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STRUCTURE OF IDEALS IN BF-ALGEBRAS

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Abstract. This paper investigates the structure of prime, maximal, irreducible, and minimal ideals in BF-algebras. Relationships among these classes of ideals are explored, and conditions under which an ideal, especially the zero ideal, becomes prime, irreducible, or maximal are established. Furthermore, prime ideals are characterized via intersections of generated ideals.

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1. Introduction

Algebraic systems motivated by non-classical logic have been widely studied for their structural and logical significance. In this direction, Imai and Iseki [4] introduced BCK- and BCI-algebras as abstract models for implication-like operations. Later, Neggers and Kim [5] proposed B-algebras as a generalization of these systems by relaxing certain axioms.

As a further extension, Walendziak [6] introduced BF-algebras, providing a broader framework for studying algebraic properties related to ideals. Recent investigations on BF-algebras have focused on structural properties, classifications of finite algebras, and the behavior of ideals.

The theory of ideals is central to understanding the internal structure of algebraic systems. In particular, maximal, prime, irreducible, and minimal ideals describe important hierarchical properties. While these notions have been extensively studied in BCK-, BCI-, and B-algebras [1,2], their relationships in BF-algebras have not yet been systematically examined.

In this paper, we investigate these classes of ideals in BF-algebras. We establish inclusion implications among maximal, prime, irreducible, and minimal ideals, provide counterexamples showing that the converses fail in general, and characterize prime ideals via intersections of generated ideals.

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Preliminaries

Definition 1. [6] *(BF-algebra) A BF-algebra is an algebra $(X;*,0)$ of type (2,0) satisfying the following axioms:*

(BF1) $x * x = 0$,

(BF2) $x * 0 = x$,

(BF3) $0 * (x * y) = y * x$ *for all* $x, y \in X$.

Example 1. *Let $X = (\{0, a, b, c\}, *, 0)$ be a BF-algebra, where the binary operation $*$ is defined by*

$*$	0	a	b	c
0	0	a	b	c
a	a	0	a	a
b	b	a	0	b
c	c	a	b	0

Example 2. *Let $Y = \{0, a, b, c\}$ be a BF-algebra, where the binary operation $*$ is defined by*

$*$	0	a	b	c
0	0	a	b	c
a	a	0	0	0
b	b	0	0	0
c	c	0	0	0

Example 3. *Let $Z = \{0, a, b, c\}$ be a BF-algebra, where the binary operation $*$ is defined by*

$*$	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

Definition 2. [6] *A subset I of X is called an **ideal** of X if*

(I_1) $0 \in I$,

(I_2) $x * y \in I$ *and* $y \in I$ *imply* $x \in I$ *for any* $x, y \in X$.

*An ideal I of X is **normal** if for any $x, y, z \in X$, $x * y \in I$ implies $(z * x) * (z * y) \in I$.*

*An ideal I of X is said to be **proper** if $I \neq X$.*

Remark 1. [6] *$\{0\}$ and X are ideals of BF-algebra X. Moreover, X is normal, but $\{0\}$ is not normal in general.*

Definition 3. *Let X be a BF-algebra and let $A \subseteq X$. The* ***ideal generated by*** *A, denoted by $\langle A \rangle$, is the least ideal of X containing A, given by*

$$\langle A \rangle = \bigcap \{I \subseteq X \mid I \text{ is an ideal of } X \text{ and } A \subseteq I\}.$$

This construction is standard in ideal theory and aligns with the approach used in BF-algebras [6].

2. Main Results

This section investigates several special classes of ideals in a BF-algebra and examines the relationships among them. In particular, we study maximal, prime, irreducible, and minimal ideals and describe how these classes are connected through inclusion and structural properties. Furthermore, characterizations of prime ideals are obtained using generated ideals and lattice-theoretic concepts. These results provide deeper insight into the internal structure of BF-algebras and motivate the introduction of finite $\cap$-structures.

Definition 4. *Let X be a BF-algebra. A proper ideal M of X is called a* ***maximal ideal*** *of X if $\langle M \cup \{x\} \rangle = X$ for every $x \in X \setminus M$, where $\langle M \cup \{x\} \rangle$ denotes the ideal generated by $M \cup \{x\}$. Equivalently, M is maximal if and only if for any ideal A of X,*

$$M \subseteq A \subseteq X \Rightarrow A = M \text{ or } A = X.$$

Example 4. Consider the BF-algebra $X = \{0, a, b, c\}$ in Example 1. The family of ideals of X is $\{\{0\}, \{0, a\}, \{0, a, b\}, X\}$. We show that $M = \{0, a, b\}$ is a maximal ideal of X. Observe that the only ideal of X properly containing M is X itself. Indeed, from the list of ideals, we have $\{0\} \subset \{0, a\} \subset \{0, a, b\} \subset X$, and there is no ideal strictly between $\{0, a, b\}$ and X. Equivalently, for the element $c \in X \setminus M$, we obtain $\langle M \cup \{c\} \rangle = \langle \{0, a, b\} \cup \{c\} \rangle = \{0, a, b, c\} = X$. Hence, M is not properly contained in any proper ideal of X, and therefore M is a maximal ideal of X. Moreover, the ideals $\{0\}$ and $\{0, a\}$ are not maximal since $\{0\} \subset \{0, a\} \subset \{0, a, b\}$ and $\{0, a\} \subset \{0, a, b\} \subset X$.

Definition 5. *A proper ideal I of BF-algebra X is called* ***irreducible ideal*** *of X if $A \cap B = I$ implies $A = I$ or $B = I$, for any ideals A and B of X.*

Example 5. Consider the BF-algebra X given in Example 1. From Example 4, the ideals of X are $\{0\}, \{0, a\}, \{0, a, b\}, X$. Let $I = \{0, a\}$. Suppose that A and B are ideals of X such that $A \cap B = I$. Since $I \subseteq A$ and $I \subseteq B$, and the ideals of X form the chain $\{0\} \subset \{0, a\} \subset \{0, a, b\} \subset X$, the only ideals containing I are $\{0, a\}$, $\{0, a, b\}$, and X. If both A and B strictly contain I, then $A, B \in \{\{0, a, b\}, X\}$. In each case, $\{0, a, b\} \cap \{0, a, b\} = \{0, a, b\} \neq I$, $\{0, a, b\} \cap X = \{0, a, b\} \neq I$, $X \cap X = X \neq I$. Thus, it is impossible for $A \cap B = I$ unless at least one of A or B equals I. Therefore, $I = \{0, a\}$ is an irreducible ideal of X. Similarly, the ideal $\{0, a, b\}$ is also irreducible.

Example 6. Let Y be the BF-algebra given in Example 2 whose family of ideals is $\{\{0\}, Y\}$. Consider the ideal $I = \{0\}$. To show irreducibility, suppose that ideals A and B of Y satisfy $A \cap B = I$. Since the only ideals containing I are I and Y, we must have $A, B \in \{I, Y\}$. If $A = Y$ and $B = Y$, then $A \cap B = Y \neq I$, which is impossible. Hence at least one of A or B equals I. Thus I is irreducible. Also, I is maximal since the only ideal strictly containing I is Y. Hence there is no ideal between I and Y. Therefore, in this BF-algebra the zero ideal is irreducible and maximal.

Theorem 1. *Every maximal ideal of a BF-algebra is irreducible.*

Proof. Let M be a maximal proper ideal of X, and suppose A and B are ideals of X such that $A \cap B = M$. Since $M = A \cap B$, it follows $M \subseteq A$ and $M \subseteq B$. By maximality of M, it follows that either $A = M$ or $A = X$, and either $B = M$ or $B = X$. If $A = X$ and $B = X$, then $A \cap B = X \neq M$, a contradiction. Thus $A = M$ or $B = M$, proving irreducibility. ☐

The converse need not be true, as the following example shows.

Example 7. Consider the BF-algebra X given in Example 1. From Example 4, the family of ideals of X is $\{\{0\}, \{0, a\}, \{0, a, b\}, X\}$. Let $I = \{0, a\}$. Then by Example 5, I is irreducible. However, I is not maximal since $\{0, a\} \subsetneq \{0, a, b\} \subsetneq X$. Thus there exists an ideal strictly between I and X, so I fails the maximality condition.

Definition 6. *A proper ideal I of BF-algebra X is called **prime** if for any ideals A and B of X, $A \cap B \subseteq I$ implies $A \subseteq I$ or $B \subseteq I$.*

Example 8. Consider the BF-algebra X given in Example 1, whose family of ideals is $\{\{0\}, \{0, a\}, \{0, a, b\}, X\}$. Let $I = \{0, a\}$. Suppose that A and B are ideals of X such that $A \cap B \subseteq I$. From the chain of ideals $\{0\} \subset \{0, a\} \subset \{0, a, b\} \subset X$, the only ideals not contained in I are $\{0, a, b\}$ and X. If $A \not\subseteq I$ and $B \not\subseteq I$, then $A, B \in \{\{0, a, b\}, X\}$. But in each case we obtain $\{0, a, b\} \cap \{0, a, b\} = \{0, a, b\} \not\subseteq I$, $\{0, a, b\} \cap X = \{0, a, b\} \not\subseteq I$, $X \cap X = X \not\subseteq I$. This contradicts the assumption that $A \cap B \subseteq I$. Hence at least one of A or B must be contained in I. Therefore, by definition, $I = \{0, a\}$ is a prime ideal of X.

Example 9. Consider the BF-algebra Y given in Example 2 whose family of ideals is $\{\{0\}, Y\}$. Let $I = \{0\}$. Suppose that A and B are ideals of Y such that $A \cap B \subseteq I$. If both A and B are not contained in I, then $A = B = Y$. But then $A \cap B = Y \not\subseteq I$, a contradiction. Hence at least one of A or B is contained in I. Therefore, I is a prime ideal.

Example 10. Let Z be the BF-algebra given in Example 3, whose family of ideals is $\{\{0\}, \{0, a\}, \{0, b\}, \{0, c\}, Z\}$. Consider the ideal $I = \{0\}$. Let $A = \{0, a\}$ and $B = \{0, b\}$. Then $A \cap B = \{0\} = I$, so $A \cap B \subseteq I$. However, $A \not\subseteq I$ and $B \not\subseteq I$. This contradicts the defining property of a prime ideal. Hence, $I = \{0\}$ is not a prime ideal of Z.

Remark 2. *As seen in Example 9 and 10, the zero ideal of a BF-algebra need not be prime.*

Theorem 2. *Every maximal ideal of a BF-algebra is prime.*

Proof. Let M be a maximal ideal of X. Then $M \neq X$. Suppose A and B are ideals such that $A \cap B \subseteq M$. Suppose, for contradiction, that $A \nsubseteq M$ and $B \nsubseteq M$. Then there exist elements $a \in A \backslash M$ and $b \in B \backslash M$. By maximality of M, we have $\langle M \cup \{a\} \rangle = X$ and $\langle M \cup \{b\} \rangle = X$. Since $a \notin M$ and M is maximal, the ideal generated by $M \cup \{a\} = X$. Because A is an ideal containing both M and a, we obtain $A = X$. Similarly, $B = X$. Hence $A \cap B = X \nsubseteq M$, a contradiction. Therefore, $A \subseteq M$ or $B \subseteq M$, and so M is prime. □

The converse is not true, see the following example.

Example 11. Consider the BF-algebra X given in Example 1. From Example 4, the family of ideals of X is $\{\{0\}, \{0, a\}, \{0, a, b\}, X\}$. Let $I = \{0, a\}$. By Example 8, I is a prime ideal. However, I is not maximal since $\{0, a\} \subsetneq \{0, a, b\} \subsetneq X$. Thus, a prime ideal of a BF-algebra need not be maximal.

Remark 3. *By set inclusion, every prime ideal of a BF-algebra is irreducible.*

Example 12. Consider the BF-algebra Z given in Example 3. The family of ideals of Z is $\{\{0\}, \{0, a\}, \{0, b\}, \{0, c\}, Z\}$. Let $J = \{0, a\}$. To verify irreducibility, suppose A and B are ideals of Z such that $A \cap B = J$. Observe that the only ideals containing J are $\{0, a\}$ and Z. Now examining all the possible intersection, $\{0, a\} \cap \{0, a\} = \{0, a\} = J$, $\{0, a\} \cap Z = \{0, a\} = J$, and $Z \cap Z = Z \neq J$. Hence, whenever $A \cap B = J$, at least one of A or B must equal J. Thus, J is irreducible. Now, take $A = \{0, b\}$ and $B = \{0, c\}$. Then $A \cap B = \{0\} \subseteq J$. However, $A \nsubseteq J$ and $B \nsubseteq J$. Therefore, J is not prime ideal.

Example 13. Consider the BF-algebra Z in Example 3. The family of ideals of Z is $\{\{0\}, \{0, a\}, \{0, b\}, \{0, c\}, Z\}$. Let $I = \{0\}$. Observe that $\{0, b\} \cap \{0, c\} = \{0\} = I$, but neither $\{0, b\}$ nor $\{0, c\}$ is contained in I. Hence I is not irreducible. Now, $A \cap B = \{0\} \subseteq I$ but $A \nsubseteq I$ and $B \nsubseteq I$. Therefore I is not a prime ideal.

Remark 4. *As seen in Example 13, the zero ideal of a BF-algebra need not be irreducible and prime.*

Proposition 1. *If the zero ideal $\{0\}$ is irreducible in a BF-algebra X, then $\{0\}$ is a prime ideal.*

Proof. Let A and B be ideals of X such that $A \cap B \subseteq \{0\}$. Then $A \cap B = \{0\}$. Since $\{0\}$ is irreducible, $A = \{0\}$ or $B = \{0\}$. In either case, it follows that $A \subseteq \{0\}$ or $B \subseteq \{0\}$. Hence, by definition of a prime ideal, $\{0\}$ is prime. □

Theorem 3. *Let I be an ideal of a BF-algebra X. Then I is a prime ideal of X if and only if $\langle x \rangle \cap \langle y \rangle \subseteq I$ implies $x \in I$ or $y \in I$, for any $x, y \in X$.*

Proof. ($\Rightarrow$) Assume I is a prime ideal of X. By Definition 6, for any ideals $A, B \subseteq X$, $A \cap B \subseteq I \implies A \subseteq I$ or $B \subseteq I$. Let $x, y \in X$ and suppose that $\langle x\rangle \cap \langle y\rangle \subseteq I$. Since $\langle x\rangle$ and $\langle y\rangle$ are ideals and I is prime, it follows that $\langle x\rangle \subseteq I$ or $\langle y\rangle \subseteq I$. Since $x \in \langle x\rangle$, the inclusion $\langle x\rangle \subseteq I$ implies $x \in I$. Hence, if $\langle x\rangle \subseteq I$, then $x \in I$. Similarly, $\langle y\rangle \subseteq I$ implies $y \in I$. Hence, $\langle x\rangle \cap \langle y\rangle \subseteq I$ implies $x \in I$ or $y \in I$.
($\Leftarrow$) Assume that for all $x, y \in X$, $\langle x\rangle \cap \langle y\rangle \subseteq I \implies x \in I$ or $y \in I$. Let A and B be ideals of X such that $A \cap B \subseteq I$. Suppose that $A \nsubseteq I$. Then there exists $x \in A \setminus I$. Let $y \in B$ be arbitrary. Since A is an ideal and $x \in A$, it follows that $\langle x\rangle \subseteq A$. Similarly, $\langle y\rangle \subseteq B$. Hence, $\langle x\rangle \cap \langle y\rangle \subseteq A \cap B \subseteq I$. Since $x \notin I$, it follows that $y \in I$. Since $y \in B$ was arbitrary, hence $B \subseteq I$. Therefore I is a prime ideal of X. ☐

Corollary 1. Let I be an ideal of a BF-algebra X. Then I is a prime ideal of X if and only if there exist $x, y \in X$ with $x \notin I$ and $y \notin I \implies \langle x\rangle \cap \langle y\rangle \nsubseteq I$.

Proof. The result follows immediately from Theorem 3 by taking the contrapositive. ☐

After discussing prime ideals, we now turn our attention to another important class of ideals that describe the lower structural layer of a BF-algebra, namely minimal ideals. These ideals play a significant role in understanding the internal structure of the algebra, as they represent the smallest non-zero ideals with respect to inclusion.

Definition 7. *Let X be a BF-algebra. A non-zero ideal M of X is called a* ***minimal ideal*** *if whenever I is an ideal of X such that $I \subseteq M$, then $I = \{0\}$ or $I = M$.*

Example 14. Let $X = (X; *, 0)$ be the BF-algebra given in Example 1, where the family of ideals is $\{\{0\}, \{0, a\}, \{0, a, b\}, X\}$. Since $\{0, a\}$ is a nonzero ideal and the only ideal properly contained in $\{0, a\}$ is $\{0\}$, it follows from Definition 7 that $\{0, a\}$ is a minimal ideal of X.

Remark 5. *The zero ideal $\{0\}$ is not a minimal ideal since minimal ideals are, by definition, required to be nonzero.*

Proposition 2. *Every minimal ideal of a BF-algebra is irreducible.*

Proof. Let I be a minimal ideal of X and suppose that $I = A \cap B$ for some ideals A and B of X. Then $I \subseteq A$ and $I \subseteq B$. If both A and B properly contain I, then $I \subsetneq A$ and $I \subsetneq B$, which contradicts the minimality of I. Hence, either $A = I$ or $B = I$, and therefore I is irreducible. ☐

The converse is not true in general.

Example 15. Consider the BF-algebra X in Example 1 whose family of ideals is $\{\{0\}, \{0, a\}, \{0, a, b\}, X\}$. Let $J = \{0, a, b\}$. By the same argument as in Example 5, J is irreducible. However, J is not minimal. Since $\{0\} \subset \{0, a\} \subset J$, hence J properly contains a nonzero ideal.

Before presenting further results, we summarize the relationships among the special classes of ideals discussed above. The following remark illustrates the hierarchy and distinctions among maximal, prime, irreducible, and minimal ideals in a BF-algebra.

Remark 6 (Ideal-theoretic relationships in a BF-algebra)**.** *Let X be a BF-algebra. The following implications hold among special classes of ideals:*

$$\textit{Maximal} \implies \textit{Prime} \implies \textit{Irreducible}.$$

Moreover, every minimal ideal is irreducible. In general, none of the converses of these implications holds.

The preceding discussion focused on the hierarchical relationships among special classes of ideals in a BF-algebra. To further understand how generated ideals interact, we now introduce the concept of finite $\cap$-structures. This notion describes how intersections of generated ideals behave with respect to subsets of elements, offering additional insight into the internal structure of the algebra.

Definition 8. *A nonempty subset F of X is called a **finite $\cap$-structure** if $(\langle x\rangle \cap \langle y\rangle) \cap F \neq \varnothing$, for all $x, y \in F$. Moreover, X is called a* finite $\cap$-structure *if the set of nonzero elements $X \setminus \{0\}$ forms a finite $\cap$-structure.*

Example 16. Let $X = (X; *, 0)$ be the BF-algebra given in Example 1, where $X = \{0, a, b, c\}$ with the given Cayley table. Let $F = X \setminus \{0\} = \{a, b, c\}$. The generated ideals of X are $\langle a\rangle = \{0, a\}$, $\langle b\rangle = \{0, b\}$, $\langle c\rangle = X$. Consider the elements a and b. Then

$$\langle a\rangle \cap \langle b\rangle = \{0\}, (\langle a\rangle \cap \langle b\rangle) \cap F = \varnothing.$$

Since the condition fails for the pair $a, b \in F$, we conclude that F is not a finite $\cap$-structure.

Remark 7. *Not every BF-algebra has $X \setminus \{0\}$ as a finite $\cap$-structure.*

Example 17. Consider the BF-algebra $Y = (Y; *, 0)$ defined in Example 2 with the generated ideals are $\langle a\rangle = \langle b\rangle = \{0, a, b\} = Y$. We check the finite $\cap$-structure condition by considering all pairs of elements in F. Let $F = Y \setminus \{0\} = \{a, b\}$.

For $x = a, y = a$: $(\langle a\rangle \cap \langle a\rangle) \cap F = \{a, b\} \neq \varnothing$

For $x = b, y = b$: $(\langle b\rangle \cap \langle b\rangle) \cap F = \{a, b\} \neq \varnothing$

For $x = a, y = b$: $(\langle a\rangle \cap \langle b\rangle) \cap F = \{a, b\} \neq \varnothing$.

Since all pairs $x, y \in F$ satisfy the condition, F is a finite $\cap$-structure. Consequently, Y is a finite $\cap$-structure.

Theorem 4. *Let X be a BF-algebra. An ideal I of X is prime if and only if $F = X \setminus I$ is a finite $\cap$-structure.*

Proof: Suppose I is a prime ideal of X. Let $F = X \setminus I$ and take any $x, y \in F$. Then $x \notin I$ and $y \notin I$. Consider the generated ideals $\langle x\rangle$ and $\langle y\rangle$. Assume, for contradiction, that $(\langle x\rangle \cap \langle y\rangle) \cap F = \varnothing$. Then

$\langle x\rangle \cap \langle y\rangle \subseteq X \setminus F = I$. Since I is prime, it follows that $\langle x\rangle \subseteq I$ or $\langle y\rangle \subseteq I$. However, $x \in \langle x\rangle$ and $x \notin I$, so $\langle x\rangle \nsubseteq I$. Similarly, $\langle y\rangle \nsubseteq I$ which is a contradiction. Hence, $(\langle x\rangle \cap \langle y\rangle) \cap F \neq \varnothing$ for all $x, y \in F$. Thus F is a finite $\cap$-structure. Conversely, suppose $F = X \setminus I$ is a finite $\cap$-structure. Let $x, y \in X$ such that $\langle x\rangle \cap \langle y\rangle \subseteq I$. If $x \notin I$ and $y \notin I$, then $x, y \in F$. By the finite $\cap$-structure property, $(\langle x\rangle \cap \langle y\rangle) \cap F \neq \varnothing$, which contradicts the assumption that $\langle x\rangle \cap \langle y\rangle \subseteq I$. Therefore, $x \in I$ or $y \in I$. By Theorem 3, I is a prime ideal of X.

Proposition 3. *Let M be maximal ideal of BF-algebra X. Then $F = X \setminus M$ is a finite $\cap$-structure.*

Proof. Since every maximal ideal of BF-algebra is prime, it follows from Theorem 4 that $X \setminus M$ forms a finite $\cap$-structure. □

Example 18. Let $X = \{0, a, b, c\}$ be the BF-algebra given in Example 1. Recall that the family of ideals of X is $\{\{0\}, \{0, a\}, \{0, a, b\}, X\}$. From Example 4, $M = \{0, a, b\}$ is a maximal ideal of X. Let $F = X \setminus M = \{c\}$. Since F contains only one element, we verify the finite $\cap$-structure condition for $x = y = c$. The generated ideal is $\langle c\rangle = X$. Then

$$(\langle c\rangle \cap \langle c\rangle) \cap F = X \cap F = F \neq \varnothing.$$

Hence, F is a finite $\cap$-structure. Therefore, the complement of the maximal ideal M forms a finite $\cap$-structure.

The preceding results highlight that the finite $\cap$-structure property naturally arises from strong ideal conditions such as primality and maximality. However, weaker notions of ideals do not necessarily preserve this property.

Remark 8. *If I is an irreducible ideal, then $X \setminus I$ need not be a finite $\cap$-structure, since irreducible ideals are not necessarily prime.*

The results obtained in this section establish several structural relationships among maximal, prime, irreducible, and minimal ideals in BF-algebras. In particular, prime ideals are characterized through generated ideals and finite $\cap$-structures, providing an alternative perspective for identifying and analyzing prime ideals. These findings highlight how lattice-theoretic properties of ideals contribute to understanding the internal algebraic structure of BF-algebras.

3. Conclusion

This paper examined special classes of ideals in BF-algebras, focusing on maximal, prime, and irreducible ideals and their structural relationships. We established that every maximal ideal is both prime and irreducible, and every prime ideal is irreducible, while the converses do not hold in general.

In addition, prime ideals are characterized using intersections of generated ideals, and the finite intersection property of their complements is established.

These results provide insight into the hierarchy of ideals and the behavior of intersections within BF-algebras, highlighting structural features of these algebraic systems. Future work may explore how these properties extend to broader classes of algebras or connect with other ideal-theoretic decompositions.

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