1	a_2	a_3
a_1	$[0; 0]$	$[-4; 7]$	$[-18; -11]$
a_2	$[-7; 4]$	$[0; 0]$	$[-18; -18]$
a_3	$[11; 18]$	$[18; 18]$	$[0; 0]$

Step 3: Calculation of δ_{ij}^k and d_{ij}^k.
These are obtained using relation 17.

Table 17, 18 shows the normalised dominance rates of any two actions of decision-maker D_1 in the cardinal criteria and ordinal criteria respectively.

TABLE 17. Normalized dominance interval according to decision-maker D_1 at the cardinal criteria level

	a_1	a_2	a_3
a_1	$[0.5; 0.5]$	$[0.634; 0.634]$	$[0.134; 0.171]$
a_2	$[0.365; 0.365]$	$[0.5; 0.5]$	$[0; 0.036]$
a_3	$[0.828; 0.865]$	$[0.963; 1]$	$[0.5; 0.5]$

TABLE 18. Normalized dominance interval according to decision maker D_1 at the level of ordinal criteria

	a_1	a_2	a_3
a_1	$[0.5; 0.5]$	$[0.611; 1]$	$[0.611; 1]$
a_2	$[0; 0.388]$	$[0.5; 0.5]$	$[0; 0.194]$
a_3	$[0; 0.388]$	$[0.611; 0.805]$	$[0.5; 0.5]$

Table 19, 20 shows the normalised dominance rates of any two actions of decision-maker D_2 in the cardinal criteria and ordinal criteria respectively.

TABLE 19. Normalized Dominance Interval According to Decision Maker D_2 at Cardinal Criteria Level

	a_1	a_2	a_3
a_1	$[0.5; 0.5]$	$[0.742; 0.958]$	$[0.242; 0.525]$
a_2	$[0.041; 0.257]$	$[0.5; 0.5]$	$[0; 0.067]$
a_3	$[0.474; 0.757]$	$[0.932; 1]$	$[0.5; 0.5]$

TABLE 20. Normalized dominance interval according to decision maker D_2 at the level of ordinal criteria

	a_1	a_2	a_3
a_1	$[0.5; 0.5]$	$[0.95; 1]$	$[0.55; 0.55]$
a_2	$[0; 0.05]$	$[0.5; 0.5]$	$[0.55; 0.55]$
a_3	$[0.45; 0.45]$	$[0.45; 0.45]$	$[0.5; 0.5]$

Table 21, 22 shows the normalised dominance rates of any two actions of decision-maker D_3 in the cardinal criteria and ordinal criteria respectively.

TABLE 21. Normalized dominance interval according to decision-maker D_3 at the cardinal criteria level

	a_1	a_2	a_3
a_1	$[0.5; 0.5]$	$[0.212; 0.401]$	$[0.; 0.281]$
a_2	$[0.598; 0.787]$	$[0.5; 0.5]$	$[0.287; 0.379]$
a_3	$[0.718; 1]$	$[0.620; 0.712]$	$[0.5; 0.5]$

TABLE 22. Normalized dominance interval according to decision maker D_3 at the level of ordinal criteria

	a_1	a_2	a_3
a_1	$[0.5; 0.5]$	$[0.388; 0.694]$	$[0; 0.194]$
a_2	$[0.305; 0.611]$	$[0.5; 0.5]$	$[0; 0]$
a_3	$[0.805; 1]$	$[1; 1]$	$[0.5; 0.5]$

Step 4: Calculation of the total dominance interval D_{ij}^k for each decision-maker. This interval matrix is obtained by applying equation 18 and presented by table 23.

Table 23. Total dominance interval according to decision makers D_1,D_2 and D_3

	d_1	d_2	d_3
D_{11}	$[10.25; 10.25]$	$[20; 20]$	$[23; 23]$
D_{12}	$[12.793; 16.293]$	$[33.637; 39.134]$	$[12.947; 23.754]$
D_{13}	$[7.043; 10.968]$	$[15.537; 21.491]$	$[3.5; 7.869]$
D_{21}	$[4.206; 7.706]$	$[0.865; 6.362]$	$[22.245; 33.052]$
D_{22}	$[10.25; 10.25]$	$[20; 20]$	$[23; 23]$
D_{23}	$[0; 2.174]$	$[10.45; 11.857]$	$[8.052; 10.61]$
D_{31}	$[9.531; 13.456]$	$[18.508; 24.462]$	$[34.630; 46]$
D_{32}	$[17; 18.325]$	$[28.142; 29.55]$	$[35.385; 37.947]$
D_{33}	$[10.25; 10.25]$	$[20; 20]$	$[23; 23]$

Step 5: Calculating the scores for each action $S^k(a_i)$

The different scores are calculated using equation 19 and presented by table 24.

Table 24. Total dominance interval according to decision makers D_1,D_2 and D_3

Makers →	d_1		d_2		d_3	
Scores ↓	$s^1(a_i)$	rank	$s^2(a_i)$	rank	$s^3(a_i)$	rank
$S(a_1)$	$[0.370; 0.372]$	2^{nd}	$[0.434; 0.451]$	2^{nd}	$[0.079; 0.121]$	3^{rd}
$S(a_2)$	$[0.086; 0.247]$	3^{rd}	$[0.023; 0.103]$	3^{rd}	$[0.167; 0.188]$	2^{nd}
$S(a_3)$	$[0.517; 0.575]$	1^{st}	$[0.438; 0.452]$	1^{st}	$[0.689; 0.752]$	1^{st}

step 6: Aggregation of overall scores and calculation of the geometric mean of performance.

Table 25 presents the calculation of the geometric mean of the different performences.

Table 25. Scores and calculation of geometric averages of performances

	a_1	a_2	a_3	$M(d_k)$
d_1	$[0.370; 0.372]$	$[0.086; 0.247]$	$[0.517; 0.575]$	$[0.164; 0.276]$
d_2	$[0.434; 0.451]$	$[0.023; 0.103]$	$[0.438; 0.452]$	$[0.164; 0.276]$
d_3	$[0.079; 0.121]$	$[0.167; 0.188]$	$[0.689; 0.752]$	$[0.209; 0.258]$
$M(a_i)$	$[0.233; 0.273]$	$[0.099; 0.119]$	$[0.558; 0.560]$	-

step 7: sorting shares into subsets and determining the winner.

Table 26 presents the construction of the sub-assemblies.

Table 26. formation of subsets: Upper group (Gsup), lower group (Ginf) and average group (Gmoy)

	Gsup	Ginf	Gmoy
d_1	$\{a_1, a_3\}$	$\{a_2\}$	$\{\}$
d_2	$\{a_1, a_3\}$	$\{a_2\}$	$\{\}$
d_3	$\{a_3\}$	$\{a_1, a_2\}$	$\{\}$

Thus, $\{a_1, a_3\} \bigcap \{a_1, a_3\} \bigcap \{a_3\} = \{a_3\}$, therefore the stock a_3 is a winner.

4. Critical Analysis of E-MACBEVID

This section presents the added value of the new method and its weaknesses with respect to certain data.

4.1. **Strengths and Innovations of E-MACBEVID.** The E-MACBEVID method presents three major innovations in the decision-making process:

- Rigorous management and aggregation of mixed data: This provides a favorable solution to the difficulties encountered by most existing methods. These methods are only effective on cardinal data, where the presence of an ordinal evaluation necessitates conversion to a numerical value before any aggregation, which can result in a real loss of information.
- Specific resolution of continuous data or problems with data entirely in interval form. This is a crucial approach that will allow for an adequate solution to decision-making problems with imprecise data. Aware of the lack of precision in these types of problems, few methods are used to solve them effectively. The E-MACBEVID method strengthens these tools with new resolution procedures that produce a satisfactory ranking of actions.
- The polynomial nature of its complexity: This demonstrates the simplicity of the calculation, taking into account the optimization of criteria, resource optimization, and the reliability of the results.

Furthermore, referring to the sensitivity analysis of the MACBEV method (see [7]), the E-MACBEVID method can be described as more or less stable when the weighting intervals of the criteria are varied, thus giving it a more or less robust nature.

In light of the above, the E-MACBEVID method is undoubtedly an indispensable tool for the decision-making process and can therefore be used efficiently to solve certain decision problems.

4.2. **Weaknesses of E-MACBEVID.** Although the E-MACBEVID method boasts significant strengths, it also has some limitations. First, it cannot solve problems with completely fuzzy data. The method shows its limitations when dealing with problems involving uncontrollable data, i.e., fuzzy numbers (see [9,17] for a deeper understanding of decision support problems in uncertain contexts). Second, the lack of computerization or implementation of the method hinders a comprehensive sensitivity analysis, which may raise some doubts about its stability. Finally, given the complexity $\mathbb{O}(\max(n^2 \times m' \times k, n^2 \times m'' \times k))$, robust calculations might only be possible with extremely large datasets. Aware of these weaknesses, providing an adequate solution remains a challenge for us.

5. Conclusion

The ongoing search for a result acceptable to all stakeholders in the decision-making process is a challenge for everyone. In this work, we have contributed by proposing an innovative model that facilitates the effective management of certain decision problems in the context of imprecise data (interval-based data). To illustrate this new method, we applied it to a problem of selecting the best action with interval-based data. This allowed us to obtain a rigorous ranking of actions from best to worst. Given the strengths and major innovations offered by the E-MACBEVID method, it could therefore be used to solve certain decision problems. However, this method is unable to solve decision problems with fuzzy data, and the lack of its implementation limits its scope, especially when the data is large. Therefore, our future research will focus on these shortcomings in order to improve the method.

Abbreviations

The following abbreviations are used in the manuscript.

VMAVA: Voting Method based on Approval Voting and Arithmetic Mean

EVAMIX: Evaluation of Mixed Data

MACBEV: Collective Aggregation Model based on the hybridization of the EVAMIX method and the VMAVA+ voting method

AHP: Analytic Hierarchy Process

MAC-AHP: Collective Aggregation Method based on the AHP approach

LON-ZO: Collective Aggregation Function based on the harmonic mean

MACUQ: Collective Aggregation Method using the Squared Mean

MWPM-GDS: Median Weighted Product Method for Group Decision Support

TOPSIS: Technique for Order Performance by Similarity to Ideal Solution.

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