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ON FUZZY TRIGENERALIZED (μ_1, μ_2, μ_3)-α-OPEN SETS

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Abstract. In this paper, a new class of α-open sets in a fuzzy trigeneralized topological space is introduced called fuzzy (μ_1, μ_2, μ_3)-α-open sets: A is a fuzzy (μ_1, μ_2, μ_3)-α-open if $A \leq int_{\mu_1}(cl_{\mu_2}(int_{\mu_3}(A)))$. Several special cases of the fuzzy open sets namely: fuzzy (μ_1, μ_2)-preopen and (μ_1, μ_2)-semiopen are also presented. The characterizations of (μ_1, μ_2, μ_3)-α-open sets, fuzzy (μ_1, μ_2)-preopen and (μ_1, μ_2)-semiopen are given. Moreover, other properties are also provided.

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1. Introduction

In 1965, L.A. Zadeh [6] introduced the concept of fuzzy sets as mathematical means of describing vagueness in linguistics. A fuzzy set is a generalization of an ordinary set and it allows a degree of membership function for each element over the interval $[0, 1]$. Fuzziness can be found in many areas in daily life, in engineering, in medicine, in meteorology, in manufacturing and many more. It is frequently observed in human judgement, evaluation, decision making and reasoning as well.

Since Zadeh introduced fuzzy sets, many approaches and theories treating imprecision and uncertainty have been proposed. In view of the fact that much attention has been paid to generalize the basic concepts of classical topology, a theory of fuzzy topology is developed. The study of fuzzy topology was first introduced by C.L. Chang [3] in 1968. Several authors then subsequently developed various concepts in fuzzy topological spaces such as the generalized fuzzy topology. In recent literature, generalized topology has been extended to bi-topological spaces wherein two topological spaces were considered.

The present stage of fuzzy topology may be termed as its expanding phase. At this stage, many existing notions of fuzzy topology are being revisited for further investigations and improvement.

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Presently, one of the most studied concepts of fuzzy topology is the concept of fuzzy open and closed sets. As a result, one is naturally prompted to study the generalized form of fuzzy open sets – the fuzzy α-open sets in a new perspective. In 1991, Singal and Bin Shahna [2] were the first to introduce the concept of fuzzy α-open sets.

In this paper, a new concept of α-open set in generalized topology are presented, wherein three generalized topologies are being considered. That is, a fuzzy set A is said to be (μ_1, μ_2, μ_3)-α-open set whenever $A \leq int_{\mu_1}(cl_{\mu_2}(int_{\mu_3}(A)))$, wherein μ_1, μ_2, μ_3 are fuzzy generalized topologies in Γ. We then investigate its behavior and relationship relative to the properties of fuzzy α-open sets. Furthermore, a characterization and some results as well are presented.

2. Preliminaries

In this section, we recall some of the basic concepts in set theory, real analysis, and in fuzzy generalized topological spaces which are necessary for the sequel of the study.

Definition 1. [1] *Let S be a nonempty subset of $\mathbb{R}$.*

1. *The set S is said to be bounded above if there exists a number $u \in \mathbb{R}$ such that $s \leq u$ for all $s \in S$. Each such number u is called an upper bound of S.*
2. *The set S is said to be bounded below if there exists a number $w \in \mathbb{R}$ such that $w \leq s$ for all $s \in S$. Each such number w is called a lower bound of S.*
3. *A set is said to be bounded if it is both bounded above and bounded below. A set is said to be unbounded if it is not bounded.*

Definition 2. [1] *Let S be a nonempty subset of $\mathbb{R}$.*

1. *If S is bounded above, then a number u is said to be a supremum (or a least upper bound) of S if it satisfies the conditions:*
   (a) *u is an upper bound of S, and*
   (b) *If v is any upper bound of S, then $u \leq v$.*
2. *If S is bounded below, then a number w is said to be an infimum (or a greatest lower bound) of S if it satisfies the conditions:*
   (a) *w is a lower bound of S, and*
   (b) *If t is any lower bound of S, then $t \leq w$.*

Theorem 1. [1] *Let A and B be sets of real numbers. Then the following statements are true:*

(i) *If $A \subseteq B$, then $inf(A) \geq inf(B)$ and $sup(A) \leq sup(B)$.*
(ii) *If $A \neq \emptyset$, then $inf(A) \leq sup(B)$.*
(iii) *If A and B are nonempty sets, then*
   (i) $inf(A + B) = inf(A) + inf(B)$

(ii) $sup(A+B) = sup(A) + sup(B)$.

(iv) *If A is a nonempty set and $c \in \mathbb{R}$ is a constant, then*

$$inf(cA) = \begin{cases} c\,inf A, & \text{if } c \geq 0 \\ c\,sup A, & \text{if } c < 0 \end{cases}$$

and

$$sup(cA) = \begin{cases} c\,sup A, & \text{if } c \geq 0 \\ c\,inf A, & \text{if } c < 0. \end{cases}$$

(v) *If A is a nonempty set and c is a constant, then $inf(c-A) = c - sup(A)$.*

(vi) *If A is a nonempty set and c is a constant, then $sup(c-A) = c - inf(A)$.*

Definition 3. [6] *Let X be a nonempty set and I be the unit interval $[0,1]$. A fuzzy set A of X is characterized by the membership function,*

$$\varphi_A : X \to I$$

and $\varphi_A(x)$ is interpreted as a degree of membership of an element x in fuzzy set A for each $x \in X$.

For convenience, we do not distinguish between a fuzzy set A and its membership function φ_A, and if X is countable, we shall denote a fuzzy set A by the notation $A(x) = \{(x_1, a_1), (x_2, a_2), (x_3, a_3), \ldots, (x_n, a_n)\}$.

The family of all fuzzy sets in X is Γ, consisting of all the mapping from X to $[0,1]$.

Example 1. *For any $A \subseteq X$ with its characteristic function $\varphi_A : X \to [0,1]$ defined by*

$$\varphi_A(x) = \begin{cases} 0, & \text{if } x \in A \\ 1, & \text{if } x \notin A. \end{cases}$$

Then φ_A is a fuzzy set of X. Characteristic functions are fuzzy sets in X.

Example 2. *Let $B : \mathbb{R} \to [0,1]$ given by*

$$B(x) = \begin{cases} 0, & \text{if } x \leq 10 \\ \frac{x-10}{30}, & \text{if } 10 < x < 100 \\ 1, & \text{if } x \geq 100 \end{cases}$$

Then B is a fuzzy set of $\mathbb{R}$.

Definition 4. [6] *The function $\tilde{O} : X \to [0,1]$ defined as $\tilde{O}(x) = 0$ for all $x \in X$ and $\tilde{I} : X \to [0,1]$ defined as $\tilde{I}(x) = 1$ for all $x \in X$ are called zero function and one function, respectively.*

Definition 5. [3] *Let A and B be fuzzy sets in a nonempty space Γ with the grade of membership of x in A and B denoted by $\varphi_A(x)$ and $\varphi_B(x)$, respectively. Then*

(i) *$A = B$ if and only if $\varphi_A(x) = \varphi_B(x)$ for all $x \in X$.*

(ii) $A \leq B$ *if and only if* $\varphi_A(x) \leq \varphi_B(x)$ *for all* $x \in X$.

(iii) $C = A \vee B$ *if and only if* $\varphi_C(x) = max[\varphi_A(x), \varphi_B(x)]$ *for all* $x \in X$.

(iv) $D = A \wedge B$ *if and only if* $\varphi_D(x) = min[\varphi_A(x), \varphi_B(x)]$ *for all* $x \in X$.

Example 3. *Let* A *and* B *be fuzzy sets defined as* $A = \{(a, 0.1), (b, 0.3), (c, 0.6)\}$ *and* $B = \{(a, 0.5), (b, 0.8), (c, 0.6)$*, respectively. Then* $A \leq B$*, since*

$$A(a) = 0.1 \leq 0.5 = B(a)$$

$$A(b) = 0.3 \leq 0.8 = B(b)$$

$$A(c) = 0.6 \leq 0.6 = B(c).$$

Definition 6. [3] *The complement of a fuzzy set* A *is* $\tilde{I} - A$ *and is defined by* $(\tilde{I} - A)(x) = \tilde{I}(x) - A(x)$*, for all* $x \in X$.

Example 4. *Let* $X = \{a, b, c\}$ *and a function* $\varphi : X \to [0, 1]$*. Suppose* A *is a fuzzy set defined as* $A = \{(a, 0.1), (b, 0.3), (c, 0.6)\}$*, then*

$$(\tilde{I} - A)(a) = 1 - 0.1 = 0.9$$

$$(\tilde{I} - A)(b) = 1 - 0.3 = 0.7$$

$$(\tilde{I} - A)(c) = 1 - 0.6 = 0.4$$

Thus, $\tilde{I} - A = \{(a, 0.9), (b, 0.7), (c, 0.4)\}$.

Definition 7. [6] *For any index set* I*, the union of the fuzzy sets* $\{A_i : i \in I\}$*, for an index set* I *is defined as*

$$\left(\bigvee_{i \in I} A_i\right)(x) = sup\{A_i(x) : i \in I\}$$

Definition 8. [6] *For any index set* I*, the intersection of the fuzzy sets* $\{A_i : i \in I\}$*, for an index set* I *is defined as follows*

$$\left(\bigwedge_{i \in I} A_i\right)(x) = inf\{A_i(x) : i \in I\}.$$

Remark 1. *If the index set* I *is finite, these suprema and infima are actually maxima and minima, respectively.*

Example 5. *Let* $X = \{a, b, c, d\}$*. Suppose* $A_i \in \Gamma X$*, for* $i = 1, 2, 3$*, where*

$$A_1 = \{(a, 0.1), (b, 0.3), (c, 0.08, (d, 0.5)\}$$

$$A_2 = \{(a, 0.4), (b, 0.5), (c, 1), (d, 0.7)\}$$

$$A_3 = \{(a, 0.2), (b, 0.8), (c, 0.5), (d, 0.2)\}.$$

Let $I = \{1, 2, 3\}$. Then the union and the intersection of A_i are

$$\left(\bigvee_{i\in I} A_i\right)(x) = max\{A_i(x) : i \in I\} = \{(a, 0.4), (b, 0.8), (c, 1), (d, 0.7)\} \text{ and}$$

$$\left(\bigwedge_{i\in I} A_i\right)(x) = min\{A_i(x) : i \in I\} = \{(a, 0.1), (b, 0.3), (c, 0.5), (d, 0.2)\},$$

respectively.

Theorem 2. *Let $\{A_i(x), i \in I\} \subseteq \Gamma$, $\{B_i(x), i \in I\} \subseteq \Gamma$, and $B \in \Gamma$. Then the following hold:*

(i) *If $A_i \leq B$ for all $i \in I$, then $sup\{A_i(x) : i \in I\} \leq B$.*
(ii) *If $A_i \geq B$ for all $i \in I$, then $inf\{A_i(x) : i \in I\} \leq B$.*
(iii) *If $A_i \leq B_i$ for all $i \in I$, then $\bigvee_{i\in I} A_i \leq \bigvee_{i\in I} B_i$.*

Definition 9. [4] *Let $X \neq \emptyset$. A collection $\mu \subseteq \Gamma$ is a fuzzy generalized topology (or FGT) in Γ if the following conditions are satisfied.*

(o_1) $\tilde{O} \in \mu$
(o_2) *If $\{A_i : i \in \mathcal{I}\} \subseteq \mu$, then $\bigvee_{i\in\mathcal{I}} A_i \in \mu$.*

The pair (Γ, μ) is called a fuzzy generalized topological space (or FGTS). The elements of μ are called fuzzy μ-open sets. A fuzzy set F is fuzzy μ-closed if $\tilde{I} - A$ is fuzzy μ-open.

Remark 2. *If $\{A_i : i \in I\}$ is a collection of fuzzy μ-closed sets, then $\bigwedge\{A_i : i \in I\}$ is also fuzzy μ-closed.*

Example 6. *Let $X = \{a, b, c, d\}$ and A, B, C be fuzzy sets in Γ, where*

$$A = \{(a, 0.1), (b, 0.3), (c, 0.6), (d, 1)\},$$

$$B = \{(a, 0.4), (b, 0.3), (c, 0.8), (d, 1)\},$$

$$C = \{(a, 0.6), (b, 0.7), (c, 0.2), (d, 0.5)\}.$$

The collection $\mu_1 = \{A, C\}$ is not a fuzzy generalized topology because the zero function $\tilde{O} \notin \mu_1$. On the other hand, the collection $\mu_2 = \{\tilde{O}, A, B, A \vee B\}$ is a fuzzy generalized topology.

To see this,

(i) $\tilde{O} \in \mu_2$
(ii) *Note that for any fuzzy set $G \in \mu_2$, we have $G \vee \tilde{O} = G$ and $G \vee G = G$.*

Now, for any fuzzy sets A and B in μ_2 we have,

$$(A \vee B)(a) = max\{A(a), B(a)\} = max\{0.1, 0.4\} = 0.4$$

$$(A \vee B)(b) = max\{A(b), B(b)\} = max\{0.3, 0.3\} = 0.3$$

$$(A \vee B)(c) = max\{A(c), B(c)\} = max\{0.6, 0.8\} = 0.8$$

$$(A \vee B)(d) = max\{A(d), B(d)\} = max\{1, 1\} = 1.$$

So, $A \vee B = \{(a, 0.4), (b, 0.3), (c, 0.8), (d, 1)\} = B \in \mu_2$. Thus, μ_2 is a fuzzy generalized topology. Considering the fuzzy set C, the $int_{\mu_2}(C) = B$ and the $cl_{\mu_2}(C) = \tilde{I}$.

Remark 3. *If $\{A_i(x), i \in I\}$ is a collection of fuzzy μ-closed sets, then $\bigvee\{A_i : i \in I\}$ is also fuzzy μ-closed.*

Definition 10. [3] *Let μ be a fuzzy generalized topology (FGT). The interior of a fuzzy set A, denoted by $int_\mu(A)$ is defined as*

$$int_\mu(A) = \bigvee\{O : O \leq A \text{ and } O \text{ is fuzzy } \mu\text{-open}\}$$

Definition 11. [3] *Let μ be a fuzzy generalized topology (FGT) in Γ. The closure of a fuzzy set A, denoted by $cl_\mu(A)$ is defined as*

$$cl_\mu(A) = \bigwedge\{F : A \leq F \text{ and } F \text{ is fuzzy } \mu\text{-closed}\}$$

Theorem 3. *Let $X \neq \emptyset$ and μ be a FGT in Γ, $A \in \Gamma$. Then*

(i) $int_\mu(A) \leq A$
(ii) *$int_\mu(A)$ is the largest μ-open subset of A.*
(iii) *$int_\mu(A) = A$ if and only if A is fuzzy μ-open*
(iv) $int_\mu(int_\mu(A)) = int_\mu(A)$

Proof.

(i) By Theorem 2(ii), $sup\{O : O \leq A$ and O is fuzzy μ-open$\} \leq A$. Since

$$\begin{aligned} int_\mu(A) &= \bigvee\{O : O \leq A \text{ and } O \text{ is fuzzy } \mu\text{-open}\} \\ &= sup\{O : O \leq A \text{ and } O \text{ is fuzzy } \mu\text{-open}\} \\ &\leq A. \end{aligned}$$

Thus, $int_\mu(A) \leq A$.

(ii) Since $int_\mu(A) = \bigvee\{O : O \leq A$ and O is fuzzy μ-open$\}$. Then $int_\mu(A)$ is fuzzy μ-open.

Suppose B is a fuzzy μ-open set such that $B \leq A$. Then

$$B \in \{O : O \leq A \text{ and } O \text{ is fuzzy } \mu\text{-open}\}.$$

By Theorem 3(i), $B \leq int_\mu(A)$. Hence, $int_\mu(A)$ is the largest fuzzy μ-open subset of A.

(iii) Let $int_\mu(A) = A$. Since $int_\mu(A) = \bigvee\{O : O \leq A$ and O is fuzzy μ-open$\}$, then $int_\mu(A)$ is fuzzy μ-open. Thus, A is fuzzy μ-open.

Suppose A is fuzzy μ-open. By Theorem 3(i), $int_\mu(A) \leq A$. Since $A \leq A$ and A is fuzzy μ-open, $A \leq int_\mu(A)$. Consequently, $int_\mu(A) = A$.

(iv) By Theorem 3(i) $int_\mu(A) \leq A$. It follows that $int_\mu(int_\mu(A)) \leq int_\mu(A)$.

By Theorem 3(ii), $int_\mu(A)$ is the largest μ-open subset A. Accordingly, $int_\mu(A) \leq int_\mu(int_\mu(A))$. Hence $int_\mu(int_\mu(A)) = int_\mu(A)$.

Theorem 4. *Let $X \neq \emptyset$ and μ be a GFT, and $A \leq \Gamma$. Then*

(i) $A \leq cl_\mu(A)$

(ii) $cl_\mu(A) = A$ *if and only if* A *is fuzzy* μ*-closed.*

Proof.

(i) By Theorem 2(ii), $inf\{F : A \leq F$ and F is fuzzy μ-closed$\} \geq A$. Since

$$\begin{aligned} cl_\mu(A) &= \bigwedge\{F : A \leq F \text{ and } F \text{ is fuzzy } \mu\text{-closed}\} \\ &= inf\{F : A \leq F \text{ and } F \text{ is fuzzy } \mu\text{-closed}\}, \end{aligned}$$

$A \leq cl_\mu(A)$.

(ii) Let $cl_\mu(A) = A$. Since $cl_\mu(A) = \bigwedge\{F : A \leq F$ and F is fuzzy μ-closed$\}$, then by Remark 3, $cl_\mu(A)$ is fuzzy μ-closed. Thus, A is fuzzy μ-closed.

Suppose A is fuzzy μ-closed. By (i), $A \leq cl_\mu(A)$. Since $A \leq A$ and A is fuzzy μ-closed, $cl_\mu(A) \leq A$. Thus, $cl_\mu(A) = A$.

Theorem 5. [3] *Let A and B be fuzzy sets of Γ such that $A \leq B$. Then the following hold:*

(i.) $int_\mu(A) \leq int_\mu(B)$

(ii.) $cl_\mu(A) \leq cl_\mu(B)$.

Theorem 6. [3] *Let $\{A_i(x), i \in I\} \subseteq \Gamma$. Then the following hold:*

(i.) $\bigvee_{i\in I} int(A_i) \leq int\left(\bigvee_{i\in I} A_i\right)$

(ii.) $\bigvee_{i\in I} cl(A_i) \leq cl\left(\bigvee_{i\in I} A_i\right)$.

Theorem 7. *Let μ be a fuzzy generalized topology. Then $\tilde{I} - int_\mu(A) = cl_\mu(\tilde{I} - A)$.*

Proof.

$$\begin{aligned} (\tilde{I} - int_\mu(A))(x) &= \tilde{I} - (int_\mu(A))(x) \\ &= \tilde{1} - sup\{O(x) : O(x) \leq A(x) \text{ and } O \text{ is fuzzy } \mu\text{-open}\} \\ &= inf\{1 - O(x) : O(x) \leq A(x) \text{ and } O \text{ is fuzzy } \mu\text{-open}\}, \quad \text{(Thm. 2.2.3 (v))} \\ &= inf\{\tilde{I}(x) - O(x) : 1 - A(x) \leq 1 - O(x), \tilde{I} - O \text{ is fuzzy } \mu\text{-closed}\} \\ &= inf\{(\tilde{I} - O)(x) : 1 - A(x) \leq 1 - O(x), \tilde{I} - O \text{ is fuzzy } \mu\text{-closed}\} \\ &= inf\{F(x) : \tilde{I}(x) - A(x) \leq F(x) \text{ and } F \text{ is fuzzy } \mu\text{-closed}\} \\ &= \bigvee\{F(x) : 1 - A(x) \leq F(x) \text{ and } F \text{ is fuzzy } \mu\text{-closed}\} \\ &= cl_\mu(\tilde{I} - A(x)). \end{aligned}$$

Corollary 1. *Let μ be a fuzzy generalized topology. Then $\tilde{I} - cl_\mu(A) = int_\mu(\tilde{I} - A)$.*

3. Main Results

This chapter shows results concerning fuzzy (μ_1, μ_2, μ_3)-α-open set.

3.1. Fuzzy (μ, μ_n)-Semiopen.

In this section we define fuzzy (μ, μ_n)-semiopen set in bigeneralized topological spaces, establish characterization and show that the collection of the fuzzy (μ, μ_n)-semiopen sets in FGTS is a fuzzy generalized topology.

Definition 12. [5] *Let μ_1 and μ_2 be fuzzy generalized topologies in Γ, then (Γ, μ_1, μ_2) is called fuzzy bigeneralized topological space.*

Definition 13. [2] *A fuzzy set A in a fuzzy bitopological space (Γ, μ_1, μ_2) is called a fuzzy (μ_1, μ_2)-semiopen set if $A \leq cl_{\mu_1}(int_{\mu_2}(A))$.*

Example 7. *Let $X = \{a, b, c\}$ and fuzzy sets A, B, and C be defined as*

$$A = \{(a, 0.5), (b, 0.3), (c, 0.7)\}$$
$$B = \{(a, 0.4), (b, 0.1), (c, 0.6)\}$$
$$C = \{(a, 0), (b, 0.1), (c, 0.2)\}.$$

Then, the complements of the fuzzy sets A, B, C are

$$(\tilde{I} - A) = \{(a, 0.5), (b, 0.7), (c, 0.3)\},$$
$$(\tilde{I} - B) = \{(a, 0.6), (b, 0.9), (c, 0.4)\}, \text{and}$$
$$(\tilde{I} - C) = \{(a, 1), (b, 0.9), (c, 0.8)\},$$

respectively. Let μ_1 and μ_2 be topologies in Γ, where $\mu_1 = \{\tilde{O}, A, C\}$, $\mu_2 = \{\tilde{O}, B, C, \tilde{I}\}$. A fuzzy set S in Γ defined as $S = \{(a, 0), (b, 0.2), (c, 0.3)\}$ is fuzzy (μ_1, μ_2)-semiopen. Since $int_{\mu_2}(S) = C$ and $S \leq cl_{\mu_1}(int_{\mu_2}(S)) = cl_{\mu_1}(C) = \{(a, 1), (b, 0.9), (c, 0.8)\}$.

Theorem 8. *The collection of fuzzy (μ_1, μ_2)-semiopen sets is a fuzzy generalized topology.*

Proof. Suppose Π is a collection of fuzzy (μ_1, μ_2)-semiopen sets. Note that,

$$\tilde{O} \leq (cl_{\mu_1}(int_{\mu_2}(\tilde{O}))).$$

Hence, $\tilde{O} \in \Pi$. Now, let $A_i, i \in I$ be fuzzy (μ_1, μ_2)-semiopen. Then,

$$\begin{aligned}\bigvee_{i \in I} A_i &\leq \bigvee_{i \in I} cl_{\mu_1}(int_{\mu_2}(A_i)) \\ &\leq cl_{\mu_1}\left(\bigvee_{i \in I}(int_{\mu_2} A_i)\right), \quad \text{(Theorem 6}(i)\text{)}\end{aligned}$$

$$\leq cl_{\mu_1}\left(int_{\mu_2}\left(\bigvee_{i\in I} A_i\right)\right), \quad \text{(Theorem 6}(ii)).$$

Thus, $\bigvee_{i\in I} A_i \in \Pi$. Therefore, the collection of fuzzy (μ_1, μ_2)-semiopen sets is an FGT.

Theorem 9. *Let (Γ, μ_1, μ_2) be a FGTS. A fuzzy set A is (μ_1, μ_2)-semiopen if and only if there exists a μ_2-open set B such that $B \leq A \leq cl_{\mu_1}(B)$.*

Proof. Let A be a fuzzy (μ_1, μ_2)-semiopen set. By definition, $A \leq cl_{\mu_1}(int_{\mu_2}(A))$. Let $B = intmu_{\mu_2}(A)$. Hence, B is μ_2-open and $int_{\mu_2}(A) \leq cl_{\mu_1}(int_{\mu_2}(A))$. Thus,

$$B \leq A \leq cl_{\mu_1}(B).$$

Suppose there exists a μ_2-open set B such that $B \leq A \leq cl_{\mu_1}(B)$. Since B is μ_2-open, $B = int_{\mu_2}(A)$. Then $int_{\mu_2}(B) \leq int_{\mu_2}(A)$ and

$$\begin{aligned} cl_{\mu_1}(B) &\leq cl_{\mu_1}(A) \\ &\leq cl_{\mu_1}(int_{\mu_2}(B)) \\ &= cl_{\mu_1}(B). \end{aligned}$$

Thus, $cl_{\mu_1}(B) = cl_{\mu_1}(A) = cl_{\mu_1}(int_{\mu_2}(B))$. Since B $\leq A$, then by Theorem 5, $cl_{\mu_1}(int_{\mu_2}(B)) \leq cl_{\mu_1}(int_{\mu_2}(A))$. Thus, $A \leq cl_{\mu_1}(B) = cl_{\mu_1}(int_{\mu_2}(B)) \leq cl_{\mu_1}(int_{\mu_2}(A))$. Therefore, A is fuzzy (μ_1, μ_2)-semiopen.

3.2. **Fuzzy (μ_1, μ_2)-Preopen.** In this section we define fuzzy (μ_1, μ_2)-preopen set in bigeneralized topological spaces, established characterization and that the collection of the fuzzy preopen sets in FGTS is a fuzzy generalized topology.

Definition 14. [2] *A fuzzy set A in a fuzzy bitopological space (Γ, μ_1, μ_2) is called fuzzy (μ_1, μ_2)-preopen set if $A \leq int_{\mu_1}(cl_{\mu_2}(A))$.*

Example 8. *Consider the fuzzy bigeneralized topological space (X, μ_1, μ_2) and the fuzzy sets A, B, and C in Example 3.1.3. A fuzzy set T defined as $T = \{(a, 0.3), (b, 0), (c, 0.5)\}$ is a fuzzy (μ_1, μ_2)-preopen. Since $cl_{\mu_2}(T) = \tilde{I} - C$ and $S \leq int_{\mu_1}(cl_{\mu_2}(T)) = int_{\mu_1}(\tilde{I} - C) = A$.*

Theorem 10. *The collection of fuzzy (μ_1, μ_2)-preopen sets is a fuzzy generalized topology.*

Proof. Suppose Σ is a collection of fuzzy (μ_1, μ_2)-preopen sets. Note that $\tilde{O} \leq (int_{\mu_1}(cl_{\mu_2}(\tilde{O})))$. Hence, $\tilde{O} \in \Sigma$. Let A_i be a fuzzy (μ_1, μ_2)-preopen, for every $i \in I$. Then $A_i \leq (int_{\mu_1}(cl_{\mu_2}(A_i))), \forall i \in I$. Thus,

$$\bigvee_{i\in I} A_i \leq \bigvee_{i\in I} int_{\mu_1}(cl_{\mu_2}(A_i))$$

$$\leq int_{\mu_1}\left(\bigvee_{i\in I}(cl_{\mu_2}(A_i))\right), \quad \text{(Theorem 6}(i))$$

$$\leq int_{\mu_1}\left(cl_{\mu_2}\left(\bigvee_{i\in I} A_i\right)\right), \quad \text{(Theorem 6}(ii))$$

Thus, $\bigvee_{i\in I} A_i \in \Sigma$. Therefore, the collection of fuzzy (μ_1, μ_2)-preopen sets is an FGT.

Theorem 11. *Let (Γ, μ_1, μ_2) be a bigeneralized topological space. A fuzzy set A is fuzzy (μ_1, μ_2)-preopen if and only if $A \leq int_{\mu_1}(F)$, whenever $A \leq F$ and F is a fuzzy μ_2-closed set.*

Proof. Suppose A is a fuzzy (μ_1, μ_2)-preopen set and F is fuzzy μ_2-closed set such that $A \leq F$. Then, $A \leq int_{\mu_1}(cl_{\mu_2}(A))$ and $int_{\mu_1}(cl_{\mu_2}(A)) \leq int_{\mu_1}(cl_{\mu_2}(F))$. Since F is fuzzy μ_2-closed, $cl_{\mu_2}(F) = F$. Thus, $A \leq int_{\mu_1}(cl_{\mu_2}(A)) \leq int_{\mu_1}(cl_{\mu_2}(F)) = int_{\mu_1}(F)$.

Suppose $A \leq int_{\mu_1}(F)$, whenever $A \leq F$ and F is μ_2-closed set. Since $A \leq (cl_{\mu_2}(A))$ and $cl_{\mu_2}(A)$ is fuzzy μ_2-closed, $A \leq int_{\mu_1}(cl_{\mu_2}(A))$. Thus, A is (μ_1, μ_2)-preopen.

Definition 15. *Let (Γ, μ_1, μ_2) be a bigeneralized topological space, $A \in \Gamma$. Then A is called a fuzzy (μ_1, μ_2)-open if $A = int_{\mu_1}(int_{\mu_2}(A))$.*

Theorem 12. *If A is (μ_1, μ_2)-open, then A is (μ_1, μ_2)-preopen.*

Proof. Suppose A is (μ_1, μ_2)-open. Then $A = int_{\mu_1}(int_{\mu_2}(A))$. Since $int_{\mu_2}(A) \leq cl_{\mu_1}(A)$, $int_{\mu_1}(int_{\mu_2}(A)) \leq int_{\mu_1}(cl_{\mu_2}(A))$. Thus, $A \leq int_{\mu_1}(cl_{\mu_2}(A))$. Hence, A is fuzzy (μ_1, μ_2)-preopen.

3.3. **Fuzzy (μ_1, μ_2, μ_3)-α-open.** A new class of fuzzy α-open set is defined in this section wherein three generalized topologies are considered.

Definition 16. *Let μ_1, μ_2, and μ_3 be fuzzy generalized topologies in Γ. Then $(\Gamma, \mu_1, \mu_2, \mu_3)$ is called a fuzzy trigeneralized topological space.*

Definition 17. *Let $(\Gamma, \mu_1, \mu_2, \mu_3)$ be a fuzzy generalized topological space and $A \in \Gamma$. A is fuzzy (μ_1, μ_2, μ_3)-open if $A = int_{\mu_1}(int_{\mu_2}(int_{\mu_3}(A)))$.*

Definition 18. *Let $(\Gamma, \mu_1, \mu_2, \mu_3)$ be a fuzzy generalized topological space and $A \in \Gamma$. Then A is called a fuzzy (μ_1, μ_2, μ_3)-α-open if $A \leq int_{\mu_1}(cl_{\mu_2}(int_{\mu_3}(A)))$, and A is called a fuzzy (μ_1, μ_2, μ_3)-α-closed if $\tilde{I} - A$ is a fuzzy (μ_1, μ_2, μ_3)-α-open.*

Example 9. *Let $X = \{a, b, c, d\}$ and $A, B, C \in \Gamma$ defined as follows:*

$$A = \{(a, 0.2), (b, 0.5), (c, 0.6), (d, 0.4)\}$$

$$B = \{(a, 0.4), (b, 0.6), (c, 0.7), (d, 1)\}$$

$$C = \{(a, 0.4), (b, 0.8), (c, 0.8), (d, 1)\}.$$

Suppose also that $\mu_1 = \{\tilde{O}, A, B\}, \mu_2 = \{\tilde{O}, A, C\}$*, and* $\mu_3 = \{\tilde{O}, A, B, C\}$ *The fuzzy complements of* $\tilde{O}$*,* A*,* B*, and* C *are:* $\tilde{I}$ *and*

$$\tilde{I} - A = \{(a, 0.8), (b, 0.5), (c, 0.4), (d, 0.6)\}$$

$$\tilde{I} - B = \{(a, 0.6), (b, 0.4), (c, 0.3), (d, 0)\}$$

$$\tilde{I} - C = \{(a, 0.6), (b, 0.2), (c, 0.2), (d, 0)\},$$

respectively. The fuzzy sets $\tilde{O}$ *and* A *are fuzzy* (μ_1, μ_2, μ_3)*-*α*-open.*

To see if the fuzzy sets B *and* C *are* (μ_1, μ_2, μ_3)*-*α*-open. We have,* (i) *since* B *is fuzzy* μ_3*-open, thus* $int_{\mu_3}(B) = B$*.* (ii) *Now, the closure of the* $int_{\mu_3}(B) = B$ *with respect to* μ_2*, that is* $cl_{\mu_2}(B) = \tilde{I}$*.* (iii) *Lastly, the interior of* $int_{\mu_1}(cl_{\mu_2}(int_{\mu_3}(B))) = int_{\mu_1}(\tilde{I})$ *with respect to* μ_1 *is* B*. So,* $B \leq int_{\mu_1}(cl_{\mu_2}(int_{\mu_3}(B))) = B$*. Thus,* B *is fuzzy* (μ_1, μ_2, μ_3)*-*α*-open.*

Considering the fuzzy set C*, we have* $int_{\mu_1}(cl_{\mu_2}(int_{\mu_3}(C))) = B$*. Since*

$$C \nleq int_{\mu_1}(cl_{\mu_2}(int_{\mu_3}(C))) = B.$$

Thus, C *is not fuzzy* (μ_1, μ_2, μ_3)*-*α*-open.*

Theorem 13. *Let* $(\Gamma, \mu_1, \mu_2, \mu_3)$ *be a fuzzy generalized topological space and* $A \in \Gamma$*. Then* A *is fuzzy* (μ_1, μ_2, μ_3)*-*α*-closed set if and only if* $cl_{\mu_1}(int_{\mu_2}(cl_{\mu_3}(A))) \leq A$*.*

Proof. Suppose A is fuzzy (μ_1, μ_2, μ_3)-α-closed. Then $\tilde{I} - A$ is fuzzy (μ_1, μ_2, μ_3)-α-open. Consequently, $\tilde{I} - A \leq int_{\mu_1}(cl_{\mu_2}(int_{\mu_3}(A)))$. Now,

$$\begin{aligned} int_{\mu_1}(cl_{\mu_2}(int_{\mu_3}(A))) &= int_{\mu_1}(cl_{\mu_2}(\tilde{I} - cl_{\mu_3}(A))), \quad \text{(Corollary 1)} \\ &= int_{\mu_1}(\tilde{I} - int_{\mu_2}(cl_{\mu_3}(A))), \quad \text{(Theorem 7)} \\ &= int_{\mu_1}(cl_{\mu_2}(\tilde{I} - cl_{\mu_3}(A))), \quad \text{(Corollary 1)} \\ &= \tilde{I} - cl_{\mu_1}(int_{\mu_2}(cl_{\mu_3}(A))). \end{aligned}$$

So, $\tilde{I} - A \leq \tilde{I} - cl_{\mu_1}(int_{\mu_2}(cl_{\mu_3}(A)))$. In effect, $cl_{\mu_1}(int_{\mu_2}(cl_{\mu_3}(A))) \leq A$.

Suppose $cl_{\mu_1}(int_{\mu_2}(cl_{\mu_3}(A))) \leq A$. Then, $\tilde{I} - A \leq \tilde{I} - cl_{\mu_1}(int_{\mu_2}(cl_{\mu_3}(A)))$. Now,

$$\begin{aligned} \tilde{I} - cl_{\mu_1}(int_{\mu_2}(cl_{\mu_3}(A))) &= int_{\mu_1}(cl_{\mu_2}(\tilde{I} - cl_{\mu_3}(A))), \quad \text{(Corollary 1)} \\ &= int_{\mu_1}(\tilde{I} - int_{\mu_2}(cl_{\mu_3}(A))), \quad \text{(Theorem 7)} \\ &= int_{\mu_1}(cl_{\mu_2}(\tilde{I} - cl_{\mu_3}(A))), \quad \text{(Corollary 1)} \\ &= int_{\mu_1}(cl_{\mu_2}(int_{\mu_3}(A))). \end{aligned}$$

So, $\tilde{I} - A \leq \tilde{I} - cl_{\mu_1}(int_{\mu_2}(cl_{\mu_3}(A)))$. Hence, $\tilde{I} - A$ is a fuzzy (μ_1, μ_2, μ_3)-α-open. Consequently, A is fuzzy (μ_1, μ_2, μ_3)-α-closed.

Theorem 14. *Let $(\Gamma, \mu_1, \mu_2, \mu_3)$ be a fuzzy trigenerated topological space and $A \in \Gamma$. Then A is fuzzy u_i-open, for all $i = 1, 2, 3$ if and only if A is fuzzy (μ_1, μ_2, μ_3)-open.*

Proof. Suppose A is fuzzy u_i-open, for every $i = 1, 2, 3$. Then

$$int_{\mu_1}(A) = int_{\mu_2}(A) = int_{\mu_3}(A).$$

Thus, $int_{\mu_1}(int_{\mu_2}(int_{\mu_3}(A))) = int_{\mu_1}(int_{\mu_2}(A)) = int_{\mu_1}(A) = A$. Hence, A is fuzzy (μ_1, μ_2, μ_3)-open.

Suppose A is fuzzy (μ_1, μ_2, μ_3)-open. Note that $int_{\mu_1}(A) \leq A$, $int_{\mu_2}(A) \leq A$, and $int_{\mu_3}(A) \leq A$. Then, $int_{\mu_1}(int_{\mu_2}(int_{\mu_3}(A))) = A$. Thus,

$$\begin{aligned} A &= int_{\mu_1}(int_{\mu_2}(int_{\mu_3}(A))) \\ &\leq int_{\mu_2}(int_{\mu_3}(A)) \\ &\leq int_{\mu_3}(A) \\ &\leq A. \end{aligned}$$

Consequently, $A = int_{\mu_3}(A) = int_{\mu_2}(int_{\mu_3}(A)) = int_{\mu_1}(int_{\mu_2}(int_{\mu_3}(A)))$. Thus,

$$A = int_{\mu_3}(A) = int_{\mu_2}(A) = int_{\mu_1}(A).$$

Hence, A is fuzzy u_i-open, for all $i = 1, 2, 3$.

Theorem 15. *Let $A \in \Gamma$. If A is fuzzy (μ_1, μ_2, μ_3)-open, then A is fuzzy (μ_1, μ_3)-open.*

Proof. Suppose A is fuzzy (μ_1, μ_2, μ_3)-open. Let $A = int_{\mu_1}(int_{\mu_2}(int_{\mu_3}(A)))$. Note that, $int_{\mu_2}(int_{\mu_3}(A)) \leq int_{\mu_3}(A)$. Thus, $A = int_{\mu_1}(int_{\mu_2}(int_{\mu_3}(A))) \leq int_{\mu_1}(int_{\mu_3}(A))$. Therefore A is fuzzy (μ_1, μ_3)-open.

Remark 4. *The converse of Theorem 15 is not true in general. Consider the example given below.*

Theorem 16. *If A is a fuzzy (μ_1, μ_3)-open set, then A is (μ_1, μ_2, μ_3)-α-open.*

Proof. Suppose A is a fuzzy (μ_1, μ_3)-open set. Then $A = int_{\mu_1}(int_{\mu_3}(A))$. Note that $int_{\mu_3}(A) \leq cl_{\mu_2}(int_{\mu_3}(A))$. Thus, $A = int_{\mu_1}(int_{\mu_3}(A)) \leq int_{\mu_1}(cl_{\mu_2}(int_{\mu_3}(A))$.

The following corollary follows directly from Theorem 15 and Theorem 16.

Corollary 2. *If A is a fuzzy (μ_1, μ_2, μ_3)-open, then A is fuzzy (μ_1, μ_2, μ_3)-α-open.*

Remark 5. *The converse is not always true. Consider the next example.*

Example 10. *Let $X = \{a, b, c\}$ and fuzzy sets A, B, and C defined as*

$$A = \{(a, 0.1), (b, 0.4), (c, 0.3)\},$$

$$B = \{(a, 0.4), (b, 0.7), (c, 0.3)\},$$

$$C = \{(a, 0.5), (b, 0.1), (c, 0.8)\}.$$

Let $\mu_1 = \{\tilde{O}, A, \tilde{I}\}, \mu_2 = \{\tilde{O}, A, B, C\}, \mu_3 = \{\tilde{O}, A, C\}$. Consider the fuzzy set S defined as $S = \{(a, 0.5), (b, 0.3), (c, 0.1)\}$. Note that S is a fuzzy (μ_1, μ_2, μ_3)-α-open. However, the $int_{\mu_1}(int_{\mu_3}(S)) = A$. Thus, S is not a fuzzy (μ_1, μ_3)-open set.

Definition 19. *The interior of a fuzzy set A in a fuzzy tri-generalized topological space $(\Gamma, \mu_1, \mu_2, \mu_3)$ denoted by $int_\alpha(A)$ is defined by*

$$int_\alpha(A) = \bigvee\{O : O \leq A \text{ and } O \text{ is fuzzy } (\mu_1, \mu_2, \mu_3)\text{-}\alpha\text{-open}\}.$$

Definition 20. *The closure of a fuzzy set A in a fuzzy tri-generalized topological space $(\Gamma, \mu_1, \mu_2, \mu_3)$ denoted by $cl_\alpha(A)$ is defined by*

$$cl_\alpha(A) = \bigwedge\{F : A \leq F \text{ and } F \text{ is fuzzy } (\mu_1, \mu_2, \mu_3)\text{-}\alpha\text{-closed}\}.$$

Theorem 17. *Let μ_1, μ_2, μ_3 be fuzzy generalized topologies. Suppose also that A is a fuzzy set in Γ. Then the following statements hold:*

(i.) $int_\alpha(A) \leq A$

(ii.) *$int_\alpha(A)$ is the largest fuzzy (μ_1, μ_2, μ_3)-α-open less than A*

(iii.) *A is fuzzy (μ_1, μ_2, μ_3)-α-open if and only if $A = int_\alpha(A)$.*

Proof.

(i) Note that

$$\begin{aligned} int_\alpha(A) &= \bigvee\{O : O \leq A \text{ and } O \text{ is fuzzy } (\mu_1, \mu_2, \mu_3)\text{-}\alpha\text{-open}\} \\ &= sup\{O : O \leq A \text{ and } O \text{ is fuzzy } (\mu_1, \mu_2, \mu_3)\text{-}\alpha\text{-open}\} \\ &\leq A. \end{aligned}$$

Thus, $int_a(A) \leq A$.

(ii) Since $int_\alpha(A) = \bigvee\{O : O \leq A \text{ and } O \text{ is fuzzy } (\mu_1, \mu_2, \mu_3)\text{-}\alpha\text{-open}\}$. Then $int_a(A)$ is fuzzy (μ_1, μ_2, μ_3)-α-open. Suppose B is a fuzzy (μ_1, μ_2, μ_3)-α-open set $B \leq A$. Then $B \in \{O : O \leq A \text{ and } O \text{ is fuzzy } (\mu_1, \mu_2, \mu_3)\text{-}\alpha\text{-open}\}$. By Theorem 17$(i)$, $B \leq int_a(A)$. Thus, $int_\alpha(A)$ is the largest fuzzy (μ_1, μ_2, μ_3)-α-open set.

(iii) Suppose A is fuzzy (μ_1, μ_2, μ_3)-α-open. By Theorem 17(i), $int_\alpha(A) \leq A$. Since $A \leq A$ and A is fuzzy (μ_1, μ_2, μ_3)-α-open, then $A \leq int_\alpha(A)$. Consequently, $A = int_\alpha(A)$.

Suppose $A = int_a(A)$, then

$$int_a(A) = \bigvee\{O : O \leq A \text{ and } O \text{ is fuzzy } (\mu_1, \mu_2, \mu_3)\text{-}\alpha\text{-open}\}.$$

Thus A is fuzzy (μ_1, μ_2, μ_3)-α-open.

Theorem 18. *Let X be a nonempty set and μ_1, μ_2, μ_3 be generalized fuzzy topologies in Γ. Suppose also that A and B are fuzzy sets in Γ. Then*

(i) *If $A \leq B$, then $int_\alpha(A) \leq int_\alpha(B)$*
(ii) *If $A \leq B$, then $cl_\alpha(A) \leq cl_\alpha(B)$*

Proof.

(i) Let A, B be fuzzy sets satisfying $A \leq B$. Note that

$$int_\alpha(A) = \bigvee\{O : O \leq A \text{ and } O \text{ is fuzzy } (\mu_1, \mu_2, \mu_3)\text{-}\alpha\text{-open}\};$$

$$int_\alpha(B) = \bigvee\{O : O \leq B \text{ and } O \text{ is fuzzy } (\mu_1, \mu_2, \mu_3)\text{-}\alpha\text{-open}\}.$$

Also, let

$$Z_1 = \{O : O \leq A \text{ and } O \text{ is fuzzy } (\mu_1, \mu_2, \mu_3)\text{-}\alpha\text{-open}\};$$

$$Z_2 = \{O : O \leq B \text{ and } O \text{ is fuzzy } (\mu_1, \mu_2, \mu_3)\text{-}\alpha\text{-open}\}.$$

Since $A \leq B$, $O \in Z_1$ implies $O \in Z_2$. Thus, $Z_1 \subseteq Z_2$. This implies $sup(Z_1) \leq sup(Z_2)$. Therefore $int_\alpha(A) \leq int_\alpha(B)$.

(ii) Let A, B be fuzzy sets satisfying $A \leq B$. Note that

$$cl_\alpha(A) = \bigwedge\{F : A \leq F \text{ and } F \text{ is fuzzy } (\mu_1, \mu_2, \mu_3)\text{-}\alpha\text{-closed}\};$$

$$cl_\alpha(B) = \bigwedge\{F : B \leq F \text{ and } F \text{ is fuzzy } (\mu_1, \mu_2, \mu_3)\text{-}\alpha\text{-closed}\}.$$

Also, let

$$Z_1 = \{F : F \geq A \text{ and } F \text{ is fuzzy } (\mu_1, \mu_2, \mu_3)\text{-}\alpha\text{-closed}\};$$

$$Z_2 = \{F : F \geq B \text{ and } F \text{ is fuzzy } (\mu_1, \mu_2, \mu_3)\text{-}\alpha\text{-closed}\}.$$

Since $A \leq B$, then $Z_1 \subseteq Z_2$. This implies that $inf(Z_1) \leq inf(Z_2)$. Therefore, $cl_\alpha(A) \leq cl_\alpha(B)$.

Theorem 19. *The collection of all fuzzy (μ_1, μ_2, μ_3)-α-open sets is a fuzzy generalized topology.*

Proof.

(o_1) Since $\tilde{\mathcal{O}} \leq int_{\mu_1}(cl_{\mu_2}(int_{\mu_3}(\tilde{\mathcal{O}})))$. Then $\tilde{\mathcal{O}}$ is a fuzzy (μ_1, μ_2, μ_3)-α-open.

(o_2) Suppose that for each $i \in I$, A_i is fuzzy (μ_1, μ_2, μ_3)-α-open. Then

$$A_i \leq int_{\mu_1}(cl_{\mu_2}(int_{\mu_3}(A_i))).$$

Thus, $\bigvee_{i\in I} A_i \leq \bigvee_{i\in I} int_{\mu_1}(cl_{\mu_2}(int_{\mu_3}(A_i)))$.

Now, by Theorem () we have,

$$\begin{aligned} A_i \leq \bigvee_{i\in I} A_i &\leq \bigvee_{i\in I} int_{\mu_1}(cl_{\mu_2}(int_{\mu_3}(A_i))) \\ &\leq int_{\mu_1}\left(\bigvee_{i\in I} cl_{\mu_2}(int_{\mu_3}(A_i))\right) \\ &\leq int_{\mu_1}\left(cl_{\mu_2}\left(\bigvee_{i\in I} int_{\mu_3}(A_i)\right)\right). \end{aligned}$$

Hence, $\bigvee_{i\in I} A_i$ is fuzzy (μ_1, μ_2, μ_3)-α-open. Therefore, the collection of all fuzzy (μ_1, μ_2, μ_3)-α-open sets is a fuzzy generalized topology.

Theorem 20. *Let $(\Gamma, \mu_1, \mu_2, \mu_3)$ be a fuzzy tri-generalized topological space and A be a fuzzy set in Γ. Then A is a fuzzy (μ_1, μ_2, μ_3)-α-open if and only if there exists a fuzzy μ_3-open set B such that $B \leq A \leq int_{\mu_1}(cl_{\mu_2}(B))$.*

Proof. Suppose A is fuzzy (μ_1, μ_2, μ_3)-α-open. Then $A \leq int_{\mu_1}(cl_{\mu_2}(int_{\mu_3}(A)))$. Let $B = int_{\mu_3}(A)$. By Theorem 2.2.19(ii), B is a fuzzy μ_3-open set. Furthermore,

$$B = int_{\mu_3}(A) \leq A \leq int_{\mu_1}(cl_{\mu_2}(int_{\mu_3}(A))) = int_{\mu_1}(cl_{\mu_2}(B)).$$

Suppose there exists a fuzzy μ_3-open set B such that $B \leq A \leq int_{\mu_1}(cl_{\mu_2}(B))$. Then, $int_{\mu_3}(B) \leq int_{\mu_3}(A)$. Consequently, $B \leq int_{\mu_3}(A)$. Thus,

$$A \leq int_{\mu_1}(cl_{\mu_2}(B)) \leq int_{\mu_1}(cl_{\mu_2}(int_{\mu_3}(A))).$$

Therefore, A is fuzzy (μ_1, μ_2, μ_3)-α-open.

Remark 6. *The converse of Theorem 20 is not necessarily true. Consider the example given below.*

Theorem 21. *If A is a fuzzy μ_1-open and μ_3-open set, then A is fuzzy (μ_1, μ_2, μ_3)-α-open.*

Proof. Since A is fuzzy μ_1-open, then $int_{\mu_1}(A) = A$. Hence $A = int_{\mu_1}(A) \leq int_{\mu_1}(cl_{\mu_2}(A))$. Since A is fuzzy μ_3-open, then $int_{\mu_3}(A) = A$. Thus, $int_{\mu_1}(cl_{\mu_2}(A)) = int_{\mu_1}(cl_{\mu_2}(int_{\mu_2}(A)))$. Consequently, $A \leq int_{\mu_1}(cl_{\mu_2}(int_{\mu_2}(A)))$. Therefore, A is fuzzy (μ_1, μ_2, μ_3)-α-open set.

Theorem 22. *A fuzzy set A is fuzzy (μ_1, μ_2, μ_3)-α-open if and only if A is fuzzy (μ_1, μ_2)-semiopen and fuzzy (μ_1, μ_2)-preopen.*

Proof. Suppose A is fuzzy (μ_1, μ_2, μ_3)-α-open set. Then,

$$A \leq int_{\mu_1}(cl_{\mu_2}(int_{\mu_3}(A))) \leq cl_{\mu_2}(int_{\mu_3}(A))$$

and

$$A \leq int_{\mu_1}(cl_{\mu_2}(int_{\mu_3}(A))) \leq int_{\mu_1}(cl_{\mu_2}(A)).$$

Thus, A is fuzzy (μ_1, μ_2)-semiopen and fuzzy (μ_1, μ_2)-preopen.

Suppose A is fuzzy (μ_1, μ_2)-semiopen and fuzzy (μ_1, μ_2)-preopen. Then $A \leq int_{\mu_1}(cl_{\mu_2}(A))$ and $A \leq cl_{\mu_2}(int_{\mu_3}(A))$. Thus,

$$\begin{aligned} A &\leq int_{\mu_1}(cl_{\mu_2}(A)) \\ &\leq int_{\mu_1}(cl_{\mu_2}(cl_{\mu_2}(int_{\mu_3}(A)))) \\ &= int_{\mu_1}(cl_{\mu_2}(int_{\mu_3}(A))). \end{aligned}$$

Therefore, A is fuzzy (μ_1, μ_2, μ_3)-α-open set.

The theorem above is illustrated using the Venn diagram as shown below.

4. Conclusion

This paper established a new class of fuzzy open sets, called fuzzy (μ_1, μ_2, μ_3)-α-open sets, in fuzzy trigeneralized topological spaces. The definition was formulated through the interaction of interior and closure operators associated with the three fuzzy topologies, allowing a deeper structural analysis of generalized fuzzy openness.

Several characterizations of fuzzy (μ_1, μ_2, μ_3)-α-open sets were derived, providing necessary and sufficient conditions that clarify their position among existing fuzzy open sets. In particular, the relationships between fuzzy (μ_1, μ_2, μ_3)-α-open sets, fuzzy (μ_1, μ_2)-preopen sets, and fuzzy (μ_1, μ_2)-semiopen sets

The study also demonstrated that several known classes of fuzzy open sets arise as special cases of the proposed concept, confirming that fuzzy (μ_1, μ_2, μ_3)-α-open sets form a proper generalization within the framework of fuzzy trigeneralized topological spaces. These results strengthen the theoretical foundation of fuzzy topology and provide tools for further exploration of related notions such as fuzzy continuity.

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