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THE CHAOTIC BEHAVIOR OF FRACTIONAL PARTIAL DIFFERENTIAL EQUATIONS

ALI LATIF ARIF[1,*], HASSAN K. JASSIM[2], NASER R. SWAIN[3,4]

[1]Fuctuly of Midicin, University of Thi-Qar, Thi-Qar, Iraq
[2]Department of Mathematics, Faculty of Education for Pure Sciences, University of Thi-Qar, Thi-Qar, Iraq
[3]Educational Directorate of Thi-Qar, Thi-Qar, Iraq
[4]College of Technical Engineering, National University of Science and Technology, Thi-Qar, Iraq
[*]Corresponding author: alilatif@utq.edu.iq

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Abstract. This paper presents a new three-dimensional continuous autonomous chaotic system with eight terms and three quadratic nonlinearities. The basic dynamical properties of the new system are analyzed by means of equilibrium points, eigenvalue structures, stability and apply it to investigate the local bifurcations of the new system. Some of the basic dynamic behavior of the system is explored further investigation in the Lyapunov exponent.

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1.. Introduction

The science of nonlinear dynamics has sparked many researchers to develop mathematical models that simulate vector fields of nonlinear chaotic physical systems. Nonlinear phenomena arise in all fields of engineering, physics, chemistry, biology, economics, and sociology [1–3].

Chen and Ueta [3] constructed a three-dimensional autonomous differential equation withoonly two quadratic terms, xy and xz. In fact, Chen system has been proved to be dual to the Lorenz system [3,4], many theoretical analysis and numerical simulation results in [5]. The Chen system [5] which takes the form:

$$\begin{aligned} x' &= a(y-x) \\ y' &= (c-a)x - xz + cy \\ z' &= xy - bz \end{aligned} \tag{1.1}$$

where the parameters a, b and c are real parameters.

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The T system has only two quadratic terms, xy and xz. The T system isorelated to the Lorenz and Chen systems [8] being a small generalization of the later one. The T system [6] is:

$$\begin{aligned} x' &= a(y-x) \\ y' &= (c-a)x - axz \\ z' &= xy - bz \end{aligned} \tag{1.2}$$

with parameters a, b and c are real parameters.

Most researchersodeveloped a new systemodepending on one chaotic system like Chenoor T system. The proposedoscheme in this paperobased on merging two chaotic systems Chen chaotic system and T chaotic system. Therefore, will be added two systems in (1.1) and (1.2), thus a new systemois shown in (1.3).

$$\begin{aligned} x' &= 2a(y-x) \\ y' &= 2(c-a)x - (a+1)xz + cy \\ z' &= 2xy - 2bz \end{aligned} \tag{1.3}$$

with parameters a, b and c are real parameters and $a \neq 0, -1$.

The history of research on PDEs goes back to the 18th century in the work of Euler, Alembert, Lagrange and Laplace as a central tool in the descripton of mechanics of continua and more generally, as the principal mode of analytical study of models in the physical science. The analysis of physical models has remaned to the present day one of the fundamntal concerns of the development of PDEs. Beginning in the middle of the 19th century, particularly with the work of Riemann, PDEs also became an esential tool in other branches of mathematics [7–11].

Recently, there are manyanalytical and numercal methods used to solve PDEs on Cantor sets such as, Adomian decomposition method [12–15], series expansion method [31,33], Fourier series method [34,35], Laplace transform method [36], Picard successive approximation method [37], homotopy perturbation method [38, 39], similarity solution [40], reduce differential transform method [41], differential transform method [42,42,44], Laplace decomposition method [45,46], and another methods [47–62] involving the local fractional operators.

2.. STABILITY ANALYSIS

Lemma 2..1. *If* $\dfrac{b(3c-2a)}{a+1} > 0$, *then the system* (1.3) *has three isolated equlibria*

$$\begin{aligned} &O(0,0,0), \\ &E_1\left(\sqrt{\frac{b(3c-2a)}{a+1}}, \frac{b(3c-2a)}{a+1}, \frac{3c-2a}{a+1}\right), \end{aligned}$$

$$E_2\left(-\sqrt{\frac{b(3c-2a)}{a+1}},-\sqrt{\frac{b(3c-2a)}{a+1}},\frac{3c-2a}{a+1}\right),$$

and for $\frac{b(3c-2a)}{a+1} \leq 0$*, it has only one isolated equilibrium .*

Proof. Solving the system(1.3)

$$\begin{aligned}
&2a(y-x)=0 \Longrightarrow x=y\\
&2(c-a)x-(a+1)xz+cy=0 \Longrightarrow x=0, z=\frac{3c-2a}{a+1}\\
&2xy-2bz=0 \Longrightarrow z=\mp\sqrt{bz}=\mp\sqrt{\frac{b(3c-2a)}{a+1}}
\end{aligned}$$

which yields $x=0, y=0, z=0$ and for $\frac{b(3c-2a)}{a+1}>0$ then $x=y=\mp\sqrt{\frac{b(3c-2a)}{a+1}}$, $z=\frac{b(3c-2a)}{a+1}$.

Therefore, the system (1.3) has only one equilibrium $O(0,0,0)$ for $\frac{b(3c-2a)}{a+1} \leq 0$, but has three isolated equlibria: O, E_1, E_2 for all $\frac{b(3c-2a)}{a+1}>0$. □

Lemma 2..2. *For the system* (1.3) *the following statements are true:*

1.If $(a>c>0, b>0)$ *or* $(a>0, b>0, c<0)$ *then* $O(0,0,0)$ *is asymptotically stable.*

2. If $(a<0, c<0)$ *or* $(a<0, c>0)$ *or* $b<0$ *or* $c>a>0$ *then* $O(0,0,0)$ *is an unstable..*

Proof. The Jacobian matrix of system (1.3) at the point $O(0,0,0)$ is:

$$J(O)=\begin{bmatrix} -2a & 2a & 0\\ 2c-2a & c & 0\\ 0 & 0 & -2b \end{bmatrix}$$

The characteristic polynomial of $J(O)$ is

$$\lambda^3+\lambda^2(2a+2b-c)+\lambda(4ab-6ac-2cb+4a^2)+4ab(2a-3c)=0.$$

Then, the eigenvalues of $J(O)$ are:

$$\begin{aligned}
\lambda_1&=-2b,\\
\lambda_2&=-a+\frac{1}{2}\left(c-\sqrt{c^2+20ac-12a^2}\right),\\
\lambda_3&=-a+\frac{1}{2}\left(c+\sqrt{c^2+20ac-12a^2}\right).
\end{aligned}$$

It is clear that if$b>0, a>c>0$ then $\lambda_1<0$ and $Re(\lambda_{2,3})<0$.

If $(a>0, b>0, c<0)$ then $\lambda_1<0$ and $Re(\lambda_{2,3})<0$. Therefore, the point $O(0,0,0)$ is asymptotically stable.

If $b>0$ then $\lambda_1>0$, if $c>a>0$ then $\lambda_2>0$, and if $(a<0, c<0)$ or $(a<0, c>0)$ then $Re(\lambda_{2,3})>0$. Consequently the point $O(0,0,0)$ is unstable. □

Next, considerothe stability ofosystem (1.3) at

$$E_1\left(\sqrt{\frac{b(3c-2a)}{a+1}}, \sqrt{\frac{b(3c-2a)}{a+1}}, \frac{3c-2a}{a+1}\right),$$
$$E_2\left(-\sqrt{\frac{b(3c-2a)}{a+1}}, -\sqrt{\frac{b(3c-2a)}{a+1}}, \frac{3c-2a}{a+1}\right),$$

For all $\dfrac{b(3c-2a)}{a+1} > 0$. Because the system is invariant under the transformation $(x, y, z) \longrightarrow (-x, -y, z)$, one only needs to consider the stability of system (1.1) at E_1.

Lemma 2..3. *The equilibrium point:*

$$E_1\left(\sqrt{\frac{b(3c-2a)}{a+1}}, \frac{b(3c-2a)}{a+1}, \frac{3c-2a}{a+1}\right),$$

is asymptotically stable if and only if :

$$2a+2b-c>0,$$
$$bc>0,$$
$$b(-4ac+2bc-c^2+4a^2>0.$$

Proof. Suppose that

$$x = X + \sqrt{\frac{b(3c-2a)}{a+1}},$$
$$y = Y + \sqrt{\frac{b(3c-2a)}{a+1}},$$
$$z = Z + \frac{3c-2a}{a+1}.$$

The system (1.3) becomes:

$$\begin{aligned} x' &= 2a(Y-X), \\ y' &= -cX-(a+1)XZ+cY-(a+1)\sqrt{\frac{b(3c-2a)}{a+1}}Z, \\ z' &= 2XY+2\sqrt{\frac{b(3c-2a)}{a+1}}(X+Y)-2bZ-\frac{2b(3c-2a)}{a+1}. \end{aligned} \tag{2.1}$$

Hence, one has to consider the stability of system (2.1) at $O(0,0,0)$. The Jacobian matrix of system (2.1) at the point $O(0,0,0)$ is:

$$J = \begin{bmatrix} -2a & 2a & 0 \\ -c & c & -(a+1)\sqrt{\dfrac{b(3c-2a)}{a+1}} \\ 2\sqrt{\dfrac{b(3c-2a)}{a+1}} & 2\sqrt{\dfrac{b(3c-2a)}{a+1}} & -2b \end{bmatrix}$$

The characteristic polynomial of J is:

$$\lambda^3 + (2a + 2b - c)\lambda^2 + 4bc\lambda + 8ab(3c - 2a) = 0. \tag{2.2}$$

Then, from Routh-Hurwits conditions, this equation has all roots with negative real parts if and only if $A > 0, C > 0$ and $AB - C > 0$, where

$$\begin{aligned} &2a + 2b - c > 0 \\ &ab(3c - 2a) > 0 \\ &b(4a^2 - c^2 + 2bc - 4ac) > 0 \end{aligned} \tag{2.3}$$

□

3.. BIFURCATIONS

1. Bifurcations at the point $O(0, 0, 0)$.

Lemma 3..1. *If* $a = \dfrac{3c}{2}$ *the equilibrium point* $O(0, 0, 0)$ *of the system* (1.3) *undergoes a pitchfork bifurcation.*

Proof. If $a \geq \dfrac{3c}{2}$ the equilibrium point $O(0, 0, 0)$ is asymptotically stable and if $a < \dfrac{3c}{2}$ equilibrium point $O(0, 0, 0)$ unstable and two equlibria birth $E_1\left(\sqrt{\dfrac{b(3c-2a)}{a+1}}, \sqrt{\dfrac{b(3c-2a)}{a+1}}, \dfrac{3c-2a}{a+1}\right)$, $E_2\left(-\sqrt{\dfrac{b(3c-2a)}{a+1}}, -\sqrt{\dfrac{b(3c-2a)}{a+1}}, \dfrac{3c-2a}{a+1}\right)$ are stable. Therefore, the system (1.3) has pitchfork bifurcation at $a = \dfrac{3c}{2}$, where $O(0, 0, 0)$ is bifurcation point . □

2. Bifurcations of equilibrium points E_1 and E_1.

The characteristic equation associated with the linearization of system (2.1) at the equilibrium $E_1(E_2)$ is given by:

$$\lambda^3 + (2a + 2b - c)\lambda^2 + 4bc\lambda + 8ab(3c - 2a) = 0. \tag{3.1}$$

It is easy to see that this equation has no zero root since $ab(3c - 2a) \neq 0$, thus we only consider the Hopfobifurcation.

Let b be the Hopf bifurcation parameter.

Suppose that $b = b_0 = \dfrac{4a^2 + 4ac + c^2}{4c}$. Then the equilibrium $E_1(E_2)$ has the following eigenvalues:

$$\begin{aligned} \lambda_1 &= -\frac{4a^2 + 6ac}{c}, \\ \lambda_{2,3} &= \pm i\sqrt{8a^2 + 8ac + 2c^2}. \end{aligned}$$

Lemma 3..2. *If* $b = b_0 = \dfrac{4a^2 + 4ac + c^2}{4c}$, *then the system* (2.1) *undergoes a Hopf bifurcation at the equilibrium point* $E_1\left(\sqrt{\dfrac{b(3c-2a)}{a+1}}, \sqrt{\dfrac{b(3c-2a)}{a+1}}, \dfrac{3c-2a}{a+1}\right)$.

Proof. If $b = b_0 = \dfrac{4a^2 + 4ac + c^2}{4c}$,then the equation (3.1) becomes

$$\left(\lambda + \frac{4a^2 + 6ac}{c}\right)\left(\lambda^2 + \sqrt{8a^2 + 8ac + 2c^2}\right) = 0.$$

Therefore, characteristic equation has a negative realoroot

$$\lambda_1 = -\frac{4a^2 + 6ac}{c},$$

and a pair of purely imaginary roots

$$\lambda_{2,3} = \pm i\sqrt{8a^2 + 8ac + 2c^2}.$$

Differentiating both sides of equation (3.1) with respect to b, we obtain

$$3\lambda^2\frac{d\lambda}{db} + 4a\lambda\frac{d\lambda}{db} + 4b\frac{d\lambda}{db} + 2\lambda^2 - 2c\lambda\frac{d\lambda}{db} + 4bc\lambda\frac{d\lambda}{db} + 4c\lambda + 24ac - 16a^2 = 0.$$

Then, we obtain

$$\frac{d\lambda}{db} = -\frac{2\lambda^2 + 4c\lambda - 16a^2 + 24ac}{3\lambda^2 + (4a + 4b - 2c)\lambda + 4bc},$$

by setting $b = b_0$ and $\lambda = i\sqrt{4bc}$

$$\lambda_b'(b_0) = \frac{2b_0c + 4a^2 - 6ac - ic\sqrt{bc}}{2b_0c - i(2a + 2b - c)\sqrt{bc}}$$

$$Re\left[\lambda_b'(b_0)\right] = \frac{6b_0c + 8a^2 - 10ac - c^2}{4b_0c + (2a + 2b - c)^2} \neq 0.$$

Therefore, $\lambda_b'(b_0) \neq =$. According to Hopf bifurcation theorem in [12], the system (2.1) has display a Hopf bifurcation at $O(0, 0, 0)$, so the system (1.3) display a Hopfobifurcation at the point $E_1\left(\sqrt{\dfrac{b(3c - 2a)}{a + 1}}, \sqrt{\dfrac{b(3c - 2a)}{a + 1}}, \dfrac{3c - 2a}{a + 1}\right)$. □

Definition 3..1. *Let V be a subset of that contains the origin in its interior. Assume that F, G and H are real valued functions on V that vanish at the origin and whose partial derivatives are continuous and also vanish at the origin. Then the system*

$$x' = a_1x + b_1y + c_1z + F(x, y, z)$$

$$y' = a_2x + b_2y + c_2z + G(x, y, z)$$

$$x' = a_3x + b_3y + c_3z + H(x, y, z)$$

is almost linear system [1].

Lemma 3..3. *The system* (1.3) *is almost linear system at the point $O(0, 0, 0)$.*

Proof. we write the system (1.3) as follows:

$$x' = 2ay - 2ax + F(x, y, z)$$

$$y' = (2c - 2a)x + cy + G(x, y, z)$$

$$z' = -2bz + H(x, y, z)$$

where $F(x, y, z) = 0, G(x, y, z) = -(a + 1)xz$ and $H(x, y, z) = 2xy$. Then $F(0, 0, 0) = G(0, 0, 0) = H(0, 0, 0) = 0$, and all first partials of F, G and H continuous and vanish atothe point $O(0, 0, 0)$. Thereforeothe system (1.3) is almost linear system at the point $O(0, 0, 0)$. ☐

4.. LYAPONUV EXPONENT

To compute the maximal Lyaponuv exponent of a system of ordinary differential equations we must integrate both the original system and its linearization $v' = A(t)v$. Essentially any initial vector v_0 can be used because almost all vectors will have some component along the direction of the maximal Lyaponuv direction. We cannot compute the limit in maximal Lyaponuv exponent:

$$\gamma(x, v) = \lim_{t \longrightarrow \infty} Sup \frac{1}{t} |\Phi(t, x)v|,$$

but instead simply integrate for some long time T and estimate:

$$\gamma_{max}(T) = \frac{1}{T} \frac{|v(T)|}{|v_0|},$$

such that $v_0 \neq 0$ [7]. This quantity will rapidly converge to the maximal exponent; to estimate the error in the computation, it is useful to plot γ_{max} as a function of T.

Next, we calculate maximal Lyaponuv exponent for system (1.3) at the critical points by using Matlab program.

1. The maximal Lyaponuv exponent for system (1.3) at the critical point $O(0, 0, 0)$ with parameters $(a > c > 0, b > 0)$ and $(a > 0, b > 0, c < 0)$is negative number.

2. The maximal Lyaponuv exponent for system (1.3) at the critical point $O(0, 0, 0)$ with parameters $(a < 0, b < 0, c < 0)$ and $(a > 0, b > 0, c > 0)$ and $(a > 0, b < 0, c < 0)$ and $(a > 0, b < 0, c > 0)$ is positive number.

3. The maximal Lyaponuv exponent for system (1.3) at the critical point $E_1\left(\sqrt{\frac{b(3c-2a)}{a+1}}, \sqrt{\frac{b(3c-2a)}{a+1}}, \frac{3c-2a}{a+1}\right)$ with parameters $(a > -1, b > 0, c > \frac{2a}{3})$ is negative number.

4. The maximal Lyaponuv exponent for system (1.3) at the critical point $E_1\left(\sqrt{\frac{b(3c-2a)}{a+1}}, \sqrt{\frac{b(3c-2a)}{a+1}}, \frac{3c-2a}{a+1}\right)$ with parameters $(a < -1, b > 0, c < \frac{2a}{3})$ and $(a < -1, b < 0, c > \frac{2a}{3})$ and $(a > -1, b < 0, c < \frac{2a}{3})$ is positive number.

5.. Conclusions

Using a rigorous mathematical analysis based on symbolic and numerical computations we have studied a relative new system. There are obtained some insights on stability and bifurcation. The system possesses a pitchfork bifurcation and Hopf bifurcation. The system has a positive Lyaponuv exponent in a special cases. Surely, there is still a lot of work, and this paper is a step in analyzing this system.

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References

[1] D. Gulick, Encounters with Chaos, McGraw-Hill, 1992.

[2] E.N. Lorenz, Deterministic Nonperiodic Flow, J. Atmospheric Sci. 20 (1963), 130–141. https://doi.org/10.1175/1520-0469(1963)020<0130:dnf>2.0.co;2.

[3] G. Chen, T. Ueta, Yet Another Chaotic Attractor, Int. J. Bifurc. Chaos 09 (1999), 1465–1466. https://doi.org/10.1142/s0218127499001024.

[4] J. Meiss, Differential Dynamical Systems, SIAM, 2007.

[5] J. LÜ, G. CHEN, A NEW CHAOTIC ATTRACTOR COINED, Int. J. Bifurc. Chaos 12 (2002), 659–661. https://doi.org/10.1142/s0218127402004620.

[6] T. Li, G. Chen, Y. Tang, On Stability and Bifurcation of Chen's System, Chaos Solitons Fractals 19 (2004), 1269–1282. https://doi.org/10.1016/S0960-0779(03)00334-5.

[7] H.K. Jassim, New Approaches for Solving Fokker Planck Equation on Cantor Sets Within Local Fractional Operators, J. Math. 2015 (2015), 684598. https://doi.org/10.1155/2015/684598.

[8] H.K. Jassim, The Approximate Solutions of Three-Dimensional Diffusion and Wave Equations Within Local Fractional Derivative Operator, Abstr. Appl. Anal. 2016 (2016), 2913539. https://doi.org/10.1155/2016/2913539.

[9] W.K. Ghafil, G.A. Al-Juaifri, A. Al-Haboobi, Exploring Chaotic Dynamics in a Fourth-Order Newton Method for Polynomial Root Finding, Math. Model. Eng. Probl. 11 (2024), 2163–2169. https://doi.org/10.18280/mmep.110818.

[10] D. Baleanu, H.K. Jassim, Approximate Solutions of the Damped Wave Equation and Dissipative Wave Equation in Fractal Strings, Fractal Fract. 3 (2019), 26. https://doi.org/10.3390/fractalfract3020026.

[11] D. Baleanu, H.K. Jassim, A Modification Fractional Homotopy Perturbation Method for Solving Helmholtz and Coupled Helmholtz Equations on Cantor Sets, Fractal Fract. 3 (2019), 30. https://doi.org/10.3390/fractalfract3020030.

[12] D. Baleanu, H.K. Jassim, M. Al Qurashi, Solving Helmholtz Equation with Local Fractional Derivative Operators, Fractal Fract. 3 (2019), 43. https://doi.org/10.3390/fractalfract3030043.

[13] H. K. Jassim, J. Vahidi, V.M. Ariyan, Solving Laplace Equation within Local Fractional Operators by Using Local Fractional Differential Transform and Laplace Variational Iteration Methods, Nonlinear Dyn. Syst. Theory, 20 (2020), 388–396.

[14] H. Kamil Jassim, M. Gassab Mohammed, H. Ali Eaued, A Modification Fractional Homotopy Analysis Method for Solving Partial Differential Equations Arising in Mathematical Physics, IOP Conf. Ser.: Mater. Sci. Eng. 928 (2020), 042021. `https://doi.org/10.1088/1757-899x/928/4/042021`.

[15] H. Ali Eaued, H. Kamil Jassim, M. Gassab Mohammed, A Novel Method for the Analytical Solution of Partial Differential Equations Arising in Mathematical Physics, IOP Conf. Ser.: Mater. Sci. Eng. 928 (2020), 042037. `https://doi.org/10.1088/1757-899x/928/4/042037`.

[16] H.K. Jassim, J. Vahidi, A New Technique of Reduce Differential Transform Method to Solve Local Fractional Pdes in Mathematical Physics, Int. J. Nonlinear Anal. Appl. 12 (2021), 37–44.

[17] Ali Lateef Arif, An Analytical Approach for Solving Fractional Partial Differential Equations, Int. J. Appl. Math. 38 (2025), 270–282. `https://doi.org/10.12732/ijam.v38i3s.145`.

[18] H.K. Jassim, M.A.S. Hussain, On Approximate Solutions for Fractional System of Differential Equations with Caputo-Fabrizio Fractional Operator, J. Math. Comput. Sci. 23 (2020), 58–66. `https://doi.org/10.22436/jmcs.023.01.06`.

[19] S.A.H. Khafif, H.K. Jassim, M.G. Mohammed, SVIM for Solving Burger's and Coupled Burger's Equations of Fractional Order, Prog. Fract. Differ. Appl. 7 (2021), 73–78. `https://doi.org/10.18576/pfda/070707`.

[20] H.K. Jassim, H. Kadmim, Fractional Sumudu Decomposition Method for Solving PDEs of Fractional Order, J. Appl. Comput. Mech. 7 (2021), 302–311. `https://doi.org/10.22055/JACM.2020.31776.1920`.

[21] H.K. Jassim, M.G. Mohammed, Natural Homotopy Perturbation Method for Solving Nonlinear Fractional Gas Dynamics Equations, Int. J. Nonlinear Anal. Appl. 12 (2021), 812–820. `https://doi.org/10.22075/IJNAA.2021.4936`.

[22] N. Rhaif Swain, H.K. Jassim, Numerical Solutions of Multi-Dimensional Fractional Telegraph Equations, Stat. Optim. Inf. Comput. 14 (2025), 1936–1963. `https://doi.org/10.19139/soic-2310-5070-2335`.

[23] H.K. Jassim, A New Approach to Find Approximate Solutions of Burger's and Coupled Burger's Equations of Fractional Order, TWMS J. Appl. Eng. Math. 11 (2021), 415–423.

[24] L.K. Alzaki, The Approximate Analytical Solutions of Nonlinear Fractional Ordinary Differential Equations, Int. J. Nonlinear Anal. Appl. 12 (2021), 527–535. `http://dx.doi.org/10.22075/ijnaa.2021.5094`.

[25] H. Jassim, H. Ahmad, A. Shamaoon, C. Cesarano, An Efficient Hybrid Technique for the Solution of Fractional-Order Partial Differential Equations, Carpathian Math. Publ. 13 (2021), 790–804. `https://doi.org/10.15330/cmp.13.3.790-804`.

[26] H.G. Taher, H. Ahmad, J. Singh, D. Kumar, H.K. Jassim, Solving Fractional PDEs by Using Daftardar-Jafari Method, AIP Conf. Proc. 2386 (2022), 060002. `https://doi.org/10.1063/5.0067177`.

[27] L.K. Alzaki, H.K. Jassim, Time-Fractional Differential Equations with an Approximate Solution, J. Niger. Soc. Phys. Sci. 4 (2022), 818. `https://doi.org/10.46481/jnsps.2022.818`.

[28] H.K. Jassim, M.A. Hussein, A Novel Formulation of the Fractional Derivative with the Order $\alpha \geq 0$ and Without the Singular Kernel, Mathematics 10 (2022), 4123. `https://doi.org/10.3390/math10214123`.

[29] S.A. Sachit, H.K. Jassim, A Modified Iterative Approach Using YJ Transform for Solving Fractional PDEs, Prog. Fract. Differ. Appl. 11 (2025), 661–676. `https://doi.org/10.18576/pfda/110403`.

[30] H.K. Jassim, R.T. Nasser, Rida-Jassim Integral Transform: A Tool for Solving Linear and Non-Linear Differential Equations, Bol. Soc. Parana. Mat. 43 (2025), 1–10. `https://doi.org/10.5269/bspm.77201`.

[31] A.L. Arif, H.K. Jassim, M.T. Yasser, N.R. Swain, The Numerical Solutions of Fractional Partial Differential Equations, Asia Pac. J. Math. 12 (2025), 84. `https://doi.org/10.28924/apjm/12-84`.

[32] H.K. Jassim, R.T. Nasser, A Novel Integral Transform (NT) and Its Applications to Fractional Differential Equations, Int. J. Sci. Math. Technol. Learn. 33 (2025), 378–389.

[33] H.K. Jassim, A.L. Arif, The Approximate Solutions and Stability Analysis for Blood Ethanol Concentration System, Prog. Fract. Differ. Appl. 11 (2025), 539–546. https://doi.org/10.18576/pfda/110308.

[34] H.K. Jassim, M.Y. Taimah, A New Integral Transform for Solving Integral and Ordinary Differential Equations, Math. Comput. Sci. 6 (2025), 32–42.

[35] N.R. Swain, H.K. Jassim, Innovation of Yang Hussein Jassim's Method in Solving Nonlinear Telegraph Equations Across Multiple Dimensions, Partial. Differ. Equ. Appl. Math. 14 (2025), 101182. https://doi.org/10.1016/j.padiff.2025.101182.

[36] N.H. Mohsin, H.K. Jassim, M.A. Alkhafaji, Approximate Analytical Solutions of Fractional Schrodinger Equations, Asia Pac. J. Math. 12 (2025), 18. https://doi.org/10.28924/apjm/12-18.

[37] M.A. Hussein, H.K. Jassim, A Novel Approach to Nonlinear Fractional Volterra Integral Equations, Acta Polytech. 64 (2024), 414–419. https://doi.org/10.14311/ap.2024.64.0414.

[38] R. Shah, H.K. Jassim, H. Ahmad, M.Y. Zayir, S.H. Mahdi, et al., An Approximation Method to Solve Atangana-Baleanu FPDEs, AIP Conf. Proc. 3219 (2024), 040004. https://doi.org/10.1063/5.0236443.

[39] H.K. Jassim, M.A. Hussein, S. Mahdi, M.Y. Zayir, S.A. Sachit, et al., Semi-Analytical Solutions of Fractional Differential Equations by Elzaki Variational Iteration Method, AIP Conf. Proc. 3219 (2024), 040003. https://doi.org/10.1063/5.0236441.

[40] M.A. Hussein, H.K. Jassim, A.K. Jassim, An Innovative Iterative Approach to Solving Volterra Integral Equations of Second Kind, Acta Polytech. 64 (2024), 87–102. https://doi.org/10.14311/ap.2024.64.0087.

[41] P. Cui, H.k. Jassim, Local Fractional Sumudu Decomposition Method to Solve Fractal PDEs Arising in Mathematical Physics, Fractals 32 (2024), 2440029. https://doi.org/10.1142/s0218348x24400292.

[42] H. Jafari, H.K. Jassim, A. Ansari, V.T. Nguyen, Local Fractional Variational Iteration Transform Method: A Tool for Solving Local Fractional Partial Differential Equations, Fractals 32 (2024), 2440022. https://doi.org/10.1142/s0218348x2440022x.

[43] H. Jafari, H.K. Jassim, C. Ünlü, V.T. Nguyen, Laplace Decomposition Method for Solving the Two-Dimensional Diffusion Problem in Fractal Heat Transfer, Fractals 32 (2024), 2440026. https://doi.org/10.1142/s0218348x24400267.

[44] Ghazwan Ali Abdul Hussein, Djelloul Ziane, A New Approximation Solutions for Fractional Order Biological Population Model, J. Educ. Pure Sci. Univ. Thi-Qar 14 (2024), 14–33. https://doi.org/10.32792/jeps.v14i3.444.

[45] H. Jassim, H. Ahmed, M. Hussein, J. Singh, D. Kumar, et al., On Efficient Method for Fractional-Order Two-Dimensional Navier-Stokes Equations, Iraqi J. Sci. 65 (2024), 5710–5726. https://doi.org/10.24996/ijs.2024.65.10.32.

[46] H.G. Taher, H.K. Jassim, N.J. Hassan, Approximate Analytical Solutions of Differential Equations with Caputo-Fabrizio Fractional Derivative via New Iterative Method, AIP Conf. Proc. 2398 (2022), 060020. https://doi.org/10.1063/5.0095338.

[47] S.A. Sachit, H.K. Jassim, N.J. Hassan, Revised Fractional Homotopy Analysis Method for Solving Nonlinear Fractional PDEs, AIP Conf. Proc. 2398 (2022), 060044. https://doi.org/10.1063/5.0093413.

[48] H.K. Jassim, H. Ahmad, J. Singh, D. Kumar, A.T. Salman, et al., New Approximate Solutions to Some of Nonlinear PDEs with Atangana-Baleanu-Caputo Operator, Prog. Fract. Differ. Appl. 10 (2024), 91–98. https://doi.org/10.18576/pfda/100109.

[49] M.Y. Zayir, H.K. Jassim, A Unique Approach for Solving the Fractional Navier–Stokes Equation, J. Interdiscip. Math. 25 (2022), 2611–2616. https://doi.org/10.1080/09720502.2022.2057050.

[50] H. Jafari, M.Y. Zair, H.K. Jassim, Analysis of Fractional Navier-Stokes Equations, Heat Transf. 52 (2023), 2859–2877. https://doi.org/10.1002/htj.22807.

[51] S.A. Sachit, H.K. Jassim, Solving Fractional PDEs by Elzaki Homotopy Analysis Method, AIP Conf. Proc. 2611 (2023), 040074. `https://doi.org/10.1063/5.0115742`.

[52] J. Singh, H.K. Jassim, D. Kumar, V.P. Dubey, Fractal Dynamics and Computational Analysis of Local Fractional Poisson Equations Arising in Electrostatics, Commun. Theor. Phys. 75 (2023), 125002. `https://doi.org/10.1088/1572-9494/ad01ad`.

[53] H.K. Jassim, M. Abdulshareef Hussein, A New Approach for Solving Nonlinear Fractional Ordinary Differential Equations, Mathematics 11 (2023), 1565. `https://doi.org/10.3390/math11071565`.

[54] D. Ziane, M.H. Cherif, K. Belghaba, H.K. Jassim, Application of Local Fractional Variational Iteration Transform Method to Solve Nonlinear Wave-Like Equations Within Local Fractional Derivative, Prog. Fract. Differ. Appl. 9 (2023), 311–318. `https://doi.org/10.18576/pfda/090211`.

[55] H.K. Jassim, A.T. Salman, H. Ahmad, M.Y. Zayir, A.H. Shuaa, Exact Analytical Solutions for Fractional Partial Differential Equations via an Analytical Approach, AIP Conf. Proc. 2899 (2023), 060007. `https://doi.org/10.1063/5.0157146`.

[56] N.H. Mohsin, H.K. Jassim, A.D. Azeez, A New Analytical Method for Solving Nonlinear Burger's and Coupled Burger's Equations, Mater. Today: Proceedings 80 (2023), 3193–3195. `https://doi.org/10.1016/j.matpr.2021.07.194`.

[57] M.A. Hussein, H.K. Jassim, Analysis of Fractional Differential Equations with Antagana-Baleanu Fractional Operator, Prog. Fract. Differ. Appl. 9 (2023), 681–686. `https://doi.org/10.18576/pfda/090411`.

[58] H.K. Jassim, M.Y. Zayir, A.H. Shuaa, N.J. Hassan, Solving Fractional PDEs by Using FADM Within Atangana-Baleanu Fractional Derivative, AIP Conf. Proc. 2899 (2023), 060004. `https://doi.org/10.1063/5.0157145`.

[59] H.K. Jassim, M.Y. Zair, H. Ahmad, L.K. Alzaki, A.H. Shuaa, et al., Approximate Analytical Solutions of Fractional Navier-Stokes Equation, AIP Conf. Proc. 2834 (2023), 080100. `https://doi.org/10.1063/5.0161550`.

[60] H.K. Jassim, M.A. Hussein, M.R. Ali, An Efficient Homotopy Permutation Technique for Solving Fractional Differential Equations Using Atangana-Baleanu-Caputo Operator, AIP Conf. Proc. 2899 (2023), 060008. `https://doi.org/10.1063/5.0157148`.

[61] H.K. Jassim, A.T. Salman, H. Ahmad, N.J. Hassan, A.E. Hashoosh, Solving Nonlinear Fractional PDEs by Elzaki Homotopy Perturbation Method, AIP Conf. Proc. 2834 (2023), 080101. `https://doi.org/10.1063/5.0161551`.

[62] H.K. Jassim, A.L. Arif, Analysis of Cauchy Reaction-Diffusion Equations Involving Atangana-Baleanu Fractional Operator, Partial. Differ. Equ. Appl. Math. 12 (2024), 100981. `https://doi.org/10.1016/j.padiff.2024.100981`.