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A NOVEL APPROACH FOR G_b^p-PARTIAL METRIC SPACES AND EXISTENCE OF FIXED POINTS

VISHAL GUPTA[1], RAJANI SAINI[2], RAHUL SHUKLA[3,*]

[1]Department of Mathematics, MMEC, Maharishi Markandeshwar (Deemed to be University), Mullana, Ambala, Haryana 133207, India

[2]Department of Mathematics, Government Post Graduate College, Ambala Cantt, Haryana 133001, India

[3]Department of Mathematical Sciences and Computing, Walter Sisulu University, Mthatha 5117, South Africa

[*]Corresponding author: rshukla@wsu.ac.za

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Abstract. This paper introduces the concept of G_b^p-partial metric spaces, a new mathematical structure that extends the frameworks of both the G_b-metric spaces and partial metric spaces. These generalized spaces provide a ground for exploring the behavior and properties of metric structures that exhibit partial symmetry and modified distance concepts. The main purpose of this study is to extend the current results by examining the fundamental properties of G_b^p-partial metric spaces and developing several fixed point theorems within this new context. Methodologically, we employ analytical techniques grounded in metric space theory to define, analyze, and validate properties specific to G_b^p-partial metric spaces with the help of examples. The principal results include new fixed point theorems, which not only generalize existing theorems but also reveal new structural insights into how such spaces behave under specific mappings. Our findings contribute to the mathematical literature by broadening the scope of metric space and deepening the understanding of fixed point properties in more generalized metric frameworks. Major conclusions suggest that G_b^p-partial metric spaces can serve as a foundation for further research in both pure and applied mathematics, offering potential applications in fields where generalized metric spaces play a role in modeling complex systems.

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1. Introduction

Metric space theory is a foundational pillar of modern mathematics, elegantly integrating analysis, topology, and geometry to create a versatile framework with extensive applications across science and engineering. Central to this theory is the concept of fixed points, which is instrumental in tackling diverse mathematical and scientific problems, such as solving differential equations and ensuring

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stability in complex systems. Over the years, the classical metric space framework has been extended through various generalizations to meet diverse theoretical and practical demands. For a deeper understanding of these advancements, readers are encouraged to explore the seminal works of Bakhtin [7], Chistyakov [9], Ege and Alaca [10], Mani [12], and Tyagi et al. [19], along with their cited references.

One notable generalization is the G_b-metric space, introduced by Aghajani et al. [1], which has inspired considerable research due to its innovative extension of classical metric properties. Another significant development is the partial metric space, proposed by Matthews [14], which offers a framework for non-classical distance measures, particularly in theoretical computer science and domain theory. This area has seen substantial contributions from researchers like Altun [2,3] and Aydi [5,6]. In 2009, Bukatin et al. [8] further broadened the scope of partial metric spaces, enhancing their applicability to diverse mathematical and computational challenges. For additional insights into related developments in metric, b-metric, and partial metric spaces, see Mani [13] and Anjana [4].

This paper introduces the G_b^p-partial metric space, a novel synthesis of G_b-metric and partial metric spaces, providing a flexible framework for studying fixed point theorems in these generalized spaces. These theorems enhance our understanding of fixed points in such contexts and hold promise for applications in fields relying on metric spaces for analyzing stability, convergence, and continuity in complex systems.

The importance of partial metric spaces was first highlighted by Matthews [14] in the context of denotational semantics for data networks, for use in program verification. More recently, Romaguera [17] proved that a partial metric space is 0-complete if and only if every Caristi-type mapping on it has a fixed point, extending Kirk's work on metric convexity [11].

The introduction of G-metric spaces by Mustafa and Sims [15,16] marked a significant advancement, inspiring numerous fixed point results under various contractive conditions. Salimi and Vetro [18] further developed this by introducing partial G-metric spaces and proving a Suzuki-type fixed point result, along with a common fixed point theorem.

By combining the strengths of G_b-metric and partial metric spaces, this paper defines G_b^p-partial metric spaces and derives fixed point theorems that deepen the theoretical understanding of generalized metric spaces. These findings contribute to the ongoing evolution of metric space theory and highlight its relevance to contemporary scientific and engineering challenges.

2. Preliminaries

Definition 2.1. [1] Assume that a mapping $G_b : \Pi \times \Pi \times \Pi \to \mathbb{R}^+$ on a nonempty set Π and $s \geq 1$ fulfills the following conditions for every $\kappa, \mu, \tau, a \in \Pi$:

(Gb1) $G_b(\kappa, \mu, \tau) = 0$ if $\kappa = \mu = \tau$;

(Gb2) $0 < G_b(\kappa, \kappa, \mu)$ with $\kappa \neq \mu$;

(Gb3) $G_b(\kappa,\kappa,\mu) \leq G_b(\kappa,\mu,\tau)$ with $\mu \neq \tau$;

(Gb4) $G_b(\kappa,\mu,\tau) = G_b(p\{\mu,\tau,\kappa\})$, where p is a permutation;

(Gb5) $G_b(\kappa,\mu,\tau) \leq s\{G_b(\kappa,a,a) + G_b(a,\mu,\tau)\}$.

The pair (Π, G_b) is called a G_b-metric space.

Definition 2.2. [8] A function $P : \Pi \times \Pi \to \mathbb{R}^+$ on a nonempty set Π fulfills the following conditions for all $\kappa,\mu,\tau \in \Pi$:

(P1) $\kappa = \mu$ if and only if $P(\kappa,\kappa) = P(\kappa,\mu) = P(\mu,\mu)$.

(P2) $P(\kappa,\kappa) \leq P(\kappa,\mu)$,

(P3) $P(\kappa,\mu) = P(\mu,\kappa)$,

(P4) $P(\kappa,\mu) \leq P(\kappa,\tau) + P(\tau,\mu) - P(\tau,\tau)$.

The pair (Π, P) is called a partial metric space.

3. Main Results

We now establish a generalization of a G_b-metric space as well as of a partial metric space.

Definition 3.1. Assume that a mapping $G_b^p : \Pi \times \Pi \times \Pi \to \mathbb{R}^+$ on a nonempty set Π and $s \geq 1$, $s \in \mathbb{R}$, fulfills the following conditions for all $\kappa,\mu,\tau,a \in \Pi$:

$$(G_bP1) \quad \kappa = \mu = \tau \text{ if } G_b^p(\kappa,\kappa,\kappa) = G_b^p(\mu,\mu,\mu) = G_b^p(\tau,\tau,\tau) = G_b^p(\kappa,\mu,\tau);$$

$$(G_bP2) \quad 0 \leq G_b^p(\kappa,\kappa,\kappa) \leq G_b^p(\kappa,\kappa,\tau) \leq G_b^p(\kappa,\mu,\tau);$$

$$(G_bP3) \quad G_b^p(\kappa,\mu,\tau) = G_b^p(p\{\mu,\tau,\kappa\}), \text{ where } p \text{ is a permutation;}$$

$$(G_bP4) \quad G_b^p(\kappa,\mu,\tau) \leq s\left\{G_b^p(\kappa,a,a) + G_b^p(a,\mu,\tau)\right\} - G_b^p(a,a,a).$$

Then, the pair $\left(\Pi, G_b^p\right)$ is called a G_b-partial metric space. Clearly, the class of G_b-partial metric spaces is broader than that of G-partial metric spaces.

Example 3.2. Let $\Pi = \mathbb{R}^+$ and let $p > 1$ be a constant. Define the function

$$G_b^p : \Pi \times \Pi \times \Pi \to \mathbb{R}^+$$

by

$$G_b^p(\kappa,\mu,\tau) = [\max\{\kappa,\mu\}]^p + |\max\{\kappa,\mu\} - \tau|^p,$$

for all $\kappa,\mu,\tau \in \Pi$.

It follows that $\left(\Pi, G_b^p\right)$ is a G_b^p-partial metric space with $s = 2p > 1$, but it is not a G-partial metric space.

To see this, consider the values $\kappa = 5$, $\mu = 2$, $\tau = 1$, and $a = 4$. Then,

$$G_b^p(5,2,1) = 5^p + 4^p.$$

Comparing this with the triangle inequality:

$$G_b^p(5,2,1) = 5^p + 4^p > \left\{G_b^p(5,4,4) + G_b^p(4,2,1)\right\} - G_b^p(4,4,4),$$

we compute

$$5^p + 4^p > 5^p + 1 + 4^p + 3^p - 4^p = 5^p + 1 + 3^p.$$

Since this inequality does not hold in general, the triangle inequality is not satisfied.

Definition 3.3. A sequence $\{\kappa_\lambda\}$ in $\left(\Pi, G_b^p\right)$ is defined as follows:

(1) Convergent if there exists $\zeta \in \Pi$ such that

$$\lim_{m,n\to\infty} G_b^p\left(\kappa_\lambda, \kappa_m, \zeta\right) = G_b^p(\zeta,\zeta,\zeta).$$

(2) Cauchy if

$$\lim_{\lambda,m,l\to\infty} G_b^p\left(\kappa_\lambda, \kappa_m, \kappa_l\right)$$

exists and is finite.

Definition 3.4. The pair $\left(\Pi, G_b^p\right)$ is called a complete G-partial metric space if every Cauchy sequence is convergent in it.

Now, we present some fixed-point results in our newly defined space.

Theorem 3.5. *Let $\left(\Pi, G_b^p\right)$ be a complete G_b^p-partial metric space with $s \geq 1$, and let $\Phi : \Pi \to \Pi$ be a function that fulfills the following condition for every $\kappa, \mu, \tau \in \Pi$ and $\xi \in (0,1)$:*

$$G_b^p(\Phi\kappa, \Phi\eta, \Phi\tau) \leq \xi G_b^p(\kappa, \eta, \tau).$$

Then, there exists a unique fixed point ζ for the function Φ, and $G_b^p(\zeta,\zeta,\zeta) = 0$.

Proof. We prove the theorem by first showing $G_b^p(\zeta,\zeta,\zeta) = 0$ and then establishing the uniqueness of ζ as a fixed point for the mapping Φ. Finally, we prove the existence of the fixed point.
Step 1: Show $G_b^p(\zeta,\zeta,\zeta) = 0$.
Assume that $G_b^p(\zeta,\zeta,\zeta) > 0$. Then:

$$G_b^p(\zeta,\zeta,\zeta) = G_b^p(\Phi\zeta, \Phi\zeta, \Phi\zeta) \leq \xi G_b^p(\zeta,\zeta,\zeta) < G_b^p(\zeta,\zeta,\zeta).$$

This is a contradiction. Hence, $G_b^p(\zeta,\zeta,\zeta) = 0$.
Step 2: Uniqueness of the Fixed Point.

Suppose there exists $\varsigma \in \Pi$ such that $\Phi\varsigma = \varsigma$ and $\varsigma \neq \zeta$. Then:

$$G_b^p(\zeta,\varsigma,\varsigma) = G_b^p(\Phi\zeta, \Phi\varsigma, \Phi\varsigma) \leq \xi G_b^p(\zeta,\varsigma,\varsigma) < G_b^p(\zeta,\varsigma,\varsigma).$$

This is also a contradiction. Therefore, $G_b^p(\zeta,\varsigma,\varsigma) = 0$, which implies $\zeta = \varsigma$.

Now, consider:

$$\begin{aligned} G_b^p\left(\kappa_\lambda, \kappa_{\lambda+1}, \kappa_{\lambda+1}\right) &= G_b^p\left(\Phi\kappa_{\lambda-1}, \Phi\kappa_\lambda, \Phi\kappa_\lambda\right) \\ &\leq \xi G_b^p\left(\kappa_{\lambda-1}, \kappa_\lambda, \kappa_\lambda\right) \\ &\leq \xi^2 G_b^p\left(\kappa_{\lambda-2}, \kappa_{\lambda-1}, \kappa_{\lambda-1}\right) \\ &\leq \dots \\ &\leq \xi^\lambda G_b^p\left(\kappa_0, \kappa_1, \kappa_1\right). \end{aligned}$$

Since $0 \leq \xi < 1$, it follows that $\xi^\lambda G_b^p\left(\kappa_0, \kappa_1, \kappa_1\right) \to 0$ as $\lambda \to \infty$.

Next, we have:

$$\begin{aligned} G_b^p\left(\kappa_\lambda, \kappa_m, \kappa_m\right) &\leq s\left[G_b^p\left(\kappa_\lambda, \kappa_{\lambda+1}, \kappa_{\lambda+1}\right) + G_b^p\left(\kappa_{\lambda+1}, \kappa_m, \kappa_m\right)\right] - G_b^p\left(\kappa_{\lambda+1}, \kappa_{\lambda+1}, \kappa_{\lambda+1}\right) \\ &\leq sG_b^p\left(\kappa_\lambda, \kappa_{\lambda+1}, \kappa_{\lambda+1}\right) + s^2 G_b^p\left(\kappa_{\lambda+1}, \kappa_{\lambda+2}, \kappa_{\lambda+2}\right) + s^3 G_b^p\left(\kappa_{\lambda+2}, \kappa_{\lambda+3}, \kappa_{\lambda+3}\right) + \dots \\ &\leq s\xi^\lambda G_b^p\left(\kappa_0, \kappa_1, \kappa_1\right) + s^2\xi^{\lambda+1} G_b^p\left(\kappa_0, \kappa_1, \kappa_1\right) + s^3\xi^{\lambda+2} G_b^p\left(\kappa_0, \kappa_1, \kappa_1\right) + \dots \\ &\leq \frac{s\xi^\lambda}{1 - s\xi} G_b^p\left(\kappa_0, \kappa_1, \kappa_1\right). \end{aligned}$$

This implies that $G_b^p\left(\kappa_\lambda, \kappa_m, \kappa_m\right) \to 0$ as $m, \lambda \to \infty$.

Hence, $\{\kappa_\lambda\}$ is a Cauchy sequence. Since Π is complete, there exists $\zeta \in \Pi$ such that $\{\kappa_\lambda\} \to \zeta$.

We now show that ζ is a fixed point.

$$\begin{aligned} G_b^p(\zeta, \Phi\zeta, \Phi\zeta) &\leq s\left[G_b^p\left(\zeta, \kappa_{\lambda+1}, \kappa_{\lambda+1}\right) + G_b^p\left(\kappa_{\lambda+1}, \Phi\zeta, \Phi\zeta\right)\right] - G_b^p\left(\kappa_{\lambda+1}, \kappa_{\lambda+1}, \kappa_{\lambda+1}\right) \\ &\leq s\left[G_b^p\left(\zeta, \kappa_{\lambda+1}, \kappa_{\lambda+1}\right) + \xi G_b^p\left(\kappa_\lambda, \zeta, \zeta\right)\right], \end{aligned}$$

this tends to zero as λ tends to infinity. Hence $G_b^p(\zeta, \Phi\zeta, \Phi\zeta) = 0$ and $\Phi\zeta = \zeta$. □

Theorem 3.6. *Let $\left(\Pi, G_b^p\right)$ be a complete G_b^p-partial metric space with $s \geq 1$ and $\Phi : \Pi \to \Pi$ be a function that fulfilling the condition*

$$G_b^p(\Phi\kappa, \Phi\eta, \Phi\tau) \leq \left[\xi_1 G_b^p(\kappa, \Phi\kappa, \Phi\kappa) + \xi_2 G_b^p(\eta, \Phi\eta, \Phi\eta) + \xi_3 G_b^p(\tau, \Phi\tau, \Phi\tau)\right]$$

for every $\kappa, \mu, \tau \in \Pi$ and $s\xi_1 + (\xi_2 + \xi_3) < 1, \xi_2 + \xi_3 \neq 1$. Then, there exist a unique fixed point ζ for the function Φ and $G_b^p(\zeta, \zeta, \zeta) = 0$.

Proof. First we will show that $G_b^p(\zeta, \zeta, \zeta) = 0$, by assuming that ζ is unique fixed point for the mapping Φ.

$$\begin{aligned} G_b^p(\zeta, \zeta, \zeta) &= G_b^p(\Phi\zeta, \Phi\zeta, \Phi\zeta) \\ &\leq \left[\xi_1 G_b^p(\zeta, \Phi\zeta, \Phi\zeta) + \xi_2 G_b^p(\zeta, \Phi\zeta, \Phi\zeta) + \xi_3 G_b^p(\zeta, \Phi\zeta, \Phi\zeta)\right] \\ &\leq \left(\xi_1 + \xi_2 + \xi_3\right) G_b^p(\zeta, \zeta, \zeta) < G_b^p(\zeta, \zeta, \zeta) \text{ as } s\xi_1 + (\xi_2 + \xi_3) < 1, \end{aligned}$$

which is possible only when

$$G_b^p(\zeta,\zeta,\zeta)=0.$$

Now, we will show the existence and uniqueness of the fixed point.

$$G_b^p\left(\kappa_\lambda,\kappa_{\lambda+1},\kappa_{\lambda+1}\right)\leq G_b^p\left(\Phi\kappa_{\lambda-1},\Phi\kappa_\lambda,\Phi\kappa_\lambda\right)$$
$$\leq\left[\xi_1G_b^p\left(\kappa_{\lambda-1},\kappa_\lambda,\kappa_\lambda\right)+\left(\xi_2+\xi_3\right)G_b^p\left(\kappa_{\lambda+1},\kappa_\lambda,\kappa_{\lambda+1}\right)\right]$$

$G_b^p\left(\kappa_\lambda,\kappa_{\lambda+1},\kappa_{\lambda+1}\right)\leq\gamma G_b^p\left(\kappa_{\lambda-1},\kappa_\lambda,\kappa_\lambda\right)$, where $\gamma=\frac{\xi_1}{(1-\xi_2-\xi_3)}<\frac{1}{s}$, continuing the same process, we get

$$G_b^p\left(\kappa_\lambda,\kappa_{\lambda+1},\kappa_{\lambda+1}\right)\leq\gamma^\lambda G_b^p\left(\kappa_0,\kappa_1,\kappa_1\right)$$

By following the same steps as in Theorem 3.5, we can show that $\{\kappa_\lambda\}$ is a Cauchy sequence, As Π is given to be complete, there must exist $\zeta\in\Pi$ such that $\{\kappa_\lambda\}\to\zeta$. Now to show that $\Phi\zeta=\zeta$.

$$G_b^p(\zeta,\Phi\zeta,\Phi\zeta)\leq s\left[G_b^p\left(\zeta,\kappa_{\lambda+1},\kappa_{\lambda+1}\right)+G_b^p\left(\kappa_{\lambda+1},\Phi\zeta,\Phi\zeta\right)\right]-G_b^p\left(\kappa_{\lambda+1},\kappa_{\lambda+1},\kappa_{\lambda+1}\right)$$
$$\leq s\left[G_b^p\left(\zeta,\kappa_{\lambda+1},\kappa_{\lambda+1}\right)+\left\{\xi_1G_b^p\left(\kappa_\lambda,\kappa_{\lambda+1},\kappa_{\lambda+1}\right)+\left(\xi_2+\xi_3\right)G_b^p(\zeta,\Phi\zeta,\Phi\zeta)\right\}\right]$$

Taking $\lambda\to\infty$, we have

$$G_b^p(\zeta,\Phi\zeta,\Phi\zeta)\leq\frac{s\left(1+\xi_1\right)}{\left(1-\xi_2-\xi_3\right)}G_b^p\left(\zeta,\kappa_{\lambda+1},\kappa_{\lambda+1}\right)\to 0,\text{ as }\xi_2+\xi_3\neq 1.\text{ Hence }\Phi\zeta=\zeta$$

Now, we will show that ζ is unique fixed point. Let ς is another fixed point such that $\Phi_\varsigma=\varsigma$.

$$G_b^p(\zeta,\varsigma,\varsigma)=G_b^p\left(\Phi\zeta,\Phi_\varsigma,\Phi\varsigma\right)\leq\left[\xi_1G_b^p(\zeta,\zeta,\zeta)+\left(\xi_1+\xi_2\right)G_b^p(\varsigma,\varsigma,\varsigma)\right]=0$$

Hence, $\zeta=\varsigma$. □

Theorem 3.7. *Let $\left(\Pi,G_b^p\right)$ be a complete G_b^p-partial metric space with $s\geq 1$ and $\Phi:\Pi\to\Pi$ be a function that fulfilling the next conditions such that for every $\kappa,\mu,\tau\in\Pi$ and $\xi\in\left[0,\frac{1}{s}\right)$,*

$$G_b^p(\Phi\kappa,\Phi\eta,\Phi\tau)\leq\xi\max\left[G_b^p(\kappa,\eta,\tau),G_b^p(\kappa,\Phi\kappa,\Phi\kappa),G_b^p(\eta,\Phi\eta,\Phi\eta),G_b^p(\tau,\Phi\tau,\Phi\tau)\right]$$

Then, there exist a unique fixed point ζ for the function Φ and $G_b^p(\zeta,\zeta,\zeta)=0$.

Proof. First is to show $G_b^p(\zeta,\zeta,\zeta)=0$. Assume that $G_b^p(\zeta,\zeta,\zeta)>0$.

$$\begin{aligned}G_b^p(\zeta,\zeta,\zeta)&=G_b^p(\Phi\zeta,\Phi\zeta,\Phi\zeta)\\&\leq\xi\max\left[G_b^p(\zeta,\zeta,\zeta),G_b^p(\zeta,\Phi\zeta,\Phi\zeta),G_b^p(\zeta,\Phi\zeta,\Phi\zeta),G_b^p(\zeta,\Phi\zeta,\Phi\zeta)\right]\\&<G_b^p(\zeta,\zeta,\zeta)\end{aligned}$$

this is not possible, hence $G_b^p(\zeta,\zeta,\zeta)=0$. Now, we shall prove the uniqueness. Let if possible there are $\varsigma\in\Pi$ such that $\Phi\varsigma=\varsigma$.

$$G_b^p(\zeta,\varsigma,\varsigma)\leq\xi\max\left[G_b^p(\zeta,\varsigma,\varsigma),\ G_b^p(\zeta,\Phi\zeta,\Phi\zeta),G_b^p(\varsigma,\Phi\varsigma,\Phi\varsigma),G_b^p(\varsigma,\Phi\varsigma,\Phi\varsigma)\right]$$
$$<G_b^p(\zeta,\varsigma,\varsigma)$$

So, we must have $G_b^p(\zeta,\varsigma,\varsigma)=0$. Hence $\zeta=\varsigma$.

Now is to show existence of of fixed point. Let $\kappa_0\in\Pi$ be any arbitrary point $\kappa_0\in\Pi$ and set $\kappa_{\lambda+1}=\Phi\kappa_\lambda$ for every $\lambda\in\Lambda$,

$$\begin{aligned}G_b^p\left(\kappa_\lambda,\kappa_{\lambda+1},\kappa_{\lambda+1}\right)&=G_b^p\left(\Phi\kappa_{\lambda-1},\Phi\kappa_\lambda,\Phi\kappa_\lambda\right)\\&\le\xi\max\left[G_b^p\left(\kappa_{\lambda-1},\kappa_\lambda,\kappa_\lambda\right),G_b^p\left(\kappa_\lambda,\kappa_{\lambda+1},\kappa_{\lambda+1}\right)\right].\end{aligned}$$

If $\max\left[G_b^p\left(\kappa_{\lambda-1},\kappa_\lambda,\kappa_\lambda\right),G_b^p\left(\kappa_\lambda,\kappa_{\lambda+1},\kappa_{\lambda+1}\right)\right]=G_b^p\left(\kappa_\lambda,\kappa_{\lambda+1},\kappa_{\lambda+1}\right)$, then we have

$$G_b^p\left(\kappa_\lambda,\kappa_{\lambda+1},\kappa_{\lambda+1}\right)\le\xi G_b^p\left(\kappa_\lambda,\kappa_{\lambda+1},\kappa_{\lambda+1}\right)<G_b^p\left(\kappa_\lambda,\kappa_{\lambda+1},\kappa_{\lambda+1}\right)$$

this is not possible. Hence,

$$G_b^p\left(\kappa_\lambda,\kappa_{\lambda+1},\kappa_{\lambda+1}\right)\le\xi G_b^p\left(\kappa_{\lambda-1},\kappa_\lambda,\kappa_\lambda\right)G_b^p\left(\kappa_\lambda,\kappa_{\lambda+1},\kappa_{\lambda+1}\right)\le\xi^\lambda G_b^p\left(\kappa_0,\kappa_1,\kappa_1\right).$$

By using the same process as in Theorem 3.5, we can easily check that $\{\kappa_\lambda\}$ is a Cauchy sequence, as Π is given to be complete, there must exist $\zeta\in\Pi$ such that $\{\kappa_\lambda\}\to\zeta$. Now, to show that ζ is fixed point of Φ.

$$\begin{aligned}G_b^p(\zeta,\Phi\zeta,\Phi\zeta)&\le s\left[G_b^p\left(\zeta,\kappa_{\lambda+1},\kappa_{\lambda+1}\right)+G_b^p\left(\kappa_{\lambda+1},\Phi\zeta,\Phi\zeta\right)\right]-G_b^p\left(\kappa_{\lambda+1},\kappa_{\lambda+1},\kappa_{\lambda+1}\right)\\&\le s\left[G_b^p\left(\zeta,\kappa_{\lambda+1},\kappa_{\lambda+1}\right)+\xi G_b^p\left(\kappa_\lambda,\zeta,\zeta\right)\right]\to 0\text{ as }\lambda\to\infty\end{aligned}$$

Hence $\Phi\zeta=\zeta$. Thus ζ is fixed point of Φ. □

Conclusion

In this paper, we introduced a new generalized structure called a G_b^p-partial metric space, which combines the main features of G_b-metric spaces and partial metric spaces. We examined its basic properties and provided an example to show that this class is strictly larger than the existing ones. The concept introduced here opens the possibility of studying further contractive mappings and other types of fixed point problems in this setting. Future work may focus on more general conditions and potential applications of this structure.

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