Asia Pac. J. Math. 2026 **13**:54

MULTIOBJECTIVE OPTIMIZATION AND MODELING OF ELECTRICAL POWER DISPATCH IN BURKINA FASO: APPLICATION TO THE KOSSODO-EAST NETWORK

AUGUSTIN KABORE[1], APPOLINAIRE TOUGMA[1,2], KOUNIHNIR SOME[1,*]

[1]Département de Mathématiques, Laboratoire de Mathématiques, Informatiques et Applications, Université Norbert ZONGO, Burkina Faso

[2]Faculté des Sciences et Technologies, Université Lédéa Bernard OUEDRAOGO, Burkina Faso

[*]Corresponding author: sokous11@gmail.com

Received Jan. 9, 2026

Abstract. In this work, we propose a modeling and optimization approach for electrical dispatching applied to the Kossodo-East network in Burkina Faso. The country's electrical energy production relies mainly on thermal power plants running on fuel. However, limited financial resources constrain production capacities, while the aging of engines leads to high carbon dioxide emissions, harmful to health and the environment. To address these two challenges, we developed a bi-objective model aiming to simultaneously minimize the energy production cost and the environmental impact related to carbon dioxide emissions. Data collected from the Kossodo-East site allowed for realistic modelling of the problem. Using an algorithm based on the Pascolitti-Serafini approach, we determined the set of Pareto optimal solutions. Furthermore, we designed a decision support algorithm to select the best compromise among these solutions, in order to improve the management of the electrical energy dispatching service for the Kossodo-East network.

2020 Mathematics Subject Classification. 90C29; 90C30; 65K05; 65D10.

Key words and phrases. modelling; multiobjective optimization; Pareto optimality; electrical power dispatching.

1. Introduction

Multiobjective optimization encompasses a set of mathematical tools for solving real-world problems in various fields such as engineering, communications, financial and administrative management, economics, health, and others. It generally involves modelling, proposing appropriate solution methods, and developing procedures for the analysis and validation of results. The presence of multiple objectives in such problems makes the search for solutions highly challenging, to the extent that exact methods are often ineffective, and heuristics and metaheuristics are therefore employed. Nowadays, the Pascoletti–Serafini approach [17, 18], which reduces the simultaneous optimization of several objectives to the optimization of a single scalar function, is widely used. Similarly, the Matlab function

DOI: 10.28924/APJM/13-54

f_{mincon} [3,7,25] is commonly employed to obtain optimal solutions. For this reason, hybridizing these two approaches facilitates the efficient resolution of multiobjective optimization problems. As this combination is well suited to solving a wide range of problems, this work focuses on the resolution of mathematical models related to challenges in electrical energy management.

However, numerous studies have already been conducted on this topic, focusing on finding solutions for the optimal planning of electric power systems in various countries. For instance, B. Zhou et al. [26] proposed an equilibrium-inspired multiple search optimization approach with synergistic learning for multiobjective electric power dispatch. W. F. Mbasso et al. [1] investigated sperm swarm optimization for many-objective power flow problems with enhanced performance evaluation in power systems. P. Ngatchou et al. [9] studied a multiobjective optimization algorithm for solving engineering problems. R. H. Bhesdadiya et al. [11] proposed an NSGA-III algorithm for solving the multiobjective economic–environmental dispatch problem. M. J. Khan et al. [12] investigated optimal power flow formulation and reactive power optimization in power systems using conventional optimization techniques. W.-T. Huang et al. [13] derived and applied a new transmission loss formula for power system economic dispatch. B. Nana et al. [16] presented an assessment of carbon dioxide emission factors from power generation in Burkina Faso. I. J. Raglend et al. [23] compared artificial intelligence techniques for solving the combined economic–emission dispatch problem with line flow constraints. Finally, M. Dashtdar et al. [4] addressed the environmental–economic dispatch problem using a hybrid FA–GA multiobjective algorithm.

In Burkina Faso, the issue of electrical energy management remains a permanent challenge that hinders socio-economic development. The national electrical system currently faces difficulties at several levels due to rapid population growth, industrialization, and the expansion of the electrical grid. In terms of energy supply, Burkina Faso remains heavily dependent on external sources, as over 49% of the electricity consumed is imported from Ghana, Togo, or Côte d'Ivoire [2]. National electricity production relies on thermal, solar, and hydroelectric sources, with a predominance of thermal energy. However, a large proportion of thermal power plants, dating from the 1970s and 1980s, are obsolete and offer limited capacities, with the highest not exceeding 18 MW. Consequently, once commissioned, these units operate directly at maximum power, leading to high consumption of fossil fuels (DDO and HFO) and generating significant costs, particularly during peak periods. This operating mode also causes technical incidents, including explosions, which further complicate energy dispatching management.

Furthermore, the continuous operation of these units at full capacity accelerates their degradation, reduces their efficiency, and compromises optimal electricity management. At the level of power lines and substations, some transmission network infrastructures dating from the 1980s were not designed to meet current demand. Moreover, insufficient network meshing limits dispatching possibilities through alternative routes. Several substations operate under overloaded conditions, increasing operational constraints. In addition, many lines initially designed for distribution are now used for transmission, resulting in significant energy losses. All these dysfunctions contribute to a decline in network performance, with losses estimated at 10.83% in the most recent ARSE report.

To address these challenges, the National Burkinabè Electricity Company (SONABEL) and researchers have traditionally focused on the single objective of minimizing energy production costs in order to achieve an economical and reliable network. In this context, dispatching establishes the priority order for commissioning thermal units, favoring the most economical units based on their specific fuel consumption. This allocation is carried out after mobilizing electricity imports as well as national solar and hydroelectric production to meet demand. Several models have been proposed to optimize the operation of Burkina Faso's power system in the presence of solar generation, using different approaches [20–22]. However, existing studies on optimal dispatching of the Burkinabè network do not adopt a multiobjective framework. For instance, Ky et al. [20] propose the use of liquefied natural gas (LNG) to replace heavy fuel oil (HFO) with the aim of reducing pollution, dependence on imports, and the intermittency of solar generation. Nevertheless, although LNG is less polluting than the fuels currently used, it remains a source of emissions and contributes to environmental impact. Consequently, energy management and dispatching constitute a fundamental problem that requires the search for compromise solutions to achieve dispatching strategies that are both economically efficient and environmentally sustainable.

To this end, we propose in this work a solution for the optimal management of electrical energy dispatching through the Kossodo-East network in Burkina Faso. A multiobjective modeling approach is proposed, integrating both the energy production cost and the environmental impact resulting from the use of highly polluting heavy fuels. Three algorithms are developed: first, Algorithm 1 estimates the economic parameters of the objective functions using the least-squares method; second, Algorithm 2 searches for Pareto-optimal solutions by hybridizing the Pascoletti–Serafini approach with the f_{mincon} function in Matlab; and finally, Algorithm 3 identifies the best compromise solution based on decision-support concepts. An analysis of the obtained solutions is provided to facilitate decision-making and support effective planning of electrical energy dispatching in Burkina Faso.

For a clear presentation of the results of this work, the remainder of the document is organized as follows: Section 1 presents the preliminaries, focusing on the fundamental concepts of energy dispatching, basic notions of multiobjective optimization, and the use of the least-squares method for parameter estimation. Section 2 is devoted to the main results; it addresses the modelling of the electrical dispatching problem of the Kossodo-East network and the numerical solutions of the problem. Section 3 presents the conclusion of this work.

2. Preliminaries

2.1. **Fundamental Concepts of Energy Dispatching.** The Economic Emission Dispatch (EED) problem aims to determine the optimal allocation of electrical energy production to minimize both the production cost and the environmental cost (expenses related to (CO_2 emissions), while respecting the production-demand-losses balance constraints and the technical limits of the production units.

Let f_1 denote the electrical energy production cost, f_2 the environmental cost, then the standard mathematical model of the EED problem is a constrained bi-objective problem that can be presented in the following form [8,11]:

$$\begin{cases} \min f_1(x) = \sum_{k=1}^{n_g} (a_k x_k^2 + b_k x_k + c_k), & \text{(i)} \\ \min f_2(x) = \sum_{k=1}^{n_g} (\alpha_k x_k^2 + \beta_k x_k + \gamma_k), & \text{(ii)} \\ \quad \sum_{k=1}^{n} x_k = x_D + x_L, & \text{(iii)} \\ \quad\quad x_k^{\min} \le x_k \le x_k^{\max}, \quad \forall k = 1, \dots, n, & \text{(iv)} \end{cases} \tag{1}$$

where: n_g denotes the number of thermal sources in the electrical network; x_D denotes the total demand of consumers connected to the electrical network; x_L the total amount of energy loss; a_k, b_k, c_k on one hand and $\alpha_k, \beta_k, \gamma_k$ on the other hand denote positive coefficients specific to each thermal source; x_k representing the energy produced by source k; x_k^{min}, x_k^{max} denote respectively the minimum and maximum production of source k.

The analytical and empirical model of the electricity production cost of a thermal source can be modelled as a polynomial. The energy cost function is written in the form [15]: $f_k(x_i) = a_k x_k^2 + b_k x_k + c_k,\ k = \overline{1, n_g}$ where a_k, b_k, c_k are positive coefficients specific to each thermal source.

The analytical and empirical model of the emission cost or rate of polluting gases (CO_2, NO_x, SO_2) from a thermal source can be modeled as a degree 2 polynomial. The model is written in the following form [23]: $f_i(x_i) = \alpha_2 x_i^2 + \alpha_1 x_i + \alpha_0$ where the are positive coefficients specific to each thermal source; $\alpha_0,\ \alpha_1,\ \alpha_2$.

Remark 1. *The coefficients of the objective functions* (a_k, b_k,c_k, α_k,β_k γ_k) *can be determined using numerical methods (interpolation methods, Newton's method, statistical, econometric,...), heuristic and metaheuristic approaches as well as artificial intelligence techniques [14,15].*

The energy losses x_L are generally modeled by the following function [4]:

$$x_L(V, \theta) = \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{R_{ij}}{R_{ij}^2 + X_{ij}^2} \left(|\underline{V_i}|^2 + |\underline{V_j}|^2 - 2|\underline{V_i}||\underline{V_j}| \cos(\theta_i - \theta_j) \right) \tag{2}$$

Where n denotes the number of nodes including generators in the network; R_{ij}, X_{ij} denote respectively the resistance and reactance of the edge ij; $|\underline{V_i}|$,$|\underline{V_j}|$ denote the respective modules of the voltage at node i and j; θ_i, θ_j denote the phase angles of the voltages at nodes i and j.

2.2. **Basic Concepts on Multiobjective Optimization.** A multiobjective optimization problem consists of simultaneously optimizing several objective functions, often conflicting, under a set of constraints. The minimization case can be mathematically modeled by the following formulation [5,6]:

$$\begin{cases} \min\{f_1(x), ..., f_m(x)\}, \text{ with } m \geq 2 \\ g_j(x) \leq 0, j = \overline{1,p} \\ x \in \mathbb{R}^n \end{cases} \tag{3}$$

where $x = (x_1, x_2, \ldots, x_n)$ denotes a vector of n decision variables; $f_i(x)$, $i = 1, \ldots, m$, represents the objective functions to be optimized (with $m \geq 2$); $g_j(x)$, $j = 1, \ldots, p$, models the constraints on the decision variables.

In this case $\mathcal{D} = \{x \in \mathbb{R}^n : g_j(x) \leq 0 \text{ for } j = 1, \ldots, p\}$ is the set of feasible solutions of the problem and $f(\mathcal{D})$ is called the set of non-dominated solutions or Pareto front [5].

Definition 2.1. [5]. *A solution $x^* \in \mathcal{D}$ is a Pareto optimal solution of problem (3) if there is no other feasible point $x \in \mathbb{D}$ such that $f_j(x) \leq f_j(x^*)$, $\forall j = 1, 2, \ldots, m$.*

Definition 2.2. [6]. *We call the ideal point any vector in the objective space, denoted by $f(x^*)$, whose components are obtained by individually optimizing each objective function, i.e., $f(x^*) = \big(\min_{x\in\mathcal{D}} f_1(x), \ldots, \min_{x\in\mathcal{D}} f_p(x)\big)$.*

Definition 2.3. [5]. *We call utopian point any vector in the objective space, denoted f^u, whose components are obtained from the ideal point in the following form: $f^u = \big(f_1^* - \epsilon,\ f_2^* - \epsilon,\ \ldots,\ f_p^* - \epsilon\big)$ where $\epsilon \in \mathbb{R}^p_+$.*

There are several methods to solve a multiobjective optimization problem, but in this work we are interested in the Pascoletti-Serafini one. The Pascoletti-Serafini method proceeds by transforming the initial problem into a single-objective problem. Its application to problem (3) gives the following formulation [17,18]:

$$\begin{cases} \min t \\ f_i(x) \leq \delta_i + t\, r_i, \quad i = 1, \ldots, p \\ x \in \mathcal{D},\ t \in \mathbb{R} \end{cases} \tag{4}$$

where $\delta = (\delta_1, \ldots, \delta_p)$ denotes a chosen reference point; $r = (r_1, \ldots, r_p)$ denotes a directional vector, with $r_i > 0$; $t \in \mathbb{R}$ is the scalar variable to be minimized (the smaller t is, the closer the solution is to the reference point δ). Let $\mathcal{D}' = \big\{(x,t) \in \mathcal{D} \times \mathbb{R} : f_i(x) \leq \delta_i + tr_i,\ i = 1, \ldots, p\big\}$

Theorem 2.1. [18,19] *Consider the single-objective optimization problem* (4). *Let* $(t^*, x^*) \in \mathcal{D}'$. *Then* (t^*, x^*) *is an optimal solution of problem* 4 *if and only if it is also a Pareto optimal solution of the multiobjective problem* (3).

2.3. **Parameter Estimation by the Least Squares Method.** In this section, we use the least squares method [14] to estimate the parameters of the analytical model of the energy production cost. Let $(x_k, z_k),\ k = 1, \ldots, n$, be a sample of size n, where x_k represents the power produced at node k (in MW), z_k the corresponding cost (in FCFA/h) and $\bar{z}$ the average energy cost. Since the cost is approximated by a quadratic polynomial of the form $\hat{z}_k = ax_k^2 + bx_k + c,\ k = 1, \ldots, n$, where a, b and c are the parameters to be estimated. The residual for observation k is $\varphi_k(a, b, c) = z_k - \hat{z}_k = z_k - (ax_k^2 + bx_k + c)$, and the sum of squared residuals is given by

$$\Phi(a, b, c) = \sum_{k=1}^{n} \big(z_k - (ax_k^2 + bx_k + c)\big)^2.$$

The search for parameter values using the function φ amounts to solving the following constrained optimization problem:

$$\begin{cases} \min \phi(a, b, c) = \displaystyle\sum_{k=1}^{n} \big(z_k - (ax_k^2 + bx_k + c)\big)^2 \\ \qquad \text{s.t.} \quad a > 0 \\ \qquad\qquad\ \ b \geq 0 \\ \qquad\qquad\ \ c \geq 0 \end{cases} \tag{5}$$

This problem (5) is convex with respect to the variables (a, b, c) (the objective function is a sum of squares affine in a, b, c), consequently, the solution is unique. The quality analysis of the parameters a,b and c is determined by the coefficient of determination defined as follows:

$$R^2 = 1 - \frac{\displaystyle\sum_{k=1}^{n} \phi_k^2}{\displaystyle\sum_{k=1}^{n} (z_k - \bar{z})^2} \tag{6}$$

By applying the sufficient conditions to reach the local minimum of system (5), we deduce the following system of equations (7):

$$\begin{cases} \dfrac{\partial \phi}{\partial a} = -2 \displaystyle\sum_{k=1}^{n} x_k^2 \big(z_k - (ax_k^2 + bx_k + c)\big) = 0 \\ \dfrac{\partial \phi}{\partial b} = -2 \displaystyle\sum_{k=1}^{n} x_k \big(z_k - (ax_k^2 + bx_k + c)\big) = 0 \\ \dfrac{\partial \phi}{\partial c} = -2 \displaystyle\sum_{k=1}^{n} \big(z_k - (ax_k^2 + bx_k + c)\big) = 0 \end{cases} \tag{7}$$

Expanding the terms of system (7), we obtain:

$$\begin{cases} a\sum_{k=1}^{n} x_k^4 + b\sum_{k=1}^{n} x_k^3 + c\sum_{k=1}^{n} x_k^2 = \sum_{k=1}^{n} x_k^2 z_k \\ a\sum_{k=1}^{n} x_k^3 + b\sum_{k=1}^{n} x_k^2 + c\sum_{k=1}^{n} x_k = \sum_{k=1}^{n} x_k z_k \\ a\sum_{k=1}^{n} x_k^2 + b\sum_{k=1}^{n} x_k + cn = \sum_{k=1}^{n} z_k \end{cases} \tag{8}$$

To determine the coefficients α_k, β_k and γ_k of the carbon dioxide emission cost for each source, we proceed in the same way by assuming $(x_k, z_k),\ k = 1, \ldots, n$, a sample of size n, where x_k denotes the energy produced at node k (in MW) and z_k the corresponding emission cost in kg/hour.

3. Main Results

3.1. Modeling the Electrical Dispatching Problem of the Kossodo-East Network.

The Kossodo-East electrical network comprises five (5) interconnected electrical substations namely i-substation33Kv, j-substation33Kv, l-substation33Kv, p-substation15Kv and k-substation33Kv. With the exception of the l-substation33Kv, which is exclusively for consumption, they are equipped with production groups. Figure 1 shows the distribution of generator groups where it is clear that G2 and G6 are currently out of service. The Kossodo-East network is interconnected with the Ouaga 1 (currently shut down) and Ouaga 2 thermal power plants, allowing energy exchanges according to the network's daily demand needs. It supplies substations n, h and m, based on instantaneous demands. The red arrows on the diagram represent these local demands.

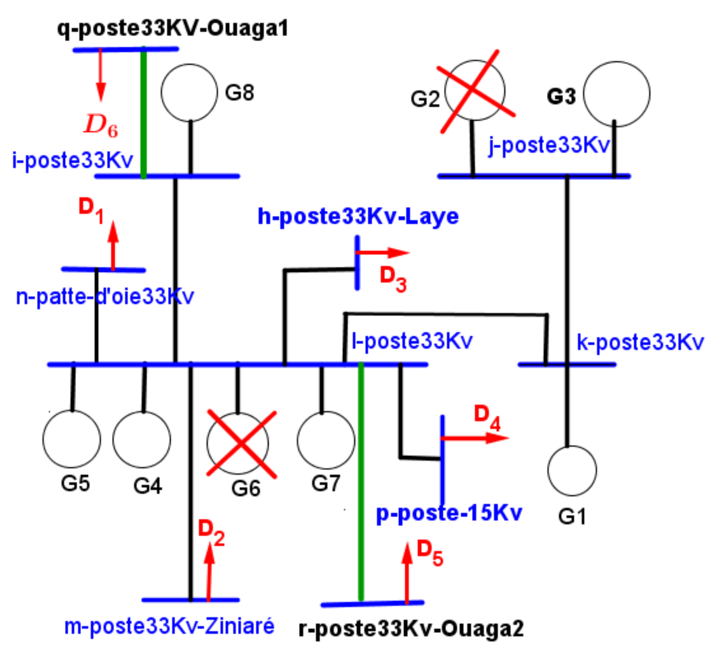

Figure 1. Kossodo East network

Table 1 shows the types of fuels used to operate the generator sets, their cost per liter and their carbon dioxide (CO2) emission factor.

Table 1. Fuel consumed by each group

Fuel	Electrical Groups	Price/liter	CO2 Emission Factor
DDO	G1-G2-G3-G4-G5-G6-G7-G8	300.00	0.95
HFO	G2-G3-G4-G5-G6-G7-G8	200.00	0.86
Oil	–	1 571.85	-

The analysis is based on production data collected over the period 2014-2024 and we used the demand schedule for December 31, 2024. The economic priority order for starting the groups of the Kossodo East power plant is as follows: G8-G7-G4-G3-G1-G5. However, this order can be modified due to the unavailability of certain units. This schedule presents a total average demand of $\overline{D}_T = 35.04$ MW distributed as follows: Patte-d'Oie ($\overline{D}_1 = 14.6$ MW), Ziniaré Substation ($\overline{D}_2 = 1.00$ MW), Laye ($\overline{D}_3 = 2.41$ MW), distribution of surrounding areas ($\overline{D}_4 = 4.93$ MW), Ouaga 2 ($\overline{D}_5 = 12.1$ MW), Ouaga 1 ($\overline{D}_6 = 0$ MW).

For the modeling of the dispatching of the Kossodo-East network, the following assumptions were retained to reflect the real daily operating conditions of the network:

- H_1 : We assume that all units of the thermal power plant supplying the Kossodo East network are in service and capable of meeting the demands directly connected to the network.
- H_2 : Sources G2 and G6 are not considered in the model, as they have been out of order for several years.
- H_3 : The injection of solar energy is not considered in the model, although the Ziniaré substation, supplied by a solar field, operates as a production source between 7 a.m. and 6 p.m. Numerical simulations will be performed between 7 p.m. and 6 a.m., a period during which all consumption substations behave solely as loads.
- H_4 : Imports are not taken into account. The maximum exploitable capacity of the power plant supplying the Kossodo East network is 51 MW, allowing it to meet consumer demand at a given time.
- H_5 : We assume that the network is autonomous and has no energy storage system.
- H_6 : In our modeling of the Kossodo East network, we only considered the costs of fuels and lubricants, excluding other costs such as maintenance, operation, and investment depreciation.

The algorithm to determine the coefficients of the objective functions is given as follows:

Algorithm 1: Determination of Objective Function Coefficients

Inputs : Raw production data

Outputs : Coefficients of objective functions $(a_k, b_k, c_k$ and $\alpha_k, \beta_k, \gamma_k)$

repeat

  Enter production data

  Determine the correlation coefficient R^2

  if $R^2 < 0.75$ **then**

    Remove outliers from production data

until $R^2 \geq 0.75$

Plot the curve of real and adjusted data

Determine the objective coefficients using system (8) to calculate a_k, b_k, c_k and $\alpha_k, \beta_k, \gamma_k$

Here are the linear adjustments of the data we obtained for each operating generator set.

Data Adjustment Model for G1: The economic production adjustment model is presented in Figure 2, while the one related to emissions is illustrated in Figure 3 for source G1.

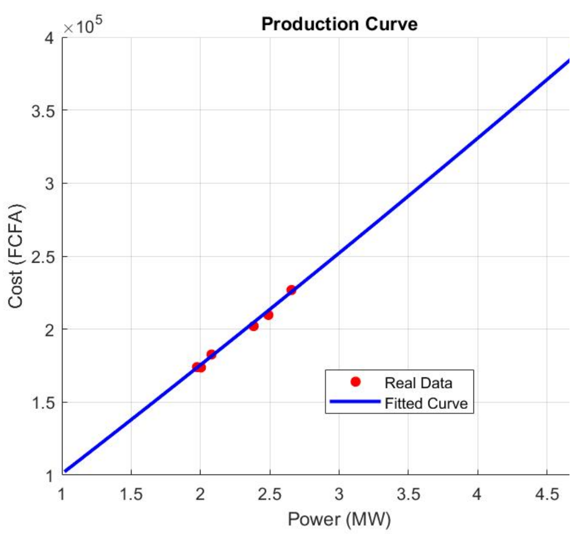

FIGURE 2. Production Model of G1

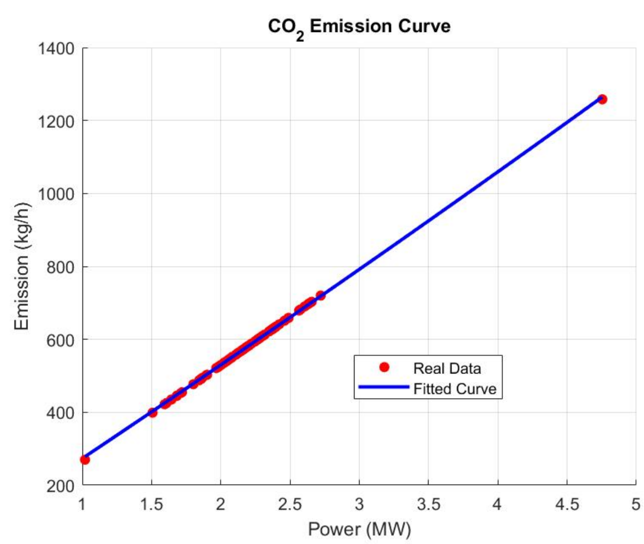

FIGURE 3. Emission Model of G1

Data Adjustment Model for G3: The economic production adjustment model is presented in Figure 4, while the one related to emissions is illustrated in Figure 5 for source G3.

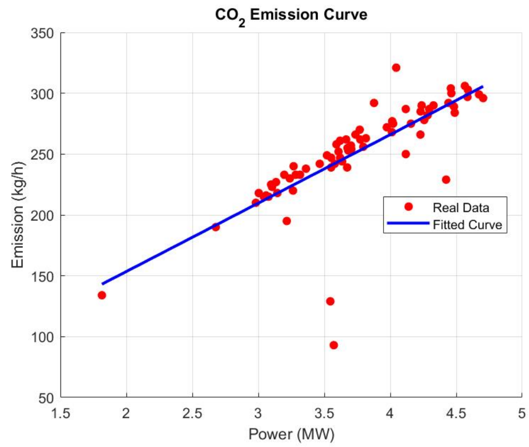

FIGURE 4. Production data

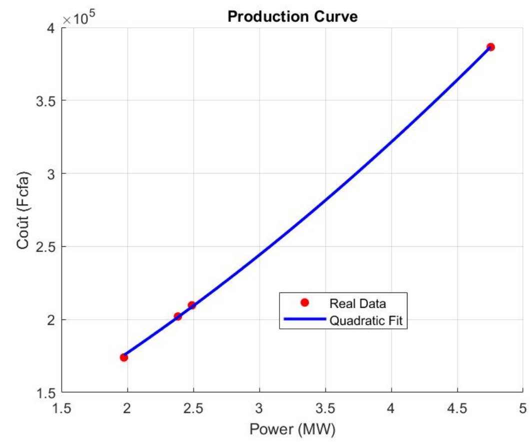

FIGURE 5. Emission data

Data Adjustment Model for G4: The economic production adjustment model is presented in Figure 6, while the one related to emissions is illustrated in Figure 7 for source G4.

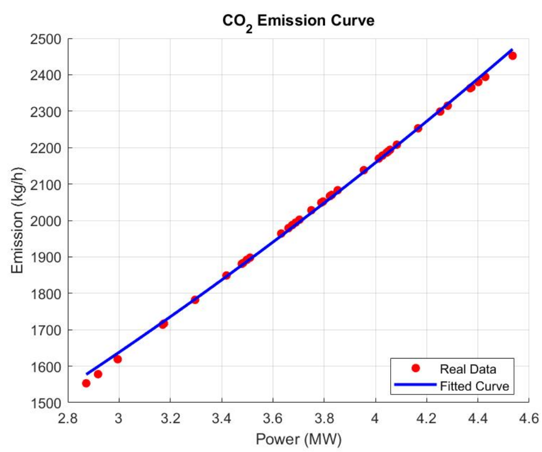

FIGURE 6. Production data

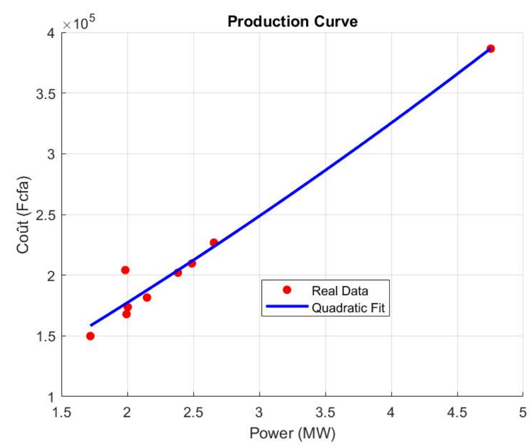

FIGURE 7. Emission data

Data Adjustment Model for G5: The economic production adjustment model is presented in Figure 8, while the one related to emissions is illustrated in Figure 9 for source G5.

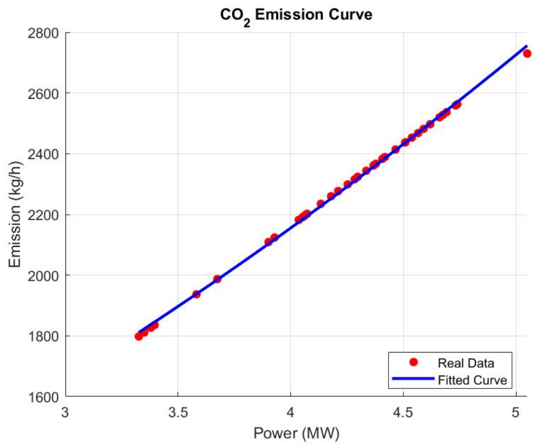

FIGURE 8. Production data

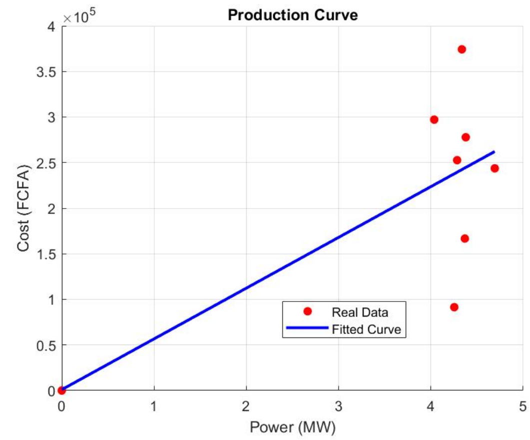

FIGURE 9. Emission data

Data Adjustment Model for G7: The economic production adjustment model is presented in Figure 10, while the one related to emissions is illustrated in Figure 11 for source G7.

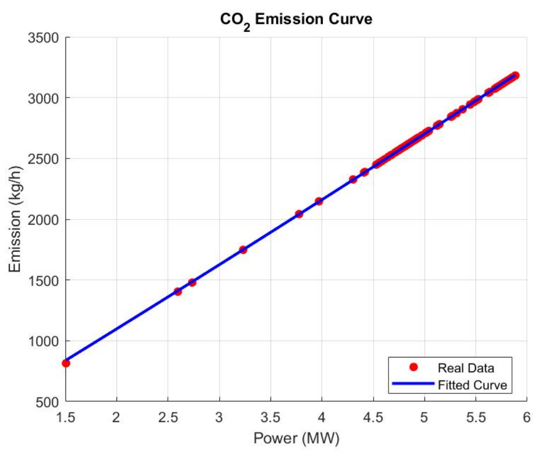

FIGURE 10. Production data

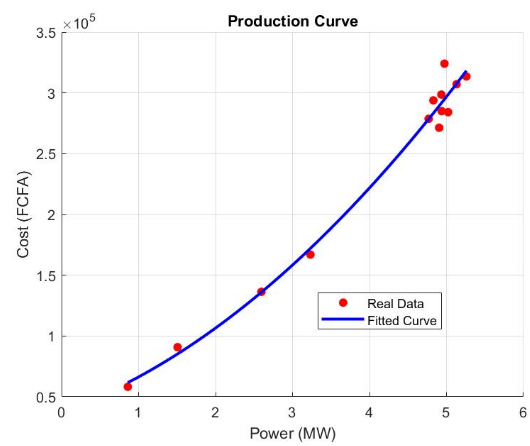

FIGURE 11. Emission data

Data Adjustment Model for G8: The economic production adjustment model is presented in Figure 12, while the one related to emissions is illustrated in Figure 13 for source G8.

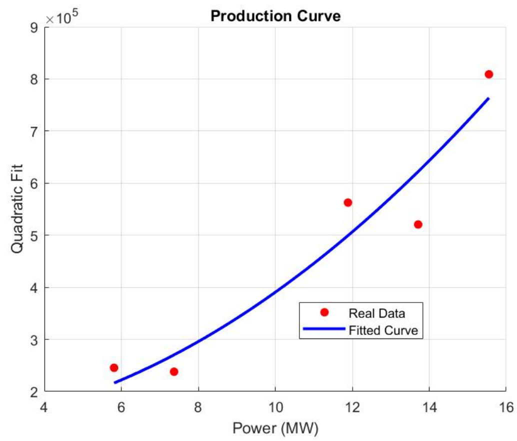

FIGURE 12. Production data

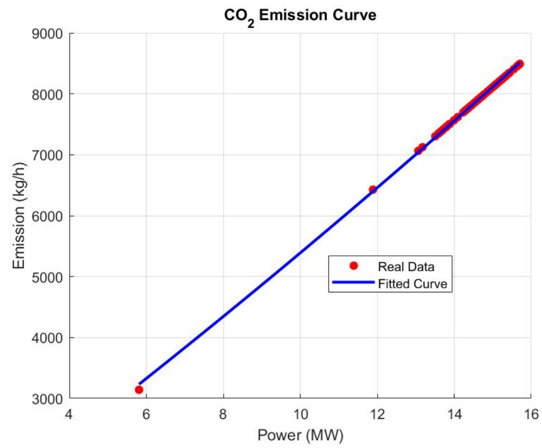

FIGURE 13. Emission data

The following Table 2 and Table 3 give the estimated values of the mathematical model parameters.

TABLE 2. Economic Production Data of Sources

Data	Data A			Data B	
Coefficients	a_k	b_k	c_k	$x^{\min}$	$x^{\max}$
G1	0.0022421	52926.34	145226.72	0.00	3.80
G3	0.0001690	6851.12	234161.13	0.00	6.25
G4	0.0043720	687116.56	64155.72	0.00	6.25
G5	0.0007250	53688.54	9144.00	0.00	6.25
G7	7904.2300000	10516.99	50549.36	0.00	7.00
G8	0.0746680	46559.23	94272.47	0.00	18.00

This allows us to formulate the production costs of the sources $(G1,G4,G3, G5,G7,G8)$ denoted respectively f_{11}, f_{13},f_{14}, f_{15}, f_{17}, f_{18} as follows: $f_{11}(x_1) = 0.002421x_1^2 + 52926.34x_1 + 145226.72$; $f_{13}(x_3) = 0.000169x_3^2 + 6851.127x_3 + 234161.137$; $f_{14}(x_4) = 0.004372x_4^2 + 687116.56x_4 + 64155.72$; $f_{15}(x_5) = 0.000725x_5^2 + 53688.54x_5 + 9144.001$; $f_{17}(x_7) = 7904.23x_7^2 + 10516.99x_7 + 50549.36$; $f_{18}(x_8) = 0.074668x_8^2 + 46559.23x_8 + 94272.47$. The total energy production cost of Kossodo East:

$$\begin{aligned} f_1(x) =& 0.0002421x_1^2 + 0.000169x_3^2 + 0.004372x_4^2 + 7904.23x_7^2 + 0.074668x_8^2 + \\ &+ 0.000725x_5^2 + 52926.34x_1 + 6851.127x_3 + 687116.56x_4 + 53688.54x_5 \\ &+ 10516.99x_7 + 46559.23x_8 + 8751294.76 \end{aligned} \tag{9}$$

TABLE 3. Economic Emission Data of Sources

Data	Data A			Data B	
Coefficients	α_k	β_k	γ_k	$x^{\min}$	$x^{\max}$
G1	2.72	248.31	22.46	0.00	3.80
G3	0.04	56.02	41.31	0.00	6.25
G4	40.51	236.79	563.12	0.00	6.25
G5	36.08	246.13	593.64	0.00	6.25
G7	3.55	509.85	63.91	0.00	7.00
G8	3.29	462.67	433.17	0.00	18.00

This allows us to formulate the gas emission rates of the sources ($G1$,$G4$,$G3$, $G5$,$G7$,$G8$) denoted respectively f_{21}, f_{23},f_{24}, f_{25}, f_{27}, f_{28} as follows: $f_{21}(x_1) = 2.72x_1^2 + 248.31x_1 + 22.46$; $f_{23}(x_3) = 0.04x_3^2 + 56.02x_3 + 41.31$; $f_{24}(x_4) = 40.51x_4^2 + 236.79x_4 + 563.12$; $f_{25}(x_5) = 36.08x_5^2 + 246.13x_5 + 593.64$; $f_{27}(x_7) = 3.55x_7^2 + 509.85x_7 + 63.91$; $f_{28}(x_8) = 3.29x_8^2 + 462.67x_8 + 433.17$. The total gas emission cost of Kossodo East:

$$
\begin{aligned}
f_2(x) =& 2.72x_1^2 + 0.04x_3^2 + 40.51x_4^2 + 36.08x_5^2 + 3.55x_7^2 \\
&+ 3.29x_8^2 + 248.31x_1 + 56.02x_3 + 236.79x_4 \\
&+ 246.13x_5 + 509.85x_7 + 462.67x_8 + 1717.61
\end{aligned} \tag{10}
$$

To complete the model, we took losses into account based on the work of Wei-Tzer et al. [13] which state that maximum losses in a transmission network are estimated at 8% of total demand.

The mathematical formulation of the electrical energy distribution optimization problem for Kossodo-East is given as follows:

$$
\begin{cases}
\min f_1(x) = 0.0002421x_1^2 + 0.000169x_3^2 + 0.004372x_4^2 + 7904.23x_7^2 + 0.074668x_8^2 + +000725x_5^2 + 52926.34x_1 \\
+6851.127x_3 + 687116.56x_4 + 53688.54x_5 + 10516.99x_7 + 46559.23x_8 + 8751294.76 \\
\min f_2(x) = 2.72x_1^2 + 0.04x_3^2 + 40.51x_4^2 + 36.08x_5^2 + 3.55x_7^2 + 3.29x_8^2 + 248.31x_1 + 56.02x_3 + 236.79x_4 \\
+246.13x_5 + 509.85x_7 + 462.67x_8 + 1717.61 \\
st \\
x_1 + x_3 + x_4 + x_5 + x_7 + x_8 = 37.84 \\
0 \leq x_1 \leq 3.8 \\
0 \leq x_3 \leq 6.25 \\
0 \leq x_4 \leq 6.25 \\
0 \leq x_5 \leq 6.25 \\
0 \leq x_7 \leq 7 \\
0 \leq x_8 \leq 18.
\end{cases} \tag{11}
$$

3.2. **Numerical Solutions.** To determine the solutions of problem 11, based on the Pascolitti-Serafini method, we used the Matlab function f_{mincom} *which is based on the interior point method* [12, 24]. *To do this, we proposed a technique for calculating the parameter* δ*. The key steps of this algorithm are as follows:*

Algorithm 2: Pareto Front Generation by the Pascoletti-Serafini Method

Inputs :*Multi-objective problem* $F(x) = (f_1(x), f_2(x))$ *and its constraints*

Number of points N *to generate for the Pareto front*

Direction vector $r = (1, 1)$

Outputs: *Set of Pareto-optimal solutions* $\mathcal{P}$

▷ `Phase 1: Determination of the objective space bounds`

Compute $f_{1,\min}, f_{1,\max}, f_{2,\min}, f_{2,\max}$ *by solving 4 single-objective optimization problems*

Define the anchor points: $p_A \leftarrow (f_{1,\max}, f_{2,\min})$ *and* $p_B \leftarrow (f_{1,\min}, f_{2,\max})$

Initialize the solution set: $\mathcal{P} \leftarrow \emptyset$

▷ `Phase 2: Iteration to generate front points`

for $k \leftarrow 0$ **to** $N - 1$ **do**

Define the weighting parameter $\alpha \leftarrow k/(N-1)$

▷ `Construct the target point "`δ`"`

$a \leftarrow p_A + \alpha(p_B - p_A)$

▷ `Solve the scalar (transformed) problem`

Find (x^*, t^*) *by solving:*

$$\min_{x,t} t \quad \textit{subject to} \quad \begin{cases} f_1(x) \leq \delta_1 + r_1 \cdot t \\ f_2(x) \leq \delta_2 + r_2 \cdot t \\ \textit{Initial constraints of model } (11) \end{cases}$$

▷ `Store the new Pareto-optimal solution`

Add the pair $(x^*, F(x^*))$ *to the set* $\mathcal{P}$

return $\mathcal{P}$

Numerical Simulations of the Proposed Model. By applying the proposed resolution method, we obtain the set of efficient solutions, which constitutes the Pareto front (Figure 14). The choice of the final optimal solution then depends on the priorities and preferences of the decision-maker.

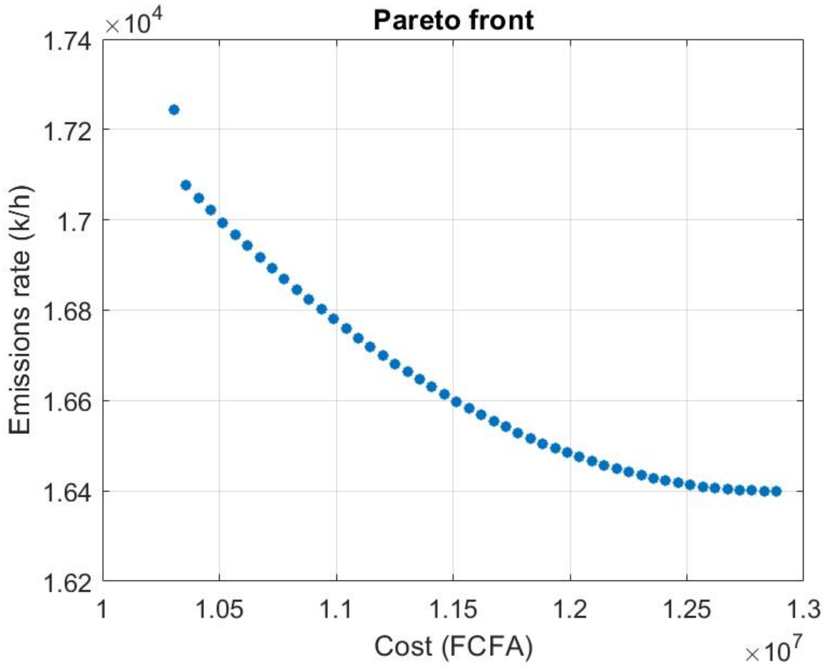

Figure 14. Pareto front

3.2.1. *Analysis and Discussion.* The obtained results indicate that it is not necessary to mobilize all production units at their maximum power to meet the demand.

In a first scenario, when the objective of the Kossodo-East dispatching operator is the exclusive minimization of production cost, without consideration for environmental impact, the optimal operating setpoints for groups G1, G3, G4, G5, G7 and G8 are established respectively at 3.80 MW; 6.25 MW; 0 MW; 6.25 MW; 3.54 MW and 18 MW. The corresponding optimal solution leads to a total production cost of 10,305,160.94 FCFA/h and an emission level of 17,243.22 kg/h. This is the extreme rightmost solution.

Conversely, in a second scenario where the priority objective is the reduction of environmental emissions, the optimal powers allocated to groups G1, G3, G4, G5, G7 and G8 are 3.80 MW; 6.25 MW; 3.90 MW; 4.25 MW; 6.00 MW and 13.65 MW. This configuration allows reaching a reduced emission level of 16,399.36 kg/h, but leads to an increase in production cost, which then amounts to 12,883,388.81 FCFA/h. This is the extreme leftmost solution.

These different solutions allow improving SONABEL's practice which consisted of continuously monitoring the specific consumption to check if the plant was operating at the economic margin. Such an approach proves limited (extreme solution) because it neglects environmental impact, and the use of heavy fuel oil (HFO) constitutes a significant source of pollution. Between the two extreme solutions, it is possible to define a dispatching of best compromises according to the obtained results (see Figure 14). For a suitable choice of a best compromise solution, we propose a decision support algorithm.

Decision Support Method. This is a decision support method to guide the choice of dispatching towards an optimal solution reconciling both economic performance and reduction of environmental footprint. It aims to guarantee more effective economic coordination, while limiting financial losses related to fuel use and environmental externalities. This consists of determining a solution close to the ideal point. The key steps of this algorithm are as follows:

Algorithm 3: Best Compromise Search Algorithm

Input: χ_E set of obtained Pareto optimal solutions.

Output: best compromise

Compute $f_1^{am} = \max(F_1) - \min(F_1)$;

Compute $f_2^{am} = \max(F_2) - \min(F_2)$;

Compute $F_{\text{nor}} = [\frac{F_1}{f_1\text{am}}; \frac{F_2}{f_2\text{am}}]$;

Compute $\text{I}_{\text{nor}} = \left[\min(F_{1\text{nor}}); \min(F_{2\text{nor}})\right]$;

for *i=1,2,$\cdots$,card(χ_E)* **do**

$$d_i = \sqrt{\sum_{k=1}^{2} \left(F^i_{\text{nor,k}} - \text{I}_{\text{nor,k}}\right)^2}$$

$\overline{d} = \min_{i=1,\cdots,card(\chi_E)} d_i$;

Compute $x_c^* = \arg\left\{F(x) : \overline{d} = min \mid F_{nor}(x) - I_{nor} \mid x \in \chi\right\}$;

return x_c^*;

By applying this decision support method, we obtain the following optimal solutions: $x_1 = 3.8MW$; $x_3 = 6.25MW$; $x_4 = 1.3705MW$; $x_5 = 4.4471MW$; $x_7 = 5.7372MW$; $x_8 = 16.235MW$. The production costs are: $f_1 = 11,252,084.68$ FCFA/h and $f_2 = 16,681.56$ kg/h. These results can serve to guide decisions regarding the operating setpoints of the Kossodo-East network. Finally, we note that units G4, G5, G7 and G8 do not need to be operated at their maximum power to meet the considered demand.

4. Conclusion

In this article, we proposed a multiobjective model to solve the electrical energy dispatching problem in Kossodo East, taking into account both production cost and environmental impact under certain assumptions. The performed simulations highlighted the existence of a set of efficient solutions (Pareto front) for the proposed model. Given the extremely high efficient solutions, a decision support method is then presented to guide the dispatching of all thermal sources. We recommend that SONABEL reconcile economic and environmental considerations to ensure effective coordination of the dispatching of the thermal plant fleet supplying the Kossodo East network, with the aim of optimizing both economic and environmental efficiency. For future work, the modeling will be enriched by integrating solar energy and network maintenance costs. The introduction of optimal power flow constraints will extend the analysis to larger networks, before applying the proposed approach to other electrical systems in the country to test its robustness.

Authors' Contributions. All authors have read and approved the final version of the manuscript. The authors contributed equally to this work.

Conflicts of Interest. The authors declare that there are no conflicts of interest regarding the publication of this paper.

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