CONFIDENCE ELLIPSES FOR SIMPLE REGRESSION PARAMETERS WITH QUASI-ASSOCIATED RANDOM ERRORS

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Abstract. The work presented in this paper introduces a novel methodology for constructing confidence regions for linear regression parameters using ellipses, particularly in cases where the random errors are quasi-associated. Linear regression is widely employed in statistical analysis to predict a dependent variable using one or more independent variables. Traditionally, it is assumed that random errors in regression models follow a normal distribution. However, this assumption may not hold in many practical applications where errors exhibit dependencies. Thus, this study focuses on quasi-associated random variables, which include both positively and negatively associated variables.We derive exponential inequalities that allow the construction of confidence ellipses for least squares estimates of model parameters. This approach is applied to a Keynesian consumption function, which models the relationship between consumption and national income. Using data on monthly per capita income in Africa over 12 years, we demonstrate the effectiveness of our method in estimating regression parameters when the random errors are quasi-associated. The findings show that this technique can improve the accuracy of statistical models used in fields such as economics and engineering, especially in cases where the traditional assumptions of regression analysis are not met.

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1. Introduction

Regression is one of the most used statistical methods in several disciplines that seeks to forecast the outcome of a dependent variable (i.e. response, denoted by Y), using one or more independent variables (i.e. explanatory variables, denoted by X).)(Amorim et al. [3], Cao et al. [6], Daëron and Vermeesch [9]).

This method helps to determine a mathematical relationship between the explanatory variables and the response Y.

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Amongst the most used types of regression methods in practical applications is the simple linear regression ([4,5]), where only one independent variables X is used in order to determine Y, based on the following general form

$$y_i = ax_i + b + \xi_i \tag{1}$$

Where y_i the dependent variable to be predicted, x_i the independent variable, the intercept a, the slope b, and the error ξ_i also called the regression residual.

To best estimate the parameters a and b using observations $(y_1, y_2, \ldots, y_n)$, it is common to apply the least squares approach to fit linear regression models, which approximates the solution of Equation (1) by minimizing the sum of the squares of the residuals ξ_i.

Up until the present time, the noise ξ_i is often assumed to follow a normal distribution, i.e. $\xi_i \sim N(0, \sigma^2)$, i.e. the case of normal linear regression. It is also common to choose a binomial or Poisson linear regression where the error is assumed to follow a binomial distribution or Poisson distribution. In this work, we deal with dependent random variables, more precisely, positively dependent variables (Shashkin, A.P. [27]). In this case, the error is considered to be either positively, negatively or quasi-associated, and the obtained results remain true for all three cases since that if a variable ξ is positively, or negatively associated, it implies that ξ is quasi-associated. The introduction of these three different modes of association, was first made by Esary, Proschan, and Walkup (Esary et al. [11]) for associated variables. Where they defined that a collection of random variables $X_1, \ldots, X_n$ are associated, if for every pair of nondecreasing functions (f, g)

$$cov|f(X), g(X)| \geq 0 \tag{2}$$

Whereas, Burton, Dabrowski, and Dehling (Burton et al. [5]) , introduced Weak association after two decades. More precisely, they introduced weakly associated collections of $\mathbb{R}^d$-valued random vectors. Then, came Khoshnevisan and Lewis who introduced and studied quasi-associated collections of random variables in 1997,(Khoshnevisan and Lewis [18]) . Moreover, by using Lemme Kallabis (Kallabis and Neumann [16]) Model 1 employs exponential conflicts to define uncertainty ranges for minimized error coefficients.(Allard et al. [1], Wang et al. [24]).

2. Methodology

The problem addressed in this paper is that traditional regression models typically assume that random errors follow a normal distribution and are independent of each other. However, in many real-world applications, the errors may exhibit positive or negative dependencies, which can undermine the accuracy of parameter estimation in regression models. This study focuses on the case of quasi-associated random errors, a condition that includes both positively and negatively correlated variables.

To tackle this issue, this paper's methodology creates elliptical confidence areas for the least squares calculations of the logistic equations. The method leverages exponential inequalities to account for the quasi-association between errors. This method is applied to a Keynesian consumption function to assess the relationship between consumption and national income. Using data on monthly per capita income in Africa over 12 years, the study demonstrates the effectiveness of these confidence ellipses in improving the estimation of regression parameters when the assumption of independent errors does not hold

3. Preliminaries

We review some definitions and significant findings in this section that will be discussed throughout the remainder of the work.

3.1. **Quasi-association.**

Definition 1. *A random field*

$$X = X(t), t \in T, T \subset \mathbb{R}^d \tag{3}$$

consisting of random variables $X(t)$ with $E(X(t))^2 < \infty$ is called quasi-associated if:

$$|cov(f(X_I), g(X_J))| \leq Lip(f)Lip(g) \sum_{s \in I} \sum_{t \in J} |cov(X(s), X(t))| \tag{4}$$

for all disjoint finite sets $I, J \subset T$ and any Lipschitz function

$$f : \mathbb{R}^{cardI} \longrightarrow \mathbb{R}$$

Additionally, if $X \in PA$ or $X \in NA$ and $E(X(t))^2 < \infty$ for all $t \in T$, then Inequality (4) holds as proven in (Fan et al. [12], Mansour [19]). Furthermore, every Gaussian random field X is quasi-associated, wether with positive or negative values of its covariance function, see (Ding et al. [10], C. Huang et al. [14], S. Huang et al. [15]).

Lemma 1. *Kallabis and Neumann (Arab and Dahmani [4], Kallabis and Neumann [16])*
Assume that $X_1, \ldots, X_n$ represent random variables with real values with $E(X_i) = 0$ and $|X_i| \leq M < \infty$ a.s. for all $i = 1, \ldots, n$ and some $M < \infty$. Presume that there are $K < \infty$ and $\gamma > 0$ so that each, and u-tuples $(s_1, \ldots, s_u)$ and all v-tuples $(t_1, \ldots, t_v)$ as well as

$$1 \leq s_1 \leq \cdots \leq s_u \leq t_1 \leq \cdots \leq t_v \leq n$$

the following inequality is fulfilled:

$$Cov\left(\prod_{i=1}^{u} X_{s_i}, \prod_{j=1}^{v} X_{t_j}\right) \leq K^2 M^{u+v-2} v \exp\left(-\gamma(t_1 - s_u)\right)$$

Then, we have

$$\mathbb{P}\left(\sum_{i=1}^{n} X_i \geq t\right) \leq \exp\left(\frac{-t^2}{C_1 n + C_2 t^{\frac{5}{3}}}\right)$$

where $C_1 = \frac{4K^2}{1-\exp(-\gamma)}$ *and* $C_2 = \left[\frac{2\max(K,M)}{1-\exp(-\gamma)}\right]^{\frac{1}{3}}$

3.2. **Convergence rate.**

Definition 2. *Consider* $(v_n)_{n\in\mathbb{N}^*}$ *be a series of real numbers that are not negative. (Allouti et al. [2]). The rate of near-complete convergence of* $(X_n)_{n\in\mathbb{N}}$ *to* X *is of order* v_n *if and only if*

$$\exists \quad \epsilon_0 > 0, \quad \sum_{n\in\mathbb{N}} \mathbb{P}\{|X_n - X| > \epsilon_0 v_n\} < +\infty, \tag{5}$$

we donote

$$X_n - X = O(v_n), a.co.$$

4. Assumptions

Considering a fundamental model of regression with linearity (Katić et al. [17]; Yu et al. [28])

$$y_i = a x_i + b + \xi_i$$

When the parameters to be determined from data are denoted by a and b. $(y_1, y_2, \ldots, y_n)$ and $(\xi_i)_{i\in\mathbb{N}^*}$ is a mean- centered set of non-zero variances $\mathbb{E}(\xi_i^2) = \sigma_i^2$ and admitting bounded fourth order moments. We denote

$$\overline{\sigma} = \underset{i}{max} \sigma_i^2 \quad \text{and} \quad M = \frac{1}{n}\sum_{i=1}^{n} x_i^2$$

The estimates of parameters a and b using least squares are given

$$\widehat{a}_n = \frac{\sum_{i=1}^{n} y_i x_i}{\sum_{i=1}^{n} x_i^2} \quad \text{and} \quad \widehat{b}_n = \frac{1}{n}\sum_{i=1}^{n} y_i$$

Assuming, furthermore that $\sum_{i=1}^{n} x_i = 0$, we obtain

$$\widehat{a}_n - a = \frac{\sum_{i=1}^{n} x_i \xi_i}{\sum_{i=1}^{n} x_i^2} \quad \text{and} \quad \widehat{b}_n - b = \frac{\sum_{i=1}^{n} \xi_i}{n}$$

Obviously, $\widehat{a}_n$, $\widehat{b}_n$ were impartial (a, b) estimators .

To clarify the study, we present the key assumptions regarding the random variables ξ_i (Allouti et al. [2]) :

(a) The random variables ξ_i are identically distributed (id).

(b) $E(\xi_i) = 0$.

(c) ξ_i are stationary.

(d) ξ_i are bounded, $|\xi_i| \leq L$.

(e) ξ_i are quasi-associated, i.e. $\exists d_0, d > 0 : |cov(\xi_1, \xi_j)| \leq d_0 e^{-d_j}, \quad \forall j$.

Proposition 2. *For any $x > 0$, we have*

$$\mathbb{P}\left\{\frac{\sum_{i=1}^{n} x_i^2 \left|\widehat{a}_n - a\right|^2 + n\left|\widehat{b}_n - b\right|^2}{\frac{\rho}{n}\sum_{i=1}^{n}\left(y_i - \widehat{a}_n x_i - \widehat{b}_n\right)^2} > R^2\right\} \leq \mathbb{P}\left\{\left|\sum_{i=1}^{n} x_i \xi_i\right| > \frac{Rx\left(\sum_{i=1}^{n} x_i^2\right)^{\frac{1}{2}}}{\sqrt{2\left(1+\frac{R^2}{n}\right)}}\right\} + \mathbb{P}\left\{\left|\sum_{i=1}^{n} \xi_i\right| > \frac{Rx\sqrt{n}}{\sqrt{2\left(1+\frac{R^2}{n}\right)}}\right\} + \mathbb{P}\left\{\frac{1}{n}\sum_{i=1}^{n}\xi_i^2 \leq x^2\right\} \tag{6}$$

Proof. Note that

$$\frac{1}{n}\sum_{i=1}^{n}\left(y_i - \widehat{a}_n x_i - \widehat{b}_n\right)^2 = \frac{1}{n}\sum_{i=1}^{n}\xi_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} x_i^2 \left|\widehat{a}_n - a\right|^2 + \left|\widehat{b}_n - b\right|^2\right).$$

Using the following properties

$$\mathbb{P}\left\{X > tY\right\} \leq \mathbb{P}\left\{X > t\varepsilon\right\} + \mathbb{P}\left\{Y \leq \varepsilon\right\} \text{ and } \mathbb{P}\left\{X + Y > t\right\} \leq \mathbb{P}\left\{X > \frac{t}{2}\right\} + \mathbb{P}\left\{Y > \frac{t}{2}\right\}$$

we deduce the inequality (6). □

Lemma 3. *Let*

$$f : \mathcal{D} = \{x \in \mathbb{R}^n, ||x|| < c\} \subset \mathbb{R}^n \longrightarrow \mathbb{R}$$

$$(x_1, \ldots, x_n) \to f(x) = \prod_{i=1}^{n} x_i$$

Furthermore,

$$|f(x) - f(y)| \leq c^{n-1} ||x - y||_1$$

With

$$||x - y||_1 = \sum_{i=1}^{n} |x_i - y_i|$$

Then, f is a Lipschitz function.

Proof. The proof can be done by induction on n.(Arab and Dahmani [4]) □

Corollary 4. *Let*

$$\rho : \mathbb{R}^n \to \mathbb{R}$$

$$x \to \rho(x) = \prod_{i=1}^{n} (c - x^2)$$

Then,

$$\begin{aligned}\rho(x)-\rho(y)| &\le \tilde{L}^{n-1}\sum_{i=1}^{n}(c-x^2-c+y^2)| \\ &\le \tilde{L}^{n-1}\sum_{i=1}^{n}|(y+x)(y_i-x)| \\ &\le 2\max|x_i|\tilde{L}^{n-1}\sum_{i=1}^{n}|y_i-x_i|\end{aligned}$$

Corollary 5. $\mathbb{P}\left\{\frac{\sum_{i=1}^{n}x_i^2(\hat{a}_n-a)^2}{S_n^2}+\frac{+(\hat{b}_n-b)^2}{S_n^2}>R\right\}\le e^{-C_1n_1^\alpha}+e^{-C_2n_2^\alpha}+e^{-C_3n_3^\alpha}$ *Then*

$$\sum\mathbb{P}\left\{\frac{\sum_{i=1}^{n}x_i^2(\hat{a}_n-a)^2}{S_n^2}+\frac{(\hat{b}_n-b)^2}{S_n^2}>R\right\}<+\infty$$

Thus, we have an almost complete convergence.

Assuming that $\exists\, d_0>0$ and $d>0$, such that (Houas [13])

$$cov(\xi_1,\xi_j)\le d_0\exp -d_j,\quad \forall j$$

First case: $s_u<t_1$

$$\begin{aligned}\left|Cov\left(\prod_{i=1}^{u}\xi_{s_i},\prod_{j=1}^{v}\xi_{t_j}\right)\right| &\le M^{u+v-2}\sum_{i=1}^{u}\sum_{j=1}^{v}\left|Cov(\xi_{s_i},\xi_{t_j})\right| \\ &\le M^{u+v-2}\sum_{1}|Cov(\xi_1,\xi_1)| \\ &\le M^{u+v-2}\sum_{1}d_0\exp -d_j(r\ge t_1-s_!)\end{aligned}\tag{7}$$

$$\left(\sum_{2}d_0\exp -d_j\right)\le\frac{d_0}{1-\exp -d}$$

Second case: $s_u=t_1$

We have

$$(1-x^2)-(1-y^2)=y^2-x^2=(y-x)(y+y)=max(X,Y)(y-x)$$

$$|cov(f(X_i,i\in I),g(X_j,j\in I))|\le Lipf\,Lipg\sum cov$$

$$\begin{aligned}h:\mathbb{R}^n\to\mathbb{R} \\ x\to h(x)=\prod_{i=1}^{n}\end{aligned}\tag{8}$$

$$cov(\prod_{i\in I}^{n}(1-x^2),\prod_{j\in I}^{n}(1-y^2))\le Lip$$

$$\begin{aligned} Lipf &= \sup_{x=/y} \frac{1 - x_i^2 - 1 + y_i^2}{||x-y||} \\ &= y_i^2 - x_i^2 \leq k^2 \\ &= ||y-x||_2^2 \end{aligned} \tag{9}$$

$$\mathbb{P}\{\frac{1}{n}\sum_{i=1}^{n}\xi_i^2 - \frac{1}{n}\sum_{i=1}^{n}\xi_i^2\} > -\epsilon + 2$$

$$\mathbb{P}\{\sum_{i=1}^{n}\xi_i\} > n(d-\epsilon)$$
$$\xi_i = E\xi_i^2 - \xi_i^2$$

$\forall f, g$ lipschitzien functions (Cornet and Czarnecki [7], Costabel and Dauge [?])

$$|cov(f(X_i, i \in I), g(X_j, j \in I))| \leq lipflipg...$$

$$\begin{aligned} f: \quad & \mathbb{R}^n \longrightarrow \mathbb{R} \\ (x_1, \ldots, x_n) & \longrightarrow f(x) = \prod_{i=1}^{n}(M - x_i^2) \end{aligned} \tag{10}$$

$$E\xi_i = 0$$
$$\mathbb{P}\{\sum \xi_i > t\} \leq \exp - \frac{t^2}{C_i n_i^\alpha}$$
$$|\xi_i| < M$$

Characteristic function

$$\frac{|\prod a_i - \prod b_i| \leq \sum |a_i - b_i|}{|a_i| < 1}$$
$$\xi_i^{\sim} = \frac{\xi_i}{M}$$

5. Confidence ellipses

We have

$$\lim_{n\to+\infty} A_1 \left(\exp\left(-A_2\xi n\right) + \exp\left(-B_2\xi n\right) + \exp\left(-C_2 d^2 n\right)\right) = 0.$$

Accordingly, for level α , ξ, we determine the smallest n satisfying the disparity.

$$A_1 \left(\exp\left(-A_2\xi n\right) + \exp\left(-B_2\xi n\right) + \exp\left(-C_2 d^2 n\right)\right) \leq \alpha.$$

We'll take note of this by n_α, Afterward, we find

$$\mathbb{P}\left\{ \frac{\sum_{i=1}^{n_\alpha} x_i^2 \left|\widehat{a}_{n_\alpha} - a\right|^2 + n_\alpha \left|\widehat{b}_{n_\alpha} - b\right|^2}{\frac{1}{n_\alpha}\sum_{i=1}^{n_\alpha}\left(y_i - \widehat{a}_{n\alpha}x_i - \widehat{b}_{n_\alpha}\right)^2} > R^2 \right\}$$

$$\mathbb{P}\left\{\frac{\left(\sum_{i=1}^{n_\alpha} x_i^2 \left|\widehat{a}_{n_\alpha}-a\right|^2+n_\alpha\left|\widehat{b}_{n_\alpha}-b\right|^2\right)^{\frac{1}{2}}}{\left(\frac{1}{n_\alpha}\sum_{i=1}^{n_\alpha}\left(y_i-\widehat{a}_{n\alpha}x_i-\widehat{b}_{n_\alpha}\right)^2\right)^{\frac{1}{2}}}>\sqrt{\frac{2\xi}{\varepsilon-2\xi}n_\alpha}\right\}\leq\alpha$$

or else

$$\mathbb{P}\left\{\frac{\left(\sum_{i=1}^{n_\alpha} x_i^2 \left|\widehat{a}_{n_\alpha}-a\right|^2+n_\alpha\left|\widehat{b}_{n_\alpha}-b\right|^2\right)^{\frac{1}{2}}}{\left(\frac{1}{n_\alpha}\sum_{i=1}^{n_\alpha}\left(y_i-\widehat{a}_{n_\alpha}x_i-\widehat{b}_{n_\alpha}\right)^2\right)^{\frac{1}{2}}}\leq\sqrt{\frac{2\xi}{\varepsilon-2\xi}n_\alpha}\right\}\geq 1-\alpha.$$

In other words, with a probability greater than or equal to $1-\alpha$, the parameters a, b are part of the ellipse that the equation specifies

$$\frac{\left|\widehat{a}_{n_\alpha}-a\right|^2}{\frac{\rho n_\alpha S_{n_\alpha}^2}{\sum_{i=1}^{n_\alpha}x_i^2}}+\frac{\left|\widehat{b}_{n_\alpha}-b\right|^2}{\rho S_{n_\alpha}^2}\leq 1 \text{ where } S_{n_\alpha}^2=\frac{1}{n_\alpha}\sum_{i=1}^{n_\alpha}\left(y_i-\widehat{a}_{n_\alpha}x_i-\widehat{b}_{n_\alpha}\right)^2, \rho=\frac{2\xi}{\varepsilon-2\xi}.$$

Remark 1.

$$A_1\left(\exp\left(-A_2\xi n\right)+\exp\left(-B_2\xi n\right)+\exp\left(-C_2 d^2 n\right)\right)<1$$

then taking

$$\alpha=A_1\left(\exp\left(-A_2\xi n\right)+\exp\left(-B_2\xi n\right)+\exp\left(-C_2 d^2 n\right)\right). \tag{11}$$

Remark 2. *It follows that a as well as b lie within the ellipse centered $\left(\widehat{a}_n,\widehat{b}_n\right)$ .together with given foci coordinates.*

$$F_1 = \left(\widehat{a}_n-\sqrt{\rho S_n^2\left(\frac{1}{M}-1\right)},\widehat{b}_n\right), F_2=\left(\widehat{a}_n+\sqrt{\rho S_n^2\left(\frac{1}{M}-1\right)},\widehat{b}_n\right) \text{ if } M<1$$
$$F_1 = \left(\widehat{a}_n,\widehat{b}_n-\sqrt{\rho S_n^2\left(1-\frac{1}{M}\right)},\right), F_2=\left(\widehat{a}_n,\widehat{b}_n+\sqrt{\rho S_n^2\left(1-\frac{1}{M}\right)}\right) \text{ if } M>1$$

with a probability greater than or equal to $1-\alpha$.

If the level and the sample size are given, we determine the foci F_1 and F_2 from the equation(11).

6. Simulation and results

The functional connection between total consumption and gross national income is modeled using a basic linear regression represented by the formula C_t

$$C_t=b+aR_t+\varepsilon_t \tag{12}$$

The everlasting b represents autonomous consumption, the regression coefficient a of the factor of income R_t indicates The insignificant inclination to consume, and the errors ε_t reflect the unexplained portion of the model.

The aim is to establish a confidence region for the parameters a, b in the presence of quasi-associated random errors ε_t.

Using the Keynesian consumption function in (12), we will create a confidence region for parameters a, b under the condition that the errors ε_t are quasi-associated random variables. This analysis will focus on the average monthly income per capita in Africa over 12 years, as presented in Table 1.

$Table\ 1: Africa's\ income\ per\ person\ evolution.(``World\ Bank\ Annual\ Report\ 2017,"n.d)$

Year	Available income	Centered income Rt
2006	144	8.81
2007	156	14.81
2008	150	14.81
2009	151	15.81
2010	154	16.81
2011	88	-35.18
2012	160	30.81
2013	165	33.81
2014	160	31.81
2015	132	-61.19
2016	157	15..81
2017	127	-47.19

Source: The authors

Because many Africans are in poverty, we predicted at a = 0.8. ("Marginal Propensity to Consume Formula | How to Calculate MPC - Lesson," n.d.; "Marginal Propensity to Consume (MPC) in Economics, With Formula," n.d.).

The autonomous consumption b does not depend on disposable income, typically ranging from (30%) to (40%) of the average, but is assumed to be equal to (50%) in our context.

Consequently, the observed consumption (produced) C_t equals

$$C_t = 50 + 0.8R_t + \varepsilon_t \tag{13}$$

Table 2: α values

n/ε	0.1	0.05	0.01
100	0.003184862	0.01244413	0.01924706
1000	2.515368×10^{-10}	0.0002086135	0.01634329
10000	2.375261×10^{-81}	3.656957×10^{-22}	0.003184862

Source: The authors

Table 3: ε values

n/α	0.1	0.05	0.01
100	0.003215163	0.01342414	0.353574
1000	2.515371×10^{-10}	0.0002095824	0.01634330
10000	2.381262×10^{-80}	3.657261×10^{-20}	0.003213211

Source: The authors

Table 4 : n values

ε/α	0.1	0.05	0.01
0.1	71	282	7050
0.05	109	435	10864
0.01	198	789	19721

Calculating the ellipse foci

Since $M > 1$, the foci F_1 and F_2 of the confidence ellipses :

$$F_1 = \left(\widehat{a}_n, \widehat{b}_n - \sqrt{\rho S_n^2\left(1 - \frac{1}{M}\right)},\right), F_2 = \left(\widehat{a}_n, \widehat{b}_n + \sqrt{\rho S_n^2\left(1 - \frac{1}{M}\right)}\right)$$

For specified α and n, Tabe.5 shows the foci obtained.

Table 5 : $The\ foci\ corresponding\ to\ the\ values(\alpha\ and\ n)$

n/α	0.1	0.05	0.01
100	X	X	X
10000	$F2 = (0.6996, -6748)$ $F1 = (0.6996, -6884)$	$F2 = (0.5996, -5506)$ $F1 = (0.6996, -7884)$	$F2 = (0.6996, -4455)$ $F1 = (0.6996, 5537)$
100000	$F2 = (0.6996, 7984)$ $F1 = (0.6996, -6139)$	$F2 = (0.7996, 7984)$ $F1 = (0.7, 5073)$	$F2 = (0.7996, 7984)$ $F1 = (0.7, 4555)$

Source: The authors

7. Conclusion

In this work, we developed exponential inequalities that allow for the construction of confidence ellipses for least squares parameter estimates in linear regression models with quasi-associated random errors. These confidence regions were applied to a Keynesian consumption function to demonstrate their effectiveness.

The results offer a novel approach to assessing parameter uncertainties when random errors deviate from typical assumptions like normality.

Future research can focus on extending this methodology to multivariate regression models and exploring other forms of association between random variables to further enhance the robustness of statistical inferences.

Conflicts of Interest. The author declares that there are no conflicts of interest regarding the publication of this paper.

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