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SEPARATION AXIOMS IN SPECIAL NEUTROSOPHIC CRISP TOPOLOGICAL SPACES

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ABSTRACT. Neutrosophic crisp sets represent a broad generalization of classical mathematical structures and have attracted increasing attention due to the flexibility and diversity of the algebraic operations defined on them. The existence of multiple forms of intersection, union, and complement, along with the variability of points and their membership, complicates distinguishing between elements and provides a rich framework for theoretical development. The notion of neutrosophic crisp points and their membership constitutes a fundamental and nontrivial aspect of this field. In this study, we examine the separation axioms $\mathfrak{S}_\mathfrak{p}\mathcal{T}_0$, $\mathfrak{S}_\mathfrak{p}\mathcal{T}_1$, and $\mathfrak{S}_\mathfrak{p}\mathcal{T}_2$ within a neutrosophic crisp topological space constructed using first-type intersections and second-type unions and complements, considering five distinct types of neutrosophic crisp points and their membership concepts. A subspace definition within this framework is also introduced. Our analysis shows that, unlike classical topology, many standard separation properties do not uniformly hold for all types of points. Comparison with prior work on stable neutrosophic crisp spaces—based on four types of points—indicates that some theorems valid there may not extend here. All relevant theorems have been rigorously proved, with explicit examples of properties that fail, highlighting essential differences among neutrosophic crisp set frameworks.

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1. INTRODUCTION

The concept of Neutrosophy emerged in 1980 as an advanced logical framework. It generalizes fuzzy logic and extends its ability to represent and handle indeterminacy and inconsistency. This concept has opened new avenues for the development of various pure mathematical structures, particularly in topology, where new topological spaces have been constructed based on neutrosophic foundations.

In 2014, A. Salama and his collaborators introduced the concept of Neutrosophic Crisp Sets, which were classified into three main types [1,2], with precise definitions provided for each type Let $\mathfrak{Y}^\mathfrak{n} =<$

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$\mathfrak{Y}_1, \mathfrak{Y}_2, \mathfrak{Y}_3 >$ be a neutrosophic crisp set. Then:
Type I: $\mathfrak{Y}_1 \cap \mathfrak{Y}_2 = \emptyset, \mathfrak{Y}_1 \cap \mathfrak{Y}_3 = \emptyset, \mathfrak{Y}_2 \cap \mathfrak{Y}_3 = \emptyset$.
Type II: $\mathfrak{Y}_1 \cap \mathfrak{Y}_2 = \emptyset, \mathfrak{Y}_1 \cap \mathfrak{Y}_3 = \emptyset, \mathfrak{Y}_2 \cap \mathfrak{Y}_3 = \emptyset$, and $\mathfrak{Y}_1 \cup \mathfrak{Y}_2 \cup \mathfrak{Y}_3 = \mathfrak{X}$.
Type III: $\mathfrak{Y}_1 \cap \mathfrak{Y}_2 \cap \mathfrak{Y}_3 = \emptyset$, and $\mathfrak{Y}_1 \cup \mathfrak{Y}_2 \cup \mathfrak{Y}_3 = \mathfrak{X}$.

They also defined the neutrosophic crisp empty and universal sets as follows:

$$\emptyset_1^{\mathfrak{n}} =< \emptyset, \emptyset, \mathfrak{X} >, \emptyset_2^{\mathfrak{n}} =< \emptyset, \mathfrak{X}, \emptyset >, \emptyset_3^{\mathfrak{n}} =< \emptyset, \mathfrak{X}, \mathfrak{X} >, \emptyset_4^{\mathfrak{n}} =< \emptyset, \emptyset, \emptyset > .$$

$$\mathfrak{X}_1^{\mathfrak{n}} =< \mathfrak{X}, \emptyset, \emptyset, >, \mathfrak{X}_2^{\mathfrak{n}} =< \mathfrak{X}, \mathfrak{X}, \emptyset >, \mathfrak{X}_3^{\mathfrak{n}} =< \mathfrak{X}, \emptyset, \mathfrak{X} >, \mathfrak{X}_4^{\mathfrak{n}} =< \mathfrak{X}, \mathfrak{X}, \mathfrak{X} > .$$

In addition, they defined three types of complements of neutrosophic crisp sets. Let $\mathrm{C}^{\mathfrak{n}} =< C_1, C_2, C_3 >$ be a neutrosophic crisp set in $\mathfrak{X}$. The complements are defined as follows:
Type I: $(C^{\mathfrak{n}})^{c_1} =< {C_1}^c, {C_2}^c, {C_3}^c >$,
Type II: $(C^{\mathfrak{n}})^{c_2} =< C_3, C_2, C_1 >$,
Type III: $(C^{\mathfrak{n}})^{c_3} =< C_3, {C_2}^c, C_1 >$.

Furthermore, the concept of neutrosophic crisp points [3] was established, which later became the basis for further extensions aimed at clarifying the membership of these points in neutrosophic crisp sets in a more rigorous and comprehensive manner. The theory of neutrosophic crisp sets has demonstrated its effectiveness in several important applications, including image processing [4], as well as in geographic information systems [5] and database processing [6], highlighting its significance from both theoretical and applied perspectives.

In the context of topological development, researchers have investigated separation axioms within neutrosophic crisp topological spaces, where the first definition of such axioms in this framework was introduced in 2018 [7], paving the way for further advanced studies in this area.

In [8], we introduced a definition of a topological space based on a Type-1 intersection, complement, and a Type-2 union, leading to a structure with distinct algebraic characteristics. In [9], the concept of a stable neutrosophic crisp topological space was proposed, while in [10,11] separation axioms were developed based on four types of points, corresponding to four types of membership relations within the stable topological framework.

In this paper, we extend these developments by introducing separation axioms based on five types of points, corresponding to five types of membership relations, according to the definition presented in [8]. Through a systematic comparison with both classical topological spaces and stable neutrosophic crisp topological spaces, it is shown that several separation properties do not hold for all types of points; they may be satisfied in certain spaces but fail in the space under consideration. Moreover, the results reveal that the choice of algebraic operations (intersection, union, and complement) plays a crucial role in determining the validity of these properties, emphasizing the sensitivity of the topological structure to the nature of the adopted operations.

This paper is organized into three main sections. Section 2 presents the fundamental definitions and supporting theorems, while Section 3 introduces the separation axioms, proves the properties that hold, and provides illustrative examples for cases where these properties fail.

Table 1. List of symbols

Symbol	Descriptions
$\mathfrak{S}_{\mathfrak{p}}\, C_s$	Neutrosophic Crisp Sets
$\mathfrak{S}\mathbb{C}\mathfrak{ms}$	Neutrosophic Crisp Point
$\mathfrak{S}_{\mathfrak{p}}\mathbb{C}\mathcal{T}_{(1,2)}-$ space	Neutrosophic Crisp Topological Spaces
$\mathfrak{S}\mathbb{C}\mathbb{C}$- set	Neutrosophic Crisp Closed Set
$\mathfrak{S}_{\mathfrak{p}}\mathbb{C}\mathcal{O}$- Set	Neutrosophic Crisp Open Set
$\mathfrak{S}_s$-$\mathfrak{S}_{\mathfrak{p}}\mathbb{C}\mathcal{T}_{(1,2)}-$ space	Subspace Neutrosophic Crisp Topology
$\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_0-$ space	Neutrosophic Crisp $\mathcal{T}_0-$– space
$\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_1-$ space	Neutrosophic crisp $\mathcal{T}_1-$– space
$\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_2-$ space	Neutrosophic Crisp $\mathcal{T}_2-$ space

2. Preliminaries

Definition 2.1. [12] Let $\mathcal{E}^{\mathfrak{n}} =< \mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3 >$ and $\mathcal{L}^{\mathfrak{n}} =< \mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3 >$ be two neutrosophic crisp sets in $\mathfrak{X}$. Their union is defined as follows:

Type I: $\mathcal{E}^{\mathfrak{n}} \cup_1 \mathcal{L}^{\mathfrak{n}} =< \mathcal{E}_1 \cup \mathcal{L}_1, \mathcal{E}_2 \cup \mathcal{L}_2, \mathcal{E}_3 \cap \mathcal{L}_3 >$.

Type II: $\mathcal{E}^{\mathfrak{n}} \cup_2 \mathcal{L}^{\mathfrak{n}} =< \mathcal{E}_1 \cup \mathcal{L}_1, \mathcal{E}_2 \cap \mathcal{L}_2,\ \mathcal{E}_3 \cap \mathcal{L}_3 >$.

Similarly, their intersection is defined as:

Type I: $\mathcal{E}^{\mathfrak{n}} \cap_1 \mathcal{L}^{\mathfrak{n}} = < \mathcal{E}_1 \cap \mathcal{L}_1, \mathcal{E}_2 \cap \mathcal{L}_2, \mathcal{E}_3 \cup \mathcal{L}_3 >$.

Type II: $\mathcal{E}^{\mathfrak{n}} \cap_2 \mathcal{L}^{\mathfrak{n}} = < \mathcal{E}_1 \cap \mathcal{L}_1,\ \mathcal{E}_2 \cup \mathcal{L}_2, \mathcal{E}_3 \cup \mathcal{L}_3 >$.

Definition 2.2. [12] For any two a neutrosophic crisp sets $\mathcal{E}^{\mathfrak{n}} =< \mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3 >$ and $\mathcal{L}^{\mathfrak{n}} =< \mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3 >$ in $\mathfrak{X}$, two subset relations are defined as:

Type I: $\mathcal{E}^{\mathfrak{n}} \subseteq_1 \mathcal{L}^{\mathfrak{n}} \Leftrightarrow \mathcal{E}_1 \subseteq \mathcal{L}_1, \mathcal{E}_2 \subseteq \mathcal{L}_2, \mathcal{L}_3 \subseteq \mathcal{E}_3$.

Type II: $\mathcal{E}^{\mathfrak{n}} \subseteq_2 \mathcal{L}^{\mathfrak{n}} \Leftrightarrow \mathcal{E}_1 \subseteq \mathcal{L}_1, \mathcal{L}_2 \subseteq \mathcal{E}_2, \mathcal{L}_3 \subseteq \mathcal{E}_3$.

Definition 2.3. [9, 10] Let $\mathfrak{X}$ be a non-empty set. A neutrosophic crisp point ($\mathfrak{S}\mathbb{C}\mathfrak{ms}$) is defined as:

i. $\mathfrak{m}^{\mathfrak{n}_1} =< \emptyset,\ \{\mathfrak{m}\}, \{\mathfrak{m}\}^c >$

ii. $\mathfrak{m}^{\mathfrak{n}_2} =< \{\mathfrak{m}\}, \emptyset, \{\mathfrak{m}\}^c >$

iii. $\mathfrak{m}^{\mathfrak{n}_3} =<, \emptyset, \emptyset, \{\mathfrak{m}\} >$, where $\{\mathfrak{m}\}$ is singleton.

iv. $\mathfrak{m}^{\mathfrak{n}4} =< \emptyset, \{\mathfrak{m}\}, \emptyset >$, where $\{\mathfrak{m}\}$ is singleton.

v. $\mathfrak{m}^{\mathfrak{n}5} =< \{\mathfrak{m}\}, \emptyset, \emptyset >$, where $\{\mathfrak{m}\}$ is singleton.

Definition 2.4. [11] Let $\mathfrak{Y}^{\mathfrak{n}} =< \mathfrak{Y}_1, \mathfrak{Y}_2, \mathfrak{Y}_3 >$ be a neutrosophic crisp set in $\mathfrak{X}$. Then

i. $\mathfrak{m}^{\mathfrak{n}_1} \in_1 \mathfrak{Y}^{\mathfrak{n}} \Leftrightarrow \mathfrak{m} \notin \mathfrak{Y}_3$.

ii. $\mathfrak{m}^{\mathfrak{n}_2} \in_2 \mathfrak{Y}^{\mathfrak{n}} \Leftrightarrow \mathfrak{m} \in \mathfrak{Y}_1$.

iii. $\mathfrak{m}^{\mathfrak{n}_3} \in_3 \mathfrak{Y}^{\mathfrak{n}} \Leftrightarrow \mathfrak{m} \in \mathfrak{Y}_3$.

iv. $\mathfrak{m}^{\mathfrak{n}_4} \in_4 \mathfrak{Y}^{\mathfrak{n}} \Leftrightarrow \mathfrak{m} \in \mathfrak{Y}_2$.

v. $\mathfrak{m}^{\mathfrak{n}_5} \in_5 \mathfrak{Y}^{\mathfrak{n}} \Leftrightarrow \mathfrak{m} \in \mathfrak{Y}_1$.

Definition 2.5. [11] Let $\mathcal{H}^{\mathfrak{n}} =< \mathcal{H}_1, \mathcal{H}_2, \mathcal{H}_3 >$ be a neutrosophic crisp set in $\mathfrak{X}$. Then the non-membership relations are defined as follows:

i. $\mathfrak{m}^{\mathfrak{n}_1} \notin_1 \mathcal{H}^{\mathfrak{n}} \Leftrightarrow \mathfrak{m} \notin \mathcal{H}_2 \wedge \mathfrak{m} \in \mathcal{H}_3$.

ii. $\mathfrak{m}^{\mathfrak{n}_2} \notin_2 \mathcal{H}^{\mathfrak{n}} \Leftrightarrow \mathfrak{m} \in \mathcal{H}_3 \wedge \mathfrak{m} \notin \mathcal{H}_1$.

iii. $\mathfrak{m}^{\mathfrak{n}_3} \notin_3 \mathcal{H}^{\mathfrak{n}} \Leftrightarrow \mathfrak{m} \notin \mathcal{H}_3$.

iv. $\mathfrak{m}^{\mathfrak{n}_4} \notin_4 \mathcal{H}^{\mathfrak{n}} \Leftrightarrow \mathfrak{m} \notin \mathcal{H}_2$

We introduce a fifth type of non-membership relation, defined as follows:

$$\mathfrak{m}^{\mathfrak{n}_5} \notin_5 \mathcal{H}^{\mathfrak{n}} \Leftrightarrow \mathfrak{m} \notin \mathcal{H}_1$$

Definition 2.6. [9] Let $\mathcal{E}^{\mathfrak{n}} =< \mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3 >$ and $\mathcal{L}^{\mathfrak{n}} =< \mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3 >$ be two neutrosophic crisp sets in $\mathfrak{X}$. Then $\mathcal{E}^{\mathfrak{n}} \neq \mathcal{L}^{\mathfrak{n}}$ if and only if $\mathfrak{m}^{\mathfrak{n}_i} \in_i \mathcal{E}^{\mathfrak{n}}, \mathfrak{m}^{\mathfrak{n}_i} \notin_i \mathcal{L}^{\mathfrak{n}}$ or $\mathfrak{m}^{\mathfrak{n}_i} \notin_i \mathcal{E}^{\mathfrak{n}}, \mathfrak{m}^{\mathfrak{n}_i} \in_i \mathcal{L}^{\mathfrak{n}}, i = 1, 2, 3, 4, 5$.

- $\mathfrak{m}^{\mathfrak{n}_i} \neq q^{\mathfrak{n}_i}$ if and only if $\mathfrak{m} \neq q$.

Definition 2.7. [13] If $\mathfrak{P}^{\mathfrak{n}} =< \mathfrak{P}_1, \mathfrak{P}_2, \mathfrak{P}_3 >$ is a neutrosophic crisp open in $\mathtt{Y}$, then the preimage of $\mathfrak{P}^{\mathfrak{n}}$ under f, defined by $f^{-1}(\mathfrak{P}^{\mathfrak{n}}) =< f^{-1}(\mathfrak{P}_1), f^{-1}(\mathfrak{P}_2), f^{-1}(\mathfrak{P}_3) >$, is a neutrosophic crisp set in $\mathfrak{X}$.

Similarly, If $\mathfrak{J}^{\mathfrak{n}} =< \mathfrak{J}_1, \mathfrak{J}_2, \mathfrak{J}_3 >$ is a neutrosophic crisp set in $\mathfrak{X}$, then the image of $\mathfrak{J}^{\mathfrak{n}}$ under f, defined by $f(\mathfrak{J}^{\mathfrak{n}}) =< f(\mathfrak{J}_1), f(\mathfrak{J}_2), f(\mathfrak{J}_3) >$, is a neutrosophic crisp set in $\mathtt{Y}$.

Corollary 2.8. [2] *Let $f : \mathfrak{X} \to \mathtt{Y}$ be a function, and let $\mathfrak{J}^{\mathfrak{n}} =< \mathfrak{J}_1, \mathfrak{J}_2, \mathfrak{J}_3 >, \mathcal{M}^{\mathfrak{n}} =< \mathcal{M}_1, \mathcal{M}_2, \mathcal{M}_3 >$ be $\mathfrak{S}_{\mathfrak{p}}\mathcal{C}_s$ in $\mathfrak{X}$. Then:*

i. $\mathfrak{J}^{\mathfrak{n}} \subseteq_1 \mathcal{M}^{\mathfrak{n}} \Leftrightarrow f(\mathfrak{J}^{\mathfrak{n}}) \subseteq_1 f(\mathcal{M}^{\mathfrak{n}})$

ii. $\mathfrak{J}^{\mathfrak{n}} \subseteq_2 \mathcal{M}^{\mathfrak{n}} \Leftrightarrow f(\mathfrak{J}^{\mathfrak{n}}) \subseteq_2 f(\mathcal{M}^{\mathfrak{n}})$

Corollary 2.9. [2] *Let $f : \mathfrak{X} \to \mathtt{Y}$ be a function, and let $\mathfrak{P}^{\mathfrak{n}} =< \mathfrak{P}_1, \mathfrak{P}_2, \mathfrak{P}_3 >$, $\mathfrak{f}^{\mathfrak{n}} =< \mathfrak{f}_1, \mathfrak{f}_2, \mathfrak{f}_3 >$ be a $\mathfrak{S}_{\mathfrak{p}}\mathcal{C}_s$ in $\mathtt{Y}$. Then:*

i. $\mathfrak{P}^{\mathfrak{n}} \subseteq_1 \mathfrak{f}^{\mathfrak{n}} \Leftrightarrow f^{-1}(\mathfrak{P}^{\mathfrak{n}}) \subseteq_1 f^{-1}(\mathfrak{f}^{\mathfrak{n}})$

ii. $\mathfrak{P}^{\mathfrak{n}} \subseteq_2 \mathfrak{f}^{\mathfrak{n}} \Leftrightarrow f^{-1}(\mathfrak{P}^{\mathfrak{n}}) \subseteq_2 f^{-1}(\mathfrak{f}^{\mathfrak{n}})$

Definition 2.10. [8] The pair $(\mathfrak{X}, \mathcal{T})$ is called a Neutrosophic Crisp Topological ($\mathfrak{S}_{\mathfrak{p}}\mathbb{C}\mathcal{T}_{(1,2)}$-space) when the following criteria are satisfied:

i. $\emptyset_1^{\mathfrak{n}} =< \emptyset, \emptyset, \mathfrak{X} > \in \mathcal{T}$, and $\mathfrak{X}_1^{\mathfrak{n}} =< \mathfrak{X}, \emptyset, \emptyset > \in \mathcal{T}$.

ii. For any $\mathfrak{A}^{\mathfrak{n}}, O^{\mathfrak{n}} \in \mathcal{T}$, the set $\mathfrak{A}^{\mathfrak{n}} \cap_1 O^{\mathfrak{n}} \in \mathcal{T}$.

iii. If $\mathfrak{A}^{\mathfrak{n}} j \in \mathcal{T} \forall j \in J$, then $\cup_{j2} \mathfrak{A}^{\mathfrak{n}} \in \mathcal{T}$.

For any $O^{\mathfrak{n}} \in \mathcal{T}$, it is a neutrosophic crisp open set ($\mathfrak{S}_{\mathfrak{p}}\mathbb{C}O$ - set), and $(O^{\mathfrak{n}})^{c_2}$ is a neutrosophic crisp closed set ($\mathfrak{S}_{\mathfrak{p}}\mathbb{C}\mathbb{C}$ - set).

Definition 2.11. [8] Let $(\mathfrak{X}, \mathcal{T}_{\mathfrak{X}})$ and $(\mathtt{Y}, \mathcal{T}_{\mathtt{Y}})$ be $\mathfrak{S}_{\mathfrak{p}}\mathbb{C}\mathcal{T}_{(1,2)}$-spaces. A function $f : \mathfrak{X} \to \mathtt{Y}$ is called neutrosophic crisp continuous$_{(1,2)}$ function if the inverse image of every $\mathfrak{S}_{\mathfrak{p}}\mathbb{C}O$- set in $\mathtt{Y}$ is a $\mathfrak{S}_{\mathfrak{p}}\mathbb{C}O$ - set in $\mathfrak{X}$.

Definition 2.12.

(1) A function $f : \mathfrak{X} \to \mathtt{Y}$ is injective if $\forall\, \mathfrak{m}^{\mathfrak{n}_i} \neq q^{\mathfrak{n}_i}, \mathfrak{m}^{\mathfrak{n}_i}, q^{\mathfrak{n}_i}$ are $\mathfrak{S}\mathbb{C}\mathfrak{ms}$ in $\mathfrak{X}$, then $f\left(\mathfrak{m}^{\mathfrak{n}_i}\right) \neq f\left(q^{\mathfrak{n}_i}\right), i = 1, 2, 3, 4, 5$.

(2) A function $f : \mathfrak{X} \to \mathtt{Y}$ is surjective if $\forall\, s^{\mathfrak{n}_i}$ is $\mathfrak{S}\mathbb{C}\mathfrak{ms}$ in $\mathtt{Y}$ $\exists\, \mathfrak{m}^{\mathfrak{n}_i}$ is $\mathfrak{S}\mathbb{C}\mathfrak{ms}$ in $\mathfrak{X}$ such that $f\left(\mathfrak{m}^{\mathfrak{n}_i}\right) = \left(s^{\mathfrak{n}_i}\right)$, $i = 1, 2, 3, 4, 5$.

Definition 2.13. Let $(\mathfrak{X}, \mathcal{T}_{\mathfrak{X}})$ and $(\mathtt{Y}, \mathcal{T}_{\mathtt{Y}})$ be $\mathfrak{S}_{\mathfrak{p}}\mathbb{C}\mathcal{T}_{(1,2)}$-spaces. A function $f : \mathfrak{X} \to \mathtt{Y}$ is called a neutrosophic crisp open$_{(1,2)}$ function if for every neutrosophic crisp open set $\mathfrak{A}^{\mathfrak{n}}$ in $\mathfrak{X}$, the image $f(\mathfrak{A}^{\mathfrak{n}})$ is a neutrosophic crisp open set in $\mathtt{Y}$.

Definition 2.14. A function $f : (\mathfrak{X}, \mathcal{T}_{\mathfrak{X}}) \to (\mathtt{Y}, \mathcal{T}_{\mathtt{Y}})$ is called neutrosophic crisp homeomorphic$_{(1,2)}$ function if f bijective, continuous$_{(1,2)}$ and open$_{(1,2)}$ function .

Theorem 2.15. *Let $\mathfrak{Y}^{\mathfrak{n}} =< \mathfrak{Y}_1, \mathfrak{Y}_2, \mathfrak{Y}_3 >, \mathrm{L}^{\mathfrak{n}} =< L_1, \emptyset, \mathrm{L}_3 >$ be neutrosophic crisp sets in $\mathfrak{X}$. if $\mathfrak{m}^{\mathfrak{n}_i} \in_i \mathrm{L}^{\mathfrak{n}}$ and $\mathfrak{m}^{\mathfrak{n}_i} \notin_i \mathfrak{Y}^{\mathfrak{n}}$, then $\mathfrak{m}^{\mathfrak{n}_i} \notin_i \mathfrak{Y}^{\mathfrak{n}} \cap_1 L^{\mathfrak{n}}, i = 1, 2, 5$.*

Proof.

Case $i = 1$

From the Definitions 2.3, 2.4 and 2.5

$$\mathfrak{m}^{\mathfrak{n}_1} =<, \emptyset, \{\mathfrak{m}\}, \{\mathfrak{m}\}^c >, \mathfrak{m}^{\mathfrak{n}_1} \in_1 L^{\mathfrak{n}} \Leftrightarrow \mathfrak{m} \notin L_3$$

So, $L^{\mathfrak{n}} \cap_1 \mathfrak{Y}^{\mathfrak{n}} =< L_1 \cap \mathfrak{Y}_1, \emptyset \cap \mathfrak{Y}_2, L_3 \cup \mathfrak{Y}_3 >$ by Definition 2.1.

Since, $\mathfrak{m}^{\mathfrak{n}_1} \notin_1 \mathfrak{Y}^{\mathfrak{n}}$. So, $\mathfrak{m} \in \mathfrak{Y}_3$ and. Hence, $\mathfrak{m} \in L_3 \cup \mathfrak{Y}_3$. Thus, $\mathfrak{m}^{\mathfrak{n}_1} \notin_1 \mathfrak{Y}^{\mathfrak{n}} \cap_1 L^{\mathfrak{n}}$.

Case $i = 2$

$\mathfrak{m}^{\mathfrak{n}_1} =< \{\mathfrak{m}\}, \emptyset, \{\mathfrak{m}\}^c > \mathfrak{m}^{\mathfrak{n}_2} \in_2 L^{\mathfrak{n}} \Leftrightarrow \mathfrak{m} \in L_1$.

So,

$$L^{\mathfrak{n}} \cap_1 \mathfrak{Y}^{\mathfrak{n}} =< L_1 \cap \mathfrak{Y}_1, \emptyset \cap \mathfrak{Y}_2, L_3 \cup \mathfrak{Y}_3 >$$

Since, $\mathfrak{m}^{\mathfrak{n}_2} \notin_2 \mathfrak{Y}^{\mathfrak{n}}$. So, $\mathfrak{m} \notin \mathfrak{Y}_1$. Hence, $\mathfrak{m} \notin L_1 \cap \mathfrak{Y}_1$. Thus, $\mathfrak{m}^{\mathfrak{n}_2} \notin_2 \mathfrak{Y}^{\mathfrak{n}} \cap_1 L^{\mathfrak{n}}$.

Case $i = 5$

$\mathfrak{m}^{\mathfrak{n}_5} =< \{\mathfrak{m}\}, \emptyset, \emptyset >, \mathfrak{m}^{\mathfrak{n}_5} \in_5 L^{\mathfrak{n}} \Leftrightarrow \mathfrak{m} \in L_1$.

So, $\mathrm{L}^{\mathfrak{n}} \cap_1 \mathfrak{Y}^{\mathfrak{n}} =< L_1 \cap \mathfrak{Y}_1, \emptyset \cap \mathfrak{Y}_2, L_3 \cup \mathfrak{Y}_3 >$.

Since, $\mathfrak{m}^{\mathfrak{n}_5} \notin_5 \mathfrak{Y}^{\mathfrak{n}}$. So, $\mathfrak{m} \notin \mathfrak{Y}_1$. Hence, $\mathfrak{m} \notin L_1 \cap \mathfrak{Y}_1$. Thus, $\mathfrak{m}^{\mathfrak{n}_5} \notin_5 \mathfrak{Y}^{\mathfrak{n}} \cap_1 L^{\mathfrak{n}}$.

□

Remark 2.16. For $i = 3, 4$, Theorem 2.15 does not necessarily hold.

Case $i = 3$

For example let $\mathfrak{m}^{\mathfrak{n}_3} =< \emptyset, \emptyset, \{c\} > \in_3 L^{\mathfrak{n}} =< \{a\}, \emptyset, \{c\} >$, $\mathfrak{m}^{\mathfrak{n}_3} \notin_3 \mathfrak{Y}^{\mathfrak{n}} =< \{a, b\}, \emptyset, \emptyset, >$. But, $\mathfrak{m}^{\mathfrak{n}_3} \in_3 \mathrm{L}^{\mathfrak{n}} \cap_1 \mathfrak{Y}^{\mathfrak{n}} =< \{a\}, \emptyset, \{c\} >$. Thus the theorem fails for $i = 3$.

Case $i = 4$

It is clear that

Theorem 2.17. *Let* $\mathfrak{Y}^{\mathfrak{n}} = < \mathfrak{Y}_1, \mathfrak{Y}_2, \mathfrak{Y}_3 >, \mathrm{L}^{\mathfrak{n}} =< \mathrm{L}_1, \mathrm{L}_2, \mathrm{L}_3 >$ *be neutrosophic crisp sets in* $\mathfrak{X}$*. If* $\mathfrak{Y}^{\mathfrak{n}} \cap_1 \mathrm{L}^{\mathfrak{n}} = \emptyset_1^{\mathfrak{n}}$*, then for any* $\mathfrak{m}^{\mathfrak{n}_i} \in_i \mathfrak{Y}^{\mathfrak{n}}$*, it is follows that then* $\mathfrak{m}^{\mathfrak{n}_i} \notin_i \mathrm{L}^{\mathfrak{n}}$*, for* $i = 1, 2, 4, 5$.

Proof. Assume that $\mathfrak{Y}^{\mathfrak{n}} \cap_1 \mathrm{L}^{\mathfrak{n}} = \emptyset_1^{\mathfrak{n}}$ and $\mathfrak{m}^{\mathfrak{n}_i} \in_i \mathfrak{Y}^{\mathfrak{n}}$ for $i = 1, 2, 4, 5$. Then by Definition 2.1 we have:

$$\mathfrak{Y}^{\mathfrak{n}} \cap_1 L^{\mathfrak{n}} = < \mathfrak{Y}_1 \cap L_1, \mathfrak{Y}_2 \cap L_2, \mathfrak{Y}_3 \cup L_3 >= \emptyset_1^{\mathfrak{n}} =< \emptyset, \emptyset, \mathfrak{X} > .$$

Case $i = 1$

If $\mathfrak{m}^{\mathfrak{n}_1} \in_1 \mathfrak{Y}^{\mathfrak{n}}$, then $\mathfrak{m} \notin \mathfrak{Y}_3$ by Definitions 2.4. Since $\mathfrak{Y}_3 \cup L_3 = \mathfrak{X}$ it is follows that $\mathfrak{m} \in \mathrm{L}_3$ which implies $\mathfrak{m}^{\mathfrak{n}_1} \notin_1 \mathrm{L}^{\mathfrak{n}}$ by Definitions 2.5.

Case $i = 2$

If $\mathfrak{m}^{\mathfrak{n}_2} \in_2 \mathfrak{Y}^{\mathfrak{n}}$, then $\mathfrak{m} \in \mathfrak{Y}_1$ by Definitions 2.3 and 2.4. Since $\mathfrak{Y}_1 \cap L_1 = \emptyset$ we obtain $\mathfrak{m} \notin \mathrm{L}_1$, hence $\mathfrak{m}^{\mathfrak{n}_2} \notin_2 \mathrm{L}^{\mathfrak{n}}$ by Definitions 2.5.

Case $i = 4, 5$

The proof it is similar to that of Case i=2 Therefore, if $\mathfrak{Y}^{\mathfrak{n}} \cap_1 \mathrm{L}^{\mathfrak{n}} = \emptyset_1^{\mathfrak{n}}$, then $\mathfrak{m}^{\mathfrak{n}_i} \notin_i \mathrm{L}^{\mathfrak{n}}$, for $i = 1, 2, 4, 5$. □

Remark 2.18. For $i = 3, 4$, Theorem 2.17 does not necessarily hold. For example, let $\mathfrak{m}^{\mathfrak{n}_3} =< \emptyset, \emptyset, \{c\} > \in_3 \mathfrak{Y}^{\mathfrak{n}} =< \emptyset, \emptyset, \{b, c\} >$, and let $\mathrm{L}^{\mathfrak{n}} =< \emptyset, \emptyset, \{a, c\} >$.

Then $\mathfrak{Y}^{\mathfrak{n}} \cap_1 \mathrm{L}^{\mathfrak{n}} = \emptyset_1^{\mathfrak{n}}$, but $\mathfrak{m}^{\mathfrak{n}_3} \in_3 \mathfrak{Y}^{n}$ does not imply $\mathfrak{m}^{\mathfrak{n}_3} \in_3 \mathrm{L}^{n}$.

3. Separation Axioms in Neutrosophic Crisp Topological Spaces ($\mathfrak{S}_\mathfrak{p}\mathbb{C}\mathcal{T}_{(1,2)}$-Space)

In this section, the separation axioms are examined based on the presented topological definition. In addition, some properties relying to five types and five types of memberships are investigated.

Definition 3.1. Let $(\mathfrak{X}, \mathcal{T})$ be a $\mathfrak{S}_\mathfrak{p}\mathbb{C}\mathcal{T}_{(1,2)}-$ space and let $\mathfrak{U}^\mathfrak{n} =< \mathfrak{U}_1, \emptyset, \mathfrak{U}_3 >\subseteq_1 \mathfrak{X}_1^\mathfrak{n}$. Then, the Neutrosophic Crisp Topology defined on $\mathfrak{U}^\mathfrak{n}$ is called the subspace Neutrosophic Crisp Topology (briefly $\mathfrak{S}_s - \mathfrak{S}_\mathfrak{p}\mathbb{C}\mathcal{T}_{(1,2)}$- space), denoted by $(\mathcal{T}_{\mathfrak{U}^\mathfrak{n}})$, where $\mathcal{T}_{\mathfrak{U}^\mathfrak{n}} = \mathfrak{U}^\mathfrak{n} \cap_1 \mathfrak{Y}^\mathfrak{n} : \mathfrak{Y}^\mathfrak{n} \in \mathcal{T}$.

Theorem 3.2. *The pair* $(\mathfrak{U}^\mathfrak{n}, \mathcal{T}_{\mathfrak{U}^\mathfrak{n}})$ *is a Neutrosophic Crisp Topological space.*

Proof.

(1) Since, $\mathfrak{X}_1^\mathfrak{n} =< \mathfrak{X}, \emptyset, \emptyset >\in \mathcal{T}, \emptyset_1^\mathfrak{n} =< \emptyset, \emptyset, \mathfrak{X} >\in \mathcal{T}$. So,$\mathfrak{U}^\mathfrak{n} =< \mathfrak{U}_1, \emptyset, \mathfrak{U}_3 >, \mathfrak{U}^\mathfrak{n} \cap_1 \emptyset_1^\mathfrak{n} = \emptyset_1^\mathfrak{n} \in \mathcal{T}_{\mathfrak{U}^\mathfrak{n}}$ and $\mathfrak{U}^\mathfrak{n} \cap_1 \mathfrak{X}_1^\mathfrak{n} =< \mathfrak{U}_1 \cap \mathfrak{X}, \emptyset \cap \emptyset, \mathfrak{U}_3 \cup \emptyset >= \mathfrak{U}^\mathfrak{n}\in \mathcal{T}_{\mathfrak{U}^\mathfrak{n}}$.

(2) If $G^\mathfrak{n}{}_t \in \mathcal{T}_{\mathfrak{U}^\mathfrak{n}}\ \forall\, t$. So, $\exists\, \mathfrak{Y}^\mathfrak{n}{}_t \in \mathcal{T} \ni\ G^\mathfrak{n}{}_t = \mathfrak{U}^\mathfrak{n} \cap_1 \mathfrak{Y}^\mathfrak{n}{}_t\ \forall\, t$.. So,

$$\begin{aligned}\cup_{2t} G^\mathfrak{n}{}_t &= \cup_{2t}(\mathfrak{U}^\mathfrak{n} \cap_1 \mathfrak{Y}^\mathfrak{n}{}_t) = \cup_{2t}(< \mathfrak{Y}_{1t} \cap \mathfrak{U}_1, \mathfrak{Y}_{2t} \cap \emptyset, \mathfrak{U}_3 \cup \mathfrak{Y}_{3t} >)\\ &= (< \cup_t(\mathfrak{Y}_{1t} \cap \mathfrak{U}_1), \emptyset, \cap_t(\mathfrak{U}_3 \cup \mathfrak{Y}_{3t}) >) = (< \mathfrak{U}_1 \cap (\cup_t \mathfrak{Y}_{3t}), \emptyset, \mathfrak{U}_3 \cup (\cap_t \mathfrak{Y}_{3t}) >)\\ &= \mathfrak{U}^\mathfrak{n} \cap_1 (\cup_{2t} \mathfrak{Y}^\mathfrak{n}{}_t).\end{aligned}$$

Hence, $\cup_{2t}\ G^\mathfrak{n}{}_t\in \mathcal{T}_{\mathfrak{U}^\mathfrak{n}}$.

(3) If $G^\mathfrak{n}{}_i \in \mathcal{T}_{\mathfrak{U}^\mathfrak{n}}\ \forall\, 1 \le i \le n$. So, $\exists\, \mathfrak{Y}^\mathfrak{n}{}_i \in \mathcal{T} \ni G^\mathfrak{n}{}_i = \mathfrak{U}^\mathfrak{n} \cap_1 \mathfrak{Y}^\mathfrak{n}{}_i\ \forall\, 1 \le i \le n$..
So,

$$\begin{aligned}\cap_{1i=1}^n G^\mathfrak{n}{}_i &= \cap_{1i=1}^n(\mathfrak{U}^\mathfrak{n} \cap_1 \mathfrak{Y}^\mathfrak{n}{}_i) = \cap_{1i=1}^n(< \mathfrak{Y}_{1t} \cap \mathfrak{U}_1, \mathfrak{Y}_{2t} \cap \emptyset, \mathfrak{U}_3 \cup \mathfrak{Y}_{3t} >)\\ &= (< \cap_{i=1}^n(\mathfrak{Y}_{1t} \cap \mathfrak{U}_1), \emptyset, \cap_{i=1}^n(\mathfrak{Y}_{1t} \cup \mathfrak{U}_1) >) =< \mathfrak{U}_1 \cap (\cap_{i=1}^n \mathfrak{Y}_{1t})\ , \emptyset, \mathfrak{U}_3 \cup (\cap_{i=1}^n \mathfrak{Y}_{3t}) >\\ &= \mathfrak{U}^\mathfrak{n} \cap (\cap_{1i=1}^n\ \mathfrak{Y}^\mathfrak{n}{}_i).\end{aligned}$$

Hence, $\cap_{1i=1}^n G^\mathfrak{n}{}_i\in \mathcal{T}_{\mathfrak{U}^\mathfrak{n}}$.

Therefore, $(\mathfrak{U}^\mathfrak{n}, \mathcal{T}_{\mathfrak{U}^\mathfrak{n}})$ is a Neutrosophic Crisp Topological space. □

Example 3.3. Let $\mathfrak{X} = \{a, b, c\}$ and define $\mathcal{T} = \{\emptyset_1^\mathfrak{n}, \mathfrak{X}_1^\mathfrak{n}, \mathfrak{A}^\mathfrak{n}, B^\mathfrak{n}, C^\mathfrak{n}, O^\mathfrak{n}, E^\mathfrak{n}, F^\mathfrak{n}, G^\mathfrak{n} H^\mathfrak{n}, I^\mathfrak{n}, J^\mathfrak{n}\}$, $\emptyset_1^\mathfrak{n} = < \emptyset, \emptyset, \mathfrak{X} >, \mathfrak{X}_1^\mathfrak{n} = < \mathfrak{X}, \emptyset, \emptyset >$, $\mathfrak{A}^\mathfrak{n} = < \{a\}, \{b\}, \{c\} >$, $B^\mathfrak{n} = < \{b\}, \{a\}, \{c\} >, C^\mathfrak{n} = < \{a\}, \{c\}, \{b\} >$, $O^\mathfrak{n} = < \{a\}, \emptyset, \{c\} >, E^\mathfrak{n} =< \{a\}, \emptyset, \{b\} >$, $F^\mathfrak{n} = < \{b\}, \emptyset, \{c\} >$,$G^\mathfrak{n} = < \emptyset, \emptyset, \{c\} >$,$H^\mathfrak{n} = < \{a\}, \emptyset, \emptyset, >$, $I^\mathfrak{n} = < \emptyset, \emptyset, \{c, b\} >$, $J^\mathfrak{n} = < \{a, b\}, \emptyset, \emptyset, >$. So, $(\mathfrak{X}, \mathcal{T})$ is a $\mathfrak{S}_\mathfrak{p}\mathbb{C}\mathcal{T}_{(1,2)}-$space (by Definition 2.10).

Let $\mathfrak{U}^\mathfrak{n} =< \{b\}, \emptyset, \{a, c\} > \subseteq_1 \mathfrak{X}_1^\mathfrak{n}$, then the subspace neutrosophic crisp topology is $\mathcal{T}_{\mathfrak{U}^\mathfrak{n}} = \emptyset_1^\mathfrak{n}, \mathfrak{U}^\mathfrak{n}, L^\mathfrak{n}, L^\mathfrak{n} =< \emptyset, \emptyset, \{a, c\} >$.

Remark 3.4. Let $\mathcal{T}_{\mathfrak{U}^{\mathfrak{n}}}$ be a $\mathfrak{S}_s - \mathfrak{S}_{\mathfrak{p}}\mathbb{C}\mathcal{T}_{(1,2)}$- space. Then $\mathfrak{m}^{\mathfrak{n}4} =< \emptyset, \{\mathfrak{m}\}, \emptyset > \notin_4 \mathrm{G}^{\mathfrak{n}}, \forall \mathrm{G}^{\mathfrak{n}} = \mathfrak{U}^{\mathfrak{n}} \cap_1 \mathfrak{Y}^{\mathfrak{n}}, \mathrm{G}^{\mathfrak{n}} \in \mathcal{T}_{\mathfrak{U}^{\mathfrak{n}}}, \mathfrak{Y}^{\mathfrak{n}} \in \mathcal{T}$. Because, $\mathfrak{U}^{\mathfrak{n}} =< \mathfrak{U}_1, \emptyset, \mathfrak{U}_3 >, \mathfrak{m} \notin \emptyset$. By Definitions 2.3, 2.4 and 2.5.

Remark 3.5. Let $(\mathfrak{U}^{\mathfrak{n}}, \mathcal{T}_{\mathfrak{U}^{\mathfrak{n}}})$ be a $\mathfrak{S}_s - \mathfrak{S}_{\mathfrak{p}}\mathbb{C}\mathcal{T}_{(1,2)}$- space. if $\mathfrak{m}^{\mathfrak{n}_i} \in_i \mathfrak{U}^{\mathfrak{n}}, i = 1, 2, 3, 5$, then $\mathfrak{m}^{\mathfrak{n}_i}$ are a $\mathfrak{S}\mathbb{C}\mathfrak{ms}$ in $\mathfrak{X}$. Because, $\mathfrak{U}_1, \mathfrak{U}_2, \mathfrak{U}_3 \subseteq \mathfrak{X}$.

Definition 3.6. Let $(\mathfrak{X}, \mathcal{T})$ be a $\mathfrak{S}_{\mathfrak{p}}\mathbb{C}\mathcal{T}_{(1,2)}-$ space. Then $(\mathfrak{X}, \mathcal{T})$ is called neutrosophic crisp $\mathcal{T}_0$-space at $\mathfrak{m}^{\mathfrak{n}i}$ (denoted by $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_0$- space) if for every pair $\mathfrak{m}^{\mathfrak{n}i} \neq q^{\mathfrak{n}i} \ \exists \ \mathfrak{Y}^{\mathfrak{n}} \in \mathcal{T} \ni \mathfrak{m}^{\mathfrak{n}i} \in_i \mathfrak{Y}^{\mathfrak{n}}, q^{\mathfrak{n}i} \notin_i \mathfrak{Y}^{\mathfrak{n}}$ or $q^{\mathfrak{n}i} \in_i \mathfrak{Y}^{\mathfrak{n}}, \mathfrak{m}^{\mathfrak{n}i} \notin_j \mathfrak{Y}^{\mathfrak{n}}, i = 1, 2, 3, 4, 5$.

Example 3.7. Based on Example 3.3, $(\mathfrak{X}, \mathcal{T})$ is a $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_0-$ space at $\mathfrak{m}^{\mathfrak{n}i}, i = 1, 2, 3, 4, 5$. For instance, when $i = 2$ we have: $\mathfrak{m}_1^{\mathfrak{n}2} =< \{a\}, \emptyset, \{b, c\} >, \mathfrak{m}_2^{\mathfrak{n}2} =< \{b\}, \emptyset, \{a, c\} >, \mathfrak{m}_3^{\mathfrak{n}2} =< \{c\}, \emptyset, \{a, b\} >$, (by Definition 2.3). Hence,

$$\mathfrak{m}_1^{\mathfrak{n}2} \neq \mathfrak{m}_2^{\mathfrak{n}2}, \mathfrak{m}_1^{\mathfrak{n}2}, \mathfrak{m}_2^{\mathfrak{n}2} \text{ are } \mathfrak{S}\mathbb{C}\mathfrak{ms} \ \exists \ C^{\mathfrak{n}} \in \mathcal{T} \ni \mathfrak{m}_1^{\mathfrak{n}2} \in_2 C^{\mathfrak{n}}, \mathfrak{m}_2^{\mathfrak{n}2} \notin_2 C^{\mathfrak{n}}$$

$$\mathfrak{m}_1^{\mathfrak{n}2} \neq \mathfrak{m}_3^{\mathfrak{n}2}, \mathfrak{m}_1^{\mathfrak{n}2}, \mathfrak{m}_3^{\mathfrak{n}2} \text{ are } \mathfrak{S}\mathbb{C}\mathfrak{ms} \ \exists \ \mathfrak{A}^{\mathfrak{n}} \in \mathcal{T} \ni \mathfrak{m}_1^{\mathfrak{n}2} \in_2 \mathfrak{A}^{\mathfrak{n}}, \mathfrak{m}_3^{\mathfrak{n}2} \notin_2 \mathfrak{A}^{\mathfrak{n}}$$

$$\mathfrak{m}_2^{\mathfrak{n}2} \neq \mathfrak{m}_3^{\mathfrak{n}2}, \mathfrak{m}_2^{\mathfrak{n}2}, \mathfrak{m}_3^{\mathfrak{n}2} \text{ are } \mathfrak{S}\mathbb{C}\mathfrak{ms} \ \exists \ \mathfrak{A}^{\mathfrak{n}} \in \mathcal{T} \ni \mathfrak{m}_2^{\mathfrak{n}2} \in_2 \mathfrak{A}^{\mathfrak{n}}, \mathfrak{m}_3^{\mathfrak{n}2} \notin_2 \mathfrak{A}^{\mathfrak{n}}$$

In the same way, we prove that the space is a $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_0-$ space at $\mathfrak{m}^{\mathfrak{n}i}, i = 1, 3, 4, 5$.

Remark 3.8. It is not necessary that all types of points and memberships satisfy the above definition.

For examples: Let $\mathfrak{X} = \{a, b, c\}$ and $\mathcal{T} = \{\emptyset_1^{\mathfrak{n}}, \mathfrak{X}_1^{\mathfrak{n}}, \mathfrak{A}^{\mathfrak{n}}, B^{\mathfrak{n}}, C^{\mathfrak{n}}, D^{\mathfrak{n}}, E^{\mathfrak{n}}, H^{\mathfrak{n}}\} \ni \emptyset_1^{\mathfrak{n}} =< \emptyset, \emptyset, \mathfrak{X} >, \mathfrak{X}_1^{\mathfrak{n}} =< \mathfrak{X}, \emptyset, \emptyset >, \mathfrak{A}^{\mathfrak{n}} = \{a, b\}, \emptyset, \{c\} >, B^{\mathfrak{n}} = < \{c\}, \emptyset, \{a, b\} >, C^{\mathfrak{n}} =< \{a, c\}, \emptyset, \{b\} >, D^{\mathfrak{n}} =< \{b\}, \emptyset, \{a, c\} >$, $E^{\mathfrak{n}} = < \{b, c\}, \emptyset, \{a\} >, \ H^{\mathfrak{n}} = < \{a\}, \emptyset, \{b, c\} >$. Then $(\mathfrak{X}, \mathcal{T})$ is $\mathfrak{S}_{\mathfrak{p}}\mathbb{C}\mathcal{T}_{(1,2)}$ – space, but it is not a $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_0$ – space at $\mathfrak{m}^{\mathfrak{n}4}$.

Theorem 3.9. *Let $(\mathfrak{U}^{\mathfrak{n}}, \mathcal{T}_{\mathfrak{U}^{\mathfrak{n}}})$ be a $\mathfrak{S}_s - \mathfrak{S}_{\mathfrak{p}}\mathbb{C}\mathcal{T}_{(1,2)}$- space, and let $(\mathfrak{X}, \mathcal{T}_{\mathfrak{X}})$ be a $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_0-$ space at $\mathfrak{m}^{\mathfrak{n}i}, i = 1, 2, 5$. Then $(\mathfrak{U}^{\mathfrak{n}}, \mathcal{T}_{\mathfrak{U}^{\mathfrak{n}}})$ is also a $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_0-$ space at $\mathfrak{m}^{\mathfrak{n}i}, i = 1, 2, 5$.*

Proof.

Case $i = 1$

Let $s_1^{\mathfrak{n}1} \neq s_2^{\mathfrak{n}1}$ where $s_1^{\mathfrak{n}1} =< \emptyset, \{s_1\}, \{s_1\}^c >, s_2^{\mathfrak{n}1} =< \emptyset, \{s_2\}, \{s_2\}^c > \in_1 \mathfrak{U}^{\mathfrak{n}}$ Since $(\mathfrak{U}^{\mathfrak{n}}, \mathcal{T}_{\mathfrak{U}^{\mathfrak{n}}})$ is a subspace of $(\mathfrak{X}, \mathcal{T}_{\mathfrak{X}})$. it is follows from Remark 3.5 that these points are $\mathfrak{S}\mathbb{C}\mathfrak{ms}$ in $\mathfrak{X}$. Because $(\mathfrak{X}, \mathcal{T}_{\mathfrak{X}})$ is a $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_0-$ space, $\exists \ \mathfrak{Y}^{\mathfrak{n}} =< \mathfrak{Y}_1, \mathfrak{Y}_2, \mathfrak{Y}_3 > \in \mathcal{T}_{\mathfrak{X}} \ni s_1^{\mathfrak{n}1} \in_1 \mathfrak{Y}^{\mathfrak{n}}, s_2^{\mathfrak{n}1} \notin_1 \mathfrak{Y}^{\mathfrak{n}}$. Consider $(\mathfrak{Y}^{\mathfrak{n}} \cap_1 \mathfrak{U}^{\mathfrak{n}} =< \mathfrak{U}_1 \cap \mathfrak{Y}_1, \emptyset \cap \mathfrak{Y}_2, \mathfrak{U}_3 \cup \mathfrak{Y}_3 >) \in \mathcal{T}_{\mathfrak{U}^{\mathfrak{n}}}$.

Therefore, $s_1^{\mathfrak{n}1} \in_1 \mathfrak{U}^{\mathfrak{n}}$ and $s_1^{\mathfrak{n}1} \in_1 \mathfrak{Y}^{\mathfrak{n}}$. So, $s_1 \notin \mathfrak{U}_3, \ s_1 \notin \mathfrak{Y}_3$. Hence, $s_1 \notin (\mathfrak{U}_3 \cup \mathfrak{Y}_3)$. Thus, $s_1^{\mathfrak{n}1} \in_1 (\mathfrak{U}^{\mathfrak{n}} \cap_1 \mathfrak{Y}^{\mathfrak{n}})$. Therefore, $s_2^{\mathfrak{n}1} \in_1 \mathfrak{U}^{\mathfrak{n}}$ and $s_2^{\mathfrak{n}1} \notin_1 \mathfrak{Y}^{\mathfrak{n}}. So, s_2 \notin \mathfrak{U}_3, s_2 \in \mathfrak{Y}_3$.

Hence, $s_2 \in (\mathfrak{U}_3 \cup \mathfrak{Y}_3)$. Thus, $s_2^{\mathfrak{n}1} \notin_1 (\mathfrak{U}^{\mathfrak{n}} \cap_1 \mathfrak{Y}^{\mathfrak{n}})$ by Theorem 2.15. Hence, $(\mathfrak{U}^{\mathfrak{n}}, \mathcal{T}_{\mathfrak{U}^{\mathfrak{n}}})$ is a $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_0-$ space at $\mathfrak{m}^{\mathfrak{n}1}$.

Case $i = 2$

$s_1^{\mathfrak{n}2} \neq s_2^{\mathfrak{n}2}$ where $s_1^{\mathfrak{n}2} =< \{s_1\}, \emptyset, \{s_1\}^c >, s_2^{\mathfrak{n}2} =< \{s_2\}, \emptyset, \{s_2\}^c > \in_2 \mathfrak{U}^{\mathfrak{n}}$.

Since $(\mathfrak{U}^{\mathfrak{n}}, \mathcal{T}_{\mathfrak{U}^{\mathfrak{n}}})$ is a subspace of $(\mathfrak{X}, \mathcal{T}_{\mathfrak{X}})$. it is follows from Remark 3.5 that these points are $\mathfrak{S}\mathbb{C}\mathfrak{ms}$ in $\mathfrak{X}$.

Because $(\mathfrak{X}, \mathcal{T}_{\mathfrak{X}})$ is $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_0-$ space. Therefore, $\exists\, \mathfrak{Y}^{\mathfrak{n}} \in \mathcal{T}_{\mathfrak{X}} \ni s_1^{\mathfrak{n}2} \in_2 \mathfrak{Y}^{\mathfrak{n}}, s_2^{\mathfrak{n}2} \notin_2 \mathfrak{Y}^{\mathfrak{n}}$.

Thus, $(\mathfrak{U}^{\mathfrak{n}} \cap_1 \mathfrak{Y}^{\mathfrak{n}}) \in \mathcal{T}_{\mathfrak{U}^{\mathfrak{n}}}$. Therefore, $s_1^{\mathfrak{n}2} \in_2 \mathfrak{U}^{\mathfrak{n}}$ and $s_1^{\mathfrak{n}2} \in_2 \mathfrak{Y}^{\mathfrak{n}}$. So, $s_1 \in \mathfrak{U}_1, s_1 \in \mathfrak{Y}_1$.

Hence, $s_1 \in (\mathfrak{U}_1 \cap \mathfrak{Y}_1)$. Thus, $s_1^{\mathfrak{n}2} \in_2 (\mathfrak{U}^{\mathfrak{n}} \cap_1 \mathfrak{Y}^{\mathfrak{n}})$.

Therefore, $s_2^{\mathfrak{n}2} \in_2 \mathfrak{U}^{\mathfrak{n}}$ and $s_2^{\mathfrak{n}2} \notin_2 \mathfrak{Y}^{\mathfrak{n}}$. So, $s_2 \in \mathfrak{U}_1, s_2 \notin \mathfrak{Y}_1$. So, $s_2 \notin (\mathfrak{U}_1 \cap \mathfrak{Y}_1)$. Thus, $s_2^{\mathfrak{n}2} \notin_2 \mathfrak{U}^{\mathfrak{n}} \cap_1 \mathfrak{Y}^{\mathfrak{n}}$ by Theorem 2.15.

Hence, $(\mathfrak{U}^{\mathfrak{n}}, \mathcal{T}_{\mathfrak{U}^{\mathfrak{n}}})$ is a $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_0-$ space at $\mathfrak{m}^{\mathfrak{n}2}$.

Case $i = 5$

$s_1^{\mathfrak{n}5} \neq s_2^{\mathfrak{n}5}$ where $s_1^{\mathfrak{n}5} =< \{s_1\}, \emptyset, \emptyset >, s_2^{\mathfrak{n}5} =< \{s_2\}, \emptyset, \emptyset > \in_5 \mathfrak{U}^{\mathfrak{n}}$.

Since $(\mathfrak{U}^{\mathfrak{n}}, \mathcal{T}_{\mathfrak{U}^{\mathfrak{n}}})$ is a subspace of $(\mathfrak{X}, \mathcal{T}_{\mathfrak{X}})$. it is follows from Remark 3.5 that these points are $\mathfrak{S}\mathbb{C}\mathfrak{ms}$ in $\mathfrak{X}$. Since, $(\mathfrak{X}, \mathcal{T}_{\mathfrak{X}})$ is $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_0-$ space. Therefore, $\exists\, \mathfrak{Y}^{\mathfrak{n}} \in \mathcal{T}_{\mathfrak{X}} \ni s_1^{\mathfrak{n}5} \in_5 \mathfrak{Y}^{\mathfrak{n}}, s_2^{\mathfrak{n}5} \notin_5 \mathfrak{Y}^{\mathfrak{n}}$.

Consider $(\mathfrak{U}^{\mathfrak{n}} \cap_1 \mathfrak{Y}^{\mathfrak{n}}) \in \mathcal{T}_{\mathfrak{U}^{\mathfrak{n}}}$, Therefore, $s_1^{\mathfrak{n}5} \in_5 \mathfrak{U}^{\mathfrak{n}}$ and $s_1^{\mathfrak{n}5} \in_5 \mathfrak{Y}^{\mathfrak{n}}. So, s_1 \in \mathfrak{U}_1, s_1 \in \mathfrak{Y}_1$. Hence, $s_1 \in (\mathfrak{U}_1 \cap \mathfrak{Y}_1)$. Thus, $s_1^{\mathfrak{n}5} \in_5 (\mathfrak{U}^{\mathfrak{n}} \cap_1 \mathfrak{Y}^{\mathfrak{n}})$. Therefore, $s_2^{\mathfrak{n}5} \in_5 \mathfrak{U}^{\mathfrak{n}}$ and $s_2^{\mathfrak{n}5} \notin_5 \mathfrak{Y}^{\mathfrak{n}}$. So, $s_2 \in \mathfrak{U}_1, s_2 \notin \mathfrak{Y}_1$. So, $s_2 \notin (\mathfrak{U}_3 \cup \mathfrak{Y}_3)$.

Thus, $s_2^{\mathfrak{n}5} \notin_5 (\mathfrak{U}^{\mathfrak{n}} \cap_1 \mathfrak{Y}^{\mathfrak{n}})$ by Theorem 2.15. Hence, $(\mathfrak{U}^{\mathfrak{n}}, \mathcal{T}_{\mathfrak{U}^{\mathfrak{n}}})$ is $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_0-$ space at $\mathfrak{m}^{\mathfrak{n}5}$. □

Remark 3.10. The result of Theorem 3.9 does not necessarily hold for $i = 3, 4$.

Case $i = 3$

In Example 3.3 $(\mathfrak{X}, \mathcal{T}_{\mathfrak{X}})$ is a $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_0-$ space at $\mathfrak{m}^{\mathfrak{n}i}, i = 1, 2, 3, 4, 5$, but $(\mathfrak{U}^{\mathfrak{n}}, \mathcal{T}_{\mathfrak{U}^{\mathfrak{n}}})$ is not $\mathfrak{S}_{\mathfrak{p}}\mathbb{C}\mathcal{T}_{(1,2)}-$ space at $\mathfrak{m}^{\mathfrak{n}i}, i = 3$.

Case $i = 4$

The flows from Remark 3.4.

Theorem 3.11. *Let* $f : (\mathfrak{X}, \mathcal{T}_{\mathfrak{X}}) \to (\mathtt{Y}, \mathcal{T}_{\mathtt{Y}})$ *be homeomorphic*$_{(1,2)}$ *neutrosophic crisp function and* $(\mathfrak{X}, \mathcal{T}_{\mathfrak{X}})$ *is a* $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_0-$ *space at* $\mathfrak{m}^{\mathfrak{n}i}$, *then* $(\mathtt{Y}, \mathcal{T}_{\mathtt{Y}})$ *is also a* $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_0-$ *space at* $\mathfrak{m}^{\mathfrak{n}i}, i = 1, 2, 3, 4, 5$.

Proof. Let $q_1^{\mathfrak{n}i} \neq q_2^{\mathfrak{n}i}$ where $q_1^{\mathfrak{n}i}, q_2^{\mathfrak{n}i}$ are $\mathfrak{S}\mathbb{C}\mathfrak{ms}$ in $\mathtt{Y}$.

Since, f is bijective there exist $\mathfrak{m}_1^{\mathfrak{n}i}, \mathfrak{m}_2^{\mathfrak{n}i}$ in $\mathfrak{X} \ni \mathfrak{m}_2^{\mathfrak{n}i} \neq \mathfrak{m}_1^{\mathfrak{n}i}, f\left(\mathfrak{m}_1^{\mathfrak{n}i}\right) = q_1^{\mathfrak{n}i}, f\left(\mathfrak{m}_2^{\mathfrak{n}i}\right) = q_2^{\mathfrak{n}i}$ (by Definition 2.12). Since, $(\mathfrak{X}, \mathcal{T}_{\mathfrak{X}})$ is a $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_0-$ space $\exists\, \mathfrak{Y}^{\mathfrak{n}} \in \mathcal{T}_{\mathfrak{X}} \ni \mathfrak{m}_1^{\mathfrak{n}i} \in_i \mathfrak{Y}^{\mathfrak{n}}, \mathfrak{m}_2^{\mathfrak{n}i} \notin_i \mathfrak{Y}^{\mathfrak{n}}, i = 1, 2, 3, 4, 5$. Because f is homeomorphic$_{(1,2)}$ it is an open$_{(1,2)}$ mapping (by Definition 2.14).

Hence, $f(\mathfrak{Y}^{\mathfrak{n}}) \in \mathcal{T}_{\Upsilon}$ (by Definition 2.14). Moreover, $f\left(\mathfrak{m}_1^{\mathfrak{n}i}\right) = q_1^{\mathfrak{n}i} \in_i f(\mathfrak{Y}^{\mathfrak{n}}), f\left(\mathfrak{m}_2^{\mathfrak{n}i}\right) = q_2^{\mathfrak{n}i} \notin_i f(\mathfrak{Y}^{\mathfrak{n}})$. Therefore, $(\Upsilon, \mathcal{T}_{\Upsilon})$ is a $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_0-$ space at $\mathfrak{m}^{\mathfrak{n}i}, i = 1,2,3,4,5$. □

Definition 3.12. Let $(\mathfrak{X}, \mathcal{T})$ is a $\mathfrak{S}_{\mathfrak{p}}\mathbb{C}\mathcal{T}_{(1,2)}$ – space. Then $(\mathfrak{X}, \mathcal{T})$ is called neutrosophic crisp $\mathcal{T}_1-$ space at $\mathfrak{m}^{\mathfrak{n}i}$ (denoted by $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_1$- space) if for every pair $\mathfrak{m}^{\mathfrak{n}i} \neq q^{\mathfrak{n}i}$ where $\mathfrak{m}^{\mathfrak{n}i}, q^{\mathfrak{n}i}$ are $\mathfrak{S}\mathbb{C}\mathfrak{ms}$ there exist $L^{\mathfrak{n}}, \mathfrak{Y}^{\mathfrak{n}} \in \mathcal{T}$ such that $\mathfrak{m}^{\mathfrak{n}i} \in_i \mathfrak{Y}^{\mathfrak{n}}$, $q^{\mathfrak{n}i} \notin_i \mathfrak{Y}^{\mathfrak{n}}$ and $q^{\mathfrak{n}i} \in_i L^{\mathfrak{n}}$, $\mathfrak{m}^{\mathfrak{n}i} \notin_i L^{\mathfrak{n}}, i = 1,2,3,4,5$.

Example 3.13. Let $\mathfrak{X} = \{a,b,c\}$ and $\mathcal{T} = \{\emptyset_1^{\mathfrak{n}}, \mathfrak{X}_1^{\mathfrak{n}}, \mathfrak{A}^{\mathfrak{n}}, B^{\mathfrak{n}}, C^{\mathfrak{n}}, O^{\mathfrak{n}}, E^{\mathfrak{n}}, H^{\mathfrak{n}}\} \ni \emptyset_1^{\mathfrak{n}} =< \emptyset, \emptyset, \mathfrak{X} >$, $\mathfrak{X}_1^{\mathfrak{n}} =< \mathfrak{X}, \emptyset, \emptyset >, \mathfrak{A}^{\mathfrak{n}} =< \{a,b\}, \emptyset, \{c\} >$, $B^{\mathfrak{n}} =< \{c\}, \emptyset, \{a,b\} >, C^{\mathfrak{n}} =< \{a,c\}, \emptyset, \{b\} >, O^{\mathfrak{n}} =< \{b\}, \emptyset, \{a,c\} >$,
$E^{\mathfrak{n}} =< \{b,c\}, \emptyset, \{a\} >, H^{\mathfrak{n}} =< \{a\}, \emptyset, \{b,c\} >$. Then $(\mathfrak{X}, \mathcal{T})$ is a $\mathfrak{S}_{\mathfrak{p}}\mathbb{C}\mathcal{T}_{(1,2)}-$ space (by Definition 2.10).

Moreover, $(\mathfrak{X}, \mathcal{T})$ is a $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_1-$ space at $\mathfrak{m}^{\mathfrak{n}i}, i = 1,2,3,5$. For instance, when $i = 2$ we have $\mathfrak{m}_1^{\mathfrak{n}2} =< \{a\}, \emptyset, \{b,c\} >$, $\mathfrak{m}_2^{\mathfrak{n}2} =< \{b\}, \emptyset, \{a,c\} >$, $\mathfrak{m}_3^{\mathfrak{n}2} =< \{c\}, \emptyset, \{a,b\} >$, (by Definition 2.3). We verify separation:

$\mathfrak{m}_1^{\mathfrak{n}2} \neq \mathfrak{m}_2^{\mathfrak{n}2}$ where $\mathfrak{m}_1^{\mathfrak{n}2}, \mathfrak{m}_2^{\mathfrak{n}2}$ are $\mathfrak{S}\mathbb{C}\mathfrak{ms}$, there exist $H^{\mathfrak{n}}, O^{\mathfrak{n}} \in \mathcal{T}$ such that

$$\mathfrak{m}_1^{\mathfrak{n}2} \in_2 H^{\mathfrak{n}}, \mathfrak{m}_2^{\mathfrak{n}2} \notin_2 H^{\mathfrak{n}} \text{ and } \mathfrak{m}_2^{\mathfrak{n}2} \in_2 O^{\mathfrak{n}}, \mathfrak{m}_1^{\mathfrak{n}2} \notin_2 O^{\mathfrak{n}}.$$

$\mathfrak{m}_1^{\mathfrak{n}2} \neq \mathfrak{m}_3^{\mathfrak{n}2}$ where $\mathfrak{m}_1^{\mathfrak{n}2}, \mathfrak{m}_3^{\mathfrak{n}2}$ are $\mathfrak{S}\mathbb{C}\mathfrak{ms}$, there exist $H^{\mathfrak{n}}, B^{\mathfrak{n}} \in \mathcal{T}$ such that

$$\mathfrak{m}_1^{\mathfrak{n}2} \in_2 H^{\mathfrak{n}}, \mathfrak{m}_3^{\mathfrak{n}2} \notin_2 H^{\mathfrak{n}} \text{ and } \mathfrak{m}_3^{\mathfrak{n}2} \in_2 B^{\mathfrak{n}}, \mathfrak{m}_1^{\mathfrak{n}2} \notin_2 B^{\mathfrak{n}}$$

$\mathfrak{m}_2^{\mathfrak{n}2} \neq \mathfrak{m}_3^{\mathfrak{n}2}$ where $\mathfrak{m}_2^{\mathfrak{n}2}, \mathfrak{m}_3^{\mathfrak{n}2}$ are $\mathfrak{S}\mathbb{C}\mathfrak{ms}$, there exist $\mathfrak{Y}^{\mathfrak{n}}, B^{\mathfrak{n}} \in \mathcal{T}$ such that

$$\mathfrak{m}_2^{\mathfrak{n}2} \in_2 \mathfrak{Y}^{\mathfrak{n}}, \mathfrak{m}_3^{\mathfrak{n}2} \notin_2 \mathfrak{Y}^{\mathfrak{n}} \text{ and } \mathfrak{m}_3^{\mathfrak{n}2} \in_2 B^{\mathfrak{n}}, \mathfrak{m}_1^{\mathfrak{n}2} \notin_2 B^{\mathfrak{n}}$$

Thus, the space is a $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_1-$ space at $\mathfrak{m}^{\mathfrak{n}2}$. Similarly, it is holds for $i = 1,3,5$.

Remark 3.14. It is not necessary that all type points and memberships satisfy the above definition. For example: In Example 3.13 $\mathfrak{m}_1^{\mathfrak{n}4} =< \emptyset, \{a\}, \emptyset >, \mathfrak{m}_2^{\mathfrak{n}4} =< \emptyset, \{b\}, \emptyset >$. Then $\mathfrak{m}_1^{\mathfrak{n}4} \neq \mathfrak{m}_2^{\mathfrak{n}4}$, but there do not exist $\mathfrak{Y}^{\mathfrak{n}}, L^{\mathfrak{n}} \in \mathcal{T} \ni \mathfrak{m}_1^{\mathfrak{n}4} \in_4 \mathfrak{Y}^{\mathfrak{n}}, \mathfrak{m}_2^{\mathfrak{n}4} \notin_4 \mathfrak{Y}^{\mathfrak{n}}$ and $\mathfrak{m}_1^{\mathfrak{n}4} \notin_4 L^{\mathfrak{n}}, \mathfrak{m}_2^{\mathfrak{n}4} \in_4 L^{\mathfrak{n}}$. Hence, the space is not a $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_1-$ space at $\mathfrak{m}^{\mathfrak{n}4}$

Example 3.15. Let $\mathfrak{X} = \{a,b,c\}$ and $\mathcal{T} = \{\emptyset_1^{\mathfrak{n}}, \mathfrak{X}_1^{\mathfrak{n}}, \mathfrak{A}^{\mathfrak{n}}, B^{\mathfrak{n}}, C^{\mathfrak{n}}, O^{\mathfrak{n}}, E^{\mathfrak{n}}, F^{\mathfrak{n}}\} \ni \emptyset_1^{\mathfrak{n}} =< \emptyset, \emptyset, \mathfrak{X} >, \mathfrak{X}_1^{\mathfrak{n}} =< \mathfrak{X}, \emptyset, \emptyset >, \mathfrak{A}^{\mathfrak{n}} =< \mathfrak{X}, \{a\}, \emptyset >, B^{\mathfrak{n}} = < \emptyset, \{a\}, \mathfrak{X} >, C^{\mathfrak{n}} =< \mathfrak{X}, \{b\}, \emptyset >, O^{\mathfrak{n}} =< \emptyset, \{b\}, \mathfrak{X} >, E^{\mathfrak{n}} =< \mathfrak{X}, \{c\}, \emptyset >, F^{\mathfrak{n}} =< \emptyset, \{c\}, \mathfrak{X} >$. Then $(\mathfrak{X}, \mathcal{T})$ is a $\mathfrak{S}_{\mathfrak{p}}\mathbb{C}\mathcal{T}_{(1,2)}-$ space (by Definition 2.10).

Moreover, $(\mathfrak{X}, \mathcal{T})$ is a $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_1-$ space at $\mathfrak{m}^{\mathfrak{n}4}$.

- it is not a $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_1-$ space at $\mathfrak{m}^{\mathfrak{n}i}, i = 1,2,3,5$

Theorem 3.16. *Every $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_1-$ space at $\mathfrak{m}^{\mathfrak{n}i}$ is a $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_0$ – space at $\mathfrak{m}^{\mathfrak{n}i}, i = 1,2,3,4,5$.*

Remark 3.17. The converse of Theorem 3.16 is not true. For example, in Example 3.7 $(\mathfrak{X}, \mathcal{T})$ is a $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_0-$ space at $\mathfrak{m}^{\mathfrak{n}i}, i = 2, 5$ but not a $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_1-$space at these indices.

Theorem 3.18. *Let $f : (\mathfrak{X}, \mathcal{T}_{\mathfrak{X}}) \to (\mathtt{Y}, \mathcal{T}_{\mathtt{Y}})$ be homeomorphic$_{(1,2)}$ neutrosophic crisp function. If $(\mathfrak{X}, \mathcal{T}_{\mathfrak{X}})$ is a $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_1-$space at $\mathfrak{m}^{\mathfrak{n}i}$. Then $(\mathtt{Y}, \mathcal{T}_{\mathtt{Y}})$ is also a $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_1-$space at $\mathfrak{m}^{\mathfrak{n}i}, i = 1, 2, 3, 4, 5$.*

The proof is analogous to the proof of Theorem 3.11.

Theorem 3.19. *Let $(\mathfrak{U}^{\mathfrak{n}}, \mathcal{T}_{\mathfrak{U}^{\mathfrak{n}}})$ be a $\mathfrak{S}_s - \mathfrak{S}_{\mathfrak{p}}\mathbb{C}\mathcal{T}_{(1,2)}$- space, and let $(\mathfrak{X}, \mathcal{T}_{\mathfrak{X}})$ be a $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_1-$space at $\mathfrak{m}^{\mathfrak{n}i}, i = 1, 2, 5$. Then $(\mathfrak{U}^{\mathfrak{n}}, \mathcal{T}_{\mathfrak{U}^{\mathfrak{n}}})$ is also a $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_1-$space at $\mathfrak{m}^{\mathfrak{n}i}, i = 1, 2, 5$.*

Proof.

Case $i = 1$

Let $s_1^{\mathfrak{n}1} \neq s_2^{\mathfrak{n}1}$ where $s_1^{\mathfrak{n}1} =< \emptyset, \{s_1\}, \{s_1\}^c >, s_2^{\mathfrak{n}1} = < \emptyset, \{s_2\}, \{s_2\}^c > \in_1 \mathfrak{U}^{\mathfrak{n}}$.

Since $(\mathfrak{U}^{\mathfrak{n}}, \mathcal{T}_{\mathfrak{U}^{\mathfrak{n}}})$ is a subspace of $(\mathfrak{X}, \mathcal{T}_{\mathfrak{X}})$. it is follows from Remark 3.5 that these points are $\mathfrak{S}\mathbb{C}\mathfrak{ms}$ in $\mathfrak{X}$. Since, $(\mathfrak{X}, \mathcal{T}_{\mathfrak{X}})$ is a $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_1-$space, there exist $\mathfrak{Y}^{\mathfrak{n}} < \mathfrak{Y}_1, \mathfrak{Y}_2, \mathfrak{Y}_3 >, L^{\mathfrak{n}} < L_1, L_2, L_3 > \in \mathcal{T}_{\mathfrak{X}}$ such that $s_1^{\mathfrak{n}1} \in_1 \mathfrak{Y}^{\mathfrak{n}}$, $s_2^{\mathfrak{n}1} \notin_1 \mathfrak{Y}^{\mathfrak{n}}$ and $s_2^{\mathfrak{n}1} \in_1 L^{\mathfrak{n}}, s_1^{\mathfrak{n}1} \notin_1 L^{\mathfrak{n}}$.

Consider

$\mathfrak{U}^{\mathfrak{n}} \cap_1 \mathfrak{Y}^{\mathfrak{n}} =< \mathfrak{U}_1, \emptyset, \mathfrak{U}_3 > \cap_1 < \mathfrak{Y}_1, \mathfrak{Y}_2, \mathfrak{Y}_3 > = < \mathfrak{U}_1 \cap \mathfrak{Y}_1, \emptyset \cap \mathfrak{Y}_2, \mathfrak{U}_3 \cup \mathfrak{Y}_3 > \in \mathcal{T}_{\mathfrak{U}^{\mathfrak{n}}}, \mathfrak{U}^{\mathfrak{n}} \cap_1 L^{\mathfrak{n}} = < \mathfrak{U}_1, \emptyset, \mathfrak{U}_3 > \cap_1 < L_1, L_2, L_3 > =< \mathfrak{U}_1 \cap L_1, \emptyset \cap L_2, \mathfrak{U}_3 \cup L_3 > \in \mathcal{T}_{\mathfrak{U}^{\mathfrak{n}}}$.

Using the same argument as in Theorem 3.9:

$s_1^{\mathfrak{n}1} \in_1 (\mathfrak{U}^{\mathfrak{n}} \cap_1 \mathfrak{Y}^{\mathfrak{n}})$, $s_2^{\mathfrak{n}1} \notin_1 (\mathfrak{U}^{\mathfrak{n}} \cap_1 \mathfrak{Y}^{\mathfrak{n}})$, $s_2^{\mathfrak{n}1} \in_1 (\mathfrak{U}^{\mathfrak{n}} \cap_1 L^{\mathfrak{n}}), s_1^{\mathfrak{n}1} \notin_1 (\mathfrak{U}^{\mathfrak{n}} \cap_1 L^{\mathfrak{n}})$. Thus, separation holds in the subspace.

Case $i = 2$

$s_1^{\mathfrak{n}2} \neq s_2^{\mathfrak{n}2}$ where $s_1^{\mathfrak{n}2} =< \{s_1\}, \emptyset, \{s_1\}^c >, s_2^{\mathfrak{n}2} =< \{s_2\}, \emptyset, \{s_2\}^c > \in_2 \mathfrak{U}^{\mathfrak{n}}$.

Since $(\mathfrak{U}^{\mathfrak{n}}, \mathcal{T}_{\mathfrak{U}^{\mathfrak{n}}})$ is a subspace of $(\mathfrak{X}, \mathcal{T}_{\mathfrak{X}})$, it is follows from Remark 3.5 that these points are $\mathfrak{S}\mathbb{C}\mathfrak{ms}$ in $\mathfrak{X}$. Since, $(\mathfrak{X}, \mathcal{T}_{\mathfrak{X}})$ is $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_1-$ space, there exist

$\mathfrak{Y}^{\mathfrak{n}} < \mathfrak{Y}_1, \mathfrak{Y}_2, \mathfrak{Y}_3 >, L^{\mathfrak{n}} < L_1, L_2, L_3 > \in \mathcal{T}_{\mathfrak{X}}$ such that $s_1^{\mathfrak{n}2} \in_2 \mathfrak{Y}^{\mathfrak{n}}, s_2^{\mathfrak{n}2} \notin_2 \mathfrak{Y}^{\mathfrak{n}}$ and $s_2^{\mathfrak{n}2} \in_2 L^{\mathfrak{n}}, s_1^{\mathfrak{n}2} \notin_2 L^{\mathfrak{n}}$.

Consider

$(\mathfrak{U}^{\mathfrak{n}} \cap_1 \mathfrak{Y}^{\mathfrak{n}} =< \mathfrak{U}_1, \emptyset, \mathfrak{U}_3 > \cap_1 < \mathfrak{Y}_1, \mathfrak{Y}_2, \mathfrak{Y}_3 > =< \mathfrak{U}_1 \cap \mathfrak{Y}_1, \emptyset \cap \mathfrak{Y}_2, \mathfrak{U}_3 \cup \mathfrak{Y}_3 >) \in \mathcal{T}_{\mathfrak{U}^{\mathfrak{n}}}, (\mathfrak{U}^{\mathfrak{n}} \cap_1 \mathfrak{Y}^{\mathfrak{n}} =< \mathfrak{U}_1, \emptyset, \mathfrak{U}_3 > \cap_1 < L_1, L_2, L_3 > =< \mathfrak{U}_1 \cap L_1, \emptyset \cap L_2, \mathfrak{U}_3 \cup L_3 >) \in \mathcal{T}_{\mathfrak{U}^{\mathfrak{n}}}$.

Using the same argument as in Theorem 3.9:

$s_1^{\mathfrak{n}2} \in_2 (\mathfrak{U}^{\mathfrak{n}} \cap_1 \mathfrak{Y}^{\mathfrak{n}}), s_2^{\mathfrak{n}2} \notin_2 (\mathfrak{U}^{\mathfrak{n}} \cap_1 \mathfrak{Y}^{\mathfrak{n}})$, $s_2^{\mathfrak{n}2} \in_2 (\mathfrak{U}^{\mathfrak{n}} \cap_1 L^{\mathfrak{n}}), s_1^{\mathfrak{n}2} \notin_2 (\mathfrak{U}^{\mathfrak{n}} \cap_1 L^{\mathfrak{n}})$. Thus, separation holds in the subspace.

Case $i = 5$

Analogous to previous cases. □

Remark 3.20. The result of Theorem 3.19 does not necessarily hold for $i = 3, 4$,

Case $i = 3$

In Example 3.13 $(\mathfrak{X}, \mathcal{T}_{\mathfrak{X}})$ is a $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_1$ – space at $\mathfrak{m}^{\mathfrak{n}3}$, but $(\mathfrak{U}^{\mathfrak{n}}, \mathcal{T}_{\mathfrak{U}^{\mathfrak{n}}})$ is not $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_1$– space at $\mathfrak{m}^{\mathfrak{n}3}$. Indeed, $\mathcal{T}_{\mathfrak{U}^{\mathfrak{n}}} =< \emptyset_1^{\mathfrak{n}}, \mathfrak{U}^{\mathfrak{n}}, L^{\mathfrak{n}}, Q^{\mathfrak{n}} >, \mathfrak{U}^{\mathfrak{n}} =< \emptyset, \emptyset, \{a\} >, L^{\mathfrak{n}} =< \emptyset, \emptyset, \{a, c\} >, Q^{\mathfrak{n}} =< \emptyset, \emptyset, \{a, b\} >$.

Case $i = 4$

This follows directly from Remark 3.4.

Theorem 3.21. *Let* $(\mathfrak{X}, \mathcal{T})$ *be a* $\mathfrak{S}_{\mathfrak{p}}\mathbb{C}\mathcal{T}_{(1,2)}$ *– space then:*

1. $\mathfrak{m}^{\mathfrak{n}2}{}_t, t = 1, 2, \ldots$ *are* $\mathfrak{S}_{\mathfrak{p}}\mathbb{C}\mathbb{C}$*– set in* $\mathfrak{X}$ *if and only if* $(\mathfrak{X}, \mathcal{T}_{\mathfrak{X}})$ *is a* $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_1$ *– space at* $\mathfrak{m}^{\mathfrak{n}2}$.
2. *If* $\mathfrak{m}^{\mathfrak{n}4}{}_t, t = 1, 2, \ldots$ *be a* $\mathfrak{S}_{\mathfrak{p}}\mathbb{C}\mathbb{C}$*– set in* $\mathfrak{X}$. *Then* $(\mathfrak{X}, \mathcal{T}_{\mathfrak{X}})$ *is a* $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_1$ *– space at* $\mathfrak{m}^{\mathfrak{n}4}$.

Proof.

(1) Let $\mathfrak{m}^{\mathfrak{n}_2} =< \{\mathfrak{m}\}, \emptyset, \{\mathfrak{m}\}^c >$ and $s^{\mathfrak{n}_2} =< \{s\}, \emptyset, \{s\}^c >$ be $\mathfrak{S}_{\mathfrak{p}}\mathbb{C}\mathbb{C}$– set in $\mathfrak{X} \ni \mathfrak{m}^{\mathfrak{n}_2} \neq s^{\mathfrak{n}_2}, \mathfrak{m} \neq s$. So, $(\mathfrak{m}^{\mathfrak{n}_2})^{c_2} =< \{\mathfrak{m}\}^c, \emptyset, \{\mathfrak{m}\} >, (s^{\mathfrak{n}_2})^{c_2} =< \{s\}^c, \emptyset, \{s\} >$ are $\mathfrak{S}_{\mathfrak{p}}\mathbb{C}O$– set in $\mathfrak{X}$. And $s \in \{\mathfrak{m}\}^c, \mathfrak{m} \in \{s\}^c$.

Since, $s \in \{\mathfrak{m}\}^c$, we have $s^{\mathfrak{n}_2} \in_2 (\mathfrak{m}^{\mathfrak{n}_2})^{c_2}, \mathfrak{m}^{\mathfrak{n}_2} \notin_2 (\mathfrak{m}^{\mathfrak{n}_2})^{c_2}$.

Similarly, $\mathfrak{m}^{\mathfrak{n}_2} \in_2 (s^{\mathfrak{n}_2})^{c_2}, s^{\mathfrak{n}_2} \notin_2 (s^{\mathfrak{n}_2})^{c_2}$.

Hence, there exist $(\mathfrak{m}^{\mathfrak{n}_2})^{c_2}, (s^{\mathfrak{n}_2})^{c_2} \in \mathcal{T}$ such that $s^{\mathfrak{n}_2} \in_2 (\mathfrak{m}^{\mathfrak{n}_2})^{c_2}, \mathfrak{m}^{\mathfrak{n}_2} \notin_2 (\mathfrak{m}^{\mathfrak{n}_2})^{c_2}$ and $\mathfrak{m}^{\mathfrak{n}_2} \in_2 (s^{\mathfrak{n}_2})^{c_2}, s^{\mathfrak{n}_2} \notin_2 (s^{\mathfrak{n}_2})^{c_2}$.

Thus, $(\mathfrak{X}, \mathcal{T}_{\mathfrak{X}})$ is a $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_1$ – space at $\mathfrak{m}^{\mathfrak{n}_2}$.

Conversely, Assume $(\mathfrak{X}, \mathcal{T})$ is a $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_1$ – space at $\mathfrak{m}^{\mathfrak{n}_2}$. Let $s \in \{\mathfrak{m}\}^c$, then $s^{\mathfrak{n}_2} \in_2 (\mathfrak{m}^{\mathfrak{n}_2})^{c_2}, \mathfrak{m} \neq s$ by Definition 2.4.

Because, $(\mathfrak{X}, \mathcal{T}_{\mathfrak{X}})$ is a $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_1$ – space at $\mathfrak{m}^{\mathfrak{n}_2}$, there exist $\mathfrak{Y}^{\mathfrak{n}}{}_{s^{\mathfrak{n}_2}} \in \mathcal{T}$ such that $s^{\mathfrak{n}_2} \in_2 \mathfrak{Y}^{\mathfrak{n}}{}_{s^{\mathfrak{n}_2}}$, $\mathfrak{m}^{\mathfrak{n}2} \notin_2 \mathfrak{Y}^{\mathfrak{n}}{}_{s^{\mathfrak{n}_2}}$.

Thus, $\{s^{\mathfrak{n}_2}\} \subseteq \{\mathfrak{m}\}^c, \{s^{\mathfrak{n}_2}\}^c \supseteq \{\mathfrak{m}\}$. Hence, $\mathfrak{Y}^{\mathfrak{n}}{}_{s^{\mathfrak{n}_2}} \subseteq_j (\mathfrak{m}^{\mathfrak{n}_2})^{c_2}, j = 1, 2$.

Therefore, $\mathfrak{m}^{\mathfrak{n}_2 c_2} = \bigcup_{2[s^{\mathfrak{n}_2} \in_2 (\mathfrak{m}^{\mathfrak{n}_2})^{c_2}]} \mathfrak{Y}^{\mathfrak{n}}{}_{s^{\mathfrak{n}_2}}$.

Thus, $(\mathfrak{m}^{\mathfrak{n}_2})^{c_2}$ is a $\mathfrak{S}_{\mathfrak{p}}\mathbb{C}O$– set. Hence, $\mathfrak{m}^{\mathfrak{n}2}$ is a $\mathfrak{S}_{\mathfrak{p}}\mathbb{C}\mathbb{C}$– set.

(2) Let $\mathfrak{m}^{\mathfrak{n}_4} =< \emptyset, \{\mathfrak{m}\}, \emptyset >$ and $s^{\mathfrak{n}_4} =< \emptyset, \{s\}, \emptyset >$ be $\mathfrak{S}_{\mathfrak{p}}\mathbb{C}\mathbb{C}$– set in $\mathfrak{X} \ni \mathfrak{m}^{\mathfrak{n}_4} \neq s^{\mathfrak{n}_4}, \mathfrak{m} \neq s$.

So, $(\mathfrak{m}^{\mathfrak{n}_4})^{c_2} =< \emptyset, \{\mathfrak{m}\}, \emptyset >, (s^{\mathfrak{n}_4})^{c_2} =< \emptyset, \{s\}, \emptyset >$ are $\mathfrak{S}_{\mathfrak{p}}\mathbb{C}O$– set in $\mathfrak{X}$.

And $s \in \{s\}$, $\mathfrak{m} \in \{\mathfrak{m}\}$.

Since, $s \in \{s\}$. we have $s^{\mathfrak{n}_4} \in_4 (s^{\mathfrak{n}_4})^{c_2}$, $\mathfrak{m}^{\mathfrak{n}_4} \notin_4 (s^{\mathfrak{n}_4})^{c_2}$.

Similarly, $\mathfrak{m}^{\mathfrak{n}_4} \in_4 (\mathfrak{m}^{\mathfrak{n}_4})^{c_2}, s^{\mathfrak{n}_4} \notin_4 (\mathfrak{m}^{\mathfrak{n}_4})^{c_2}$.

Hence, there exist $(\mathfrak{m}^{\mathfrak{n}_4})^{c_2}, (s^{\mathfrak{n}_4})^{c_2} \in \mathcal{T}$, such that $s^{\mathfrak{n}_4} \in_4 (s^{\mathfrak{n}_4})^{c_2}$, $\mathfrak{m}^{\mathfrak{n}_4} \notin_4 (s^{\mathfrak{n}_4})^{c_2}$ and $\mathfrak{m}^{\mathfrak{n}_4} \in_4 (\mathfrak{m}^{\mathfrak{n}_4})^{c_2}, s^{\mathfrak{n}_4} \notin_4 (\mathfrak{m}^{\mathfrak{n}_4})^{c_2}$, by Definition 2.5.

Hence, $(\mathfrak{X}, \mathcal{T}_{\mathfrak{X}})$ is a $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_1-$ space at $\mathfrak{m}^{\mathfrak{n}_4}$.

□

Remark 3.22.

(1) if $\mathfrak{m}^{\mathfrak{n}i}, i = 1, 3, 5$ is a $\mathfrak{S}_{\mathfrak{p}}\mathbb{CC}-$ set in $\mathfrak{X}$, it does not necessarily imply that the space is $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_1$. For examples.

Case $i = 1, 3$

Let $\mathfrak{X} = \{a, b, c\}$ and $\mathcal{T} = \{\emptyset_1^{\mathfrak{n}}, \mathfrak{X}_1^{\mathfrak{n}}, \mathfrak{A}^{\mathfrak{n}}, B^{\mathfrak{n}}, C^{\mathfrak{n}}, O^{\mathfrak{n}}, E^{\mathfrak{n}}, F^{\mathfrak{n}}, G^{\mathfrak{n}}, H^{\mathfrak{n}}, I^{\mathfrak{n}}\}, \emptyset_1^{\mathfrak{n}} =< \emptyset, \emptyset, \mathfrak{X} >$, $\mathfrak{X}_1^{\mathfrak{n}} =< \mathfrak{X}, \emptyset, \emptyset >$ $\mathfrak{A}^{\mathfrak{n}} =< \{b, c\}, \{a\}, \emptyset >, B^{\mathfrak{n}} = < \{a, c\}, \{b\}, \emptyset >, C^{\mathfrak{n}} =< \{a, b\}, \{c\}, \emptyset >$, $O^{\mathfrak{n}} = < \{b, c\}, \emptyset, \emptyset >$ $E^{\mathfrak{n}} = < \{a, c\}, \emptyset, \emptyset >$ $, F^{\mathfrak{n}} =< \{a, b\}, \emptyset, \emptyset >$ $, G^{\mathfrak{n}} =< \{c\}, \emptyset, \emptyset >$ $, H^{\mathfrak{n}} =< \{b\}, \emptyset, \emptyset >, I^{\mathfrak{n}} =< \{a\}, \emptyset, \emptyset >$. Then $(\mathfrak{X}, \mathcal{T})$ is a $\mathfrak{S}_{\mathfrak{p}}\mathbb{C}\mathcal{T}_{(1,2)}$ – space (by Definition 2.10).

If $i = 1$

From Definition 2.3, we obtain $a^{\mathfrak{n}_1} =< \emptyset, \{a\}, \{b, c\} >, b^{\mathfrak{n}_1} =< \emptyset, \{b\}, \{a, c\} >$, $c^{\mathfrak{n}_1} =< \emptyset, \{c\}, \{a, b\} >$ are $\mathfrak{S}_{\mathfrak{p}}\mathbb{CC}-$ set in $\mathfrak{X}$. However, $(\mathfrak{X}, \mathcal{T})$ is not a $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_1$ – space at $\mathfrak{m}^{\mathfrak{n}1}$, since for $a^{\mathfrak{n}_1} \neq b^{\mathfrak{n}_1}$, there do not exist $\mathfrak{Y}^{\mathfrak{n}}, L^{\mathfrak{n}} \in \mathcal{T}$, such that $a^{\mathfrak{n}_1} \in_1 \mathfrak{Y}^{\mathfrak{n}}$, $b^{\mathfrak{n}_1} \notin_1 \mathfrak{Y}^{\mathfrak{n}}$ and $b^{\mathfrak{n}_1} \in_2 L^{\mathfrak{n}}$,$a^{\mathfrak{n}_1} \notin_2 L^{\mathfrak{n}}$, by Definitions 2.4, and 2.5.

If $i = 3$.

From Definition 2.3, we obtain $a^{\mathfrak{n}_3} =< \emptyset, \emptyset, \{a\} >, b^{\mathfrak{n}_3} =< \emptyset, \emptyset, \{b\} >$, $c^{\mathfrak{n}_3} =< \emptyset, \emptyset, \{c\} >$ are $\mathfrak{S}_{\mathfrak{p}}\mathbb{CC}-$ set in $\mathfrak{X}$. However, $(\mathfrak{X}, \mathcal{T})$ is not a $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_1$ – space at $\mathfrak{m}^{\mathfrak{n}3}$, since for $a^{\mathfrak{n}_3} \neq b^{\mathfrak{n}_3}$ there do not exist $\mathfrak{Y}^{\mathfrak{n}}, L^{\mathfrak{n}} \in \mathcal{T}$ such that $a^{\mathfrak{n}_3} \in_3 \mathfrak{Y}^{\mathfrak{n}}, b^{\mathfrak{n}_3} \notin_3 \mathfrak{Y}^{\mathfrak{n}}$ and $b^{\mathfrak{n}_3} \in_3 L^{\mathfrak{n}}$, $a^{\mathfrak{n}_3} \notin_3 L^{\mathfrak{n}}$, by Definitions 2.4, and 2.5.

If $i = 5$

Let $\mathfrak{X} = \{a, b, c\}$ and $\mathcal{T} = \{\emptyset_1^{\mathfrak{n}}, \mathfrak{X}_1^{\mathfrak{n}}, \mathfrak{A}^{\mathfrak{n}}, B^{\mathfrak{n}}, C^{\mathfrak{n}}, O^{\mathfrak{n}}, E^{\mathfrak{n}}, F^{\mathfrak{n}}, G^{\mathfrak{n}}\}, \emptyset_1^{\mathfrak{n}} =< \emptyset, \emptyset, \mathfrak{X} >$ $\mathfrak{X}_1^{\mathfrak{n}} =< \mathfrak{X}, \emptyset, \emptyset >$ $\mathfrak{A}^{\mathfrak{n}} =< \emptyset, \emptyset, \{a\} >, B^{\mathfrak{n}} =< \emptyset, \emptyset, \{b\} >, C^{\mathfrak{n}} =< \emptyset, \emptyset, \{c\} >$ $, O^{\mathfrak{n}} =< \emptyset, \emptyset, \{a, c\} >, E^{\mathfrak{n}} =< \emptyset, \emptyset, \{a, b\} >$ $, F^{\mathfrak{n}} =< \emptyset, \emptyset, \{b, c\} >, G^{\mathfrak{n}} =< \emptyset, \emptyset, \emptyset >$.

Then $(\mathfrak{X}, \mathcal{T})$ is a $\mathfrak{S}_{\mathfrak{p}}\mathbb{C}\mathcal{T}_{(1,2)}$ – space (by Definition 2.10).

From Definition 2.3, we obtain $a^{\mathfrak{n}_5} =< \{a\}, \emptyset, \emptyset >, b^{\mathfrak{n}_5} =< \{b\}, \emptyset, \emptyset >, c^{\mathfrak{n}_5} =< \{c\}, \emptyset, \emptyset >$ are $\mathfrak{S}_{\mathfrak{p}}\mathbb{CC}-$ set in $\mathfrak{X}$. However, $(\mathfrak{X}, \mathcal{T})$ is not a $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_1$ – space at $\mathfrak{m}^{\mathfrak{n}5}$, since for $a^{\mathfrak{n}_5} \neq b^{\mathfrak{n}_5}$ there do not exist $\mathfrak{Y}^{\mathfrak{n}}, L^{\mathfrak{n}} \in \mathcal{T}$ such that $a^{\mathfrak{n}_5} \in_5 \mathfrak{Y}^{\mathfrak{n}}, b^{\mathfrak{n}_5} \notin_5 \mathfrak{Y}^{\mathfrak{n}}$ and $b^{\mathfrak{n}_5} \in_5 L^{\mathfrak{n}}$, $a^{\mathfrak{n}_5} \notin_5 L^{\mathfrak{n}}$, by Definitions 2.4, and 2.5

(2) The converse of statement (1) in Theorem 3.21 does not hold. is not true; refer to the Remark 3.22 (1), Case 1. Where $(\mathfrak{X}, \mathcal{T}_{\mathfrak{X}})$ is a $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_1$ – space at $\mathfrak{m}^{\mathfrak{n}_4}$. But $\mathfrak{m}^{\mathfrak{n}_4}$ are not $\mathfrak{S}_{\mathfrak{p}}\mathbb{CC}$– set in $\mathfrak{X}$.

Definition 3.23. Let $(\mathfrak{X}, \mathcal{T})$ be a $\mathfrak{S}_{\mathfrak{p}}\mathbb{C}\mathcal{T}_{(1,2)}$ – space. Then it is called a neutrosophic crisp $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_2$– space at $\mathfrak{m}^{\mathfrak{n}i}$ if for every $\mathfrak{m}^{\mathfrak{n}i} \neq q^{\mathfrak{n}i}$, there exist $\mathfrak{Y}^{\mathfrak{n}}, L^{\mathfrak{n}} \in \mathcal{T}$ such that $\mathfrak{m}^{\mathfrak{n}i} \in_i \mathfrak{Y}^{\mathfrak{n}}, q^{\mathfrak{n}i} \in_i L^{\mathfrak{n}}$ and $\mathfrak{Y}^{\mathfrak{n}} \cap_1 L^{\mathfrak{n}} = \emptyset_1^{\mathfrak{n}}$, $i = 1, 2, 3, 4, 5$.

Example 3.24. Based on Example 3.13

- $(\mathfrak{X}, \mathcal{T})$ is a $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_2$– space at $\mathfrak{m}^{\mathfrak{n}i}$, $i = 1, 2, 3, 5$.
- $(\mathfrak{X}, \mathcal{T})$ is not a $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_2$ – space at $\mathfrak{m}^{\mathfrak{n}4}$.

Example 3.25. Based on Example 3.15

- $(\mathfrak{X}, \mathcal{T})$ is a $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_2$ – space at $\mathfrak{m}^{\mathfrak{n}i}$, $i = 3, 4$.
- $(\mathfrak{X}, \mathcal{T})$ is not a $\mathfrak{S}_{\mathfrak{p}}\mathbb{C}\mathcal{T}_{(1,2)}$– space at $\mathfrak{m}^{\mathfrak{n}i}$, $i = 1, 2, 5$.

Theorem 3.26. *Every $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_2$– space at $\mathfrak{m}^{\mathfrak{n}i}$ is a $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_1$– space at $\mathfrak{m}^{\mathfrak{n}i}$, $i = 1, 2, 4, 5$.*

Proof. Let $(\mathfrak{X}, \mathcal{T})$ is a $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_2$– space at $\mathfrak{m}^{\mathfrak{n}i}$, $i = 1, 2, 4, 5$, and let $\mathfrak{m}^{\mathfrak{n}i} \neq q^{\mathfrak{n}i}$, $\mathfrak{m}^{\mathfrak{n}i}, q^{\mathfrak{n}i}$ are $\mathfrak{S}\mathbb{C}\mathfrak{ms}$ in $\mathfrak{X}$. Since $(\mathfrak{X}, \mathcal{T})$ is a $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_2$– space. So, there exist $\mathfrak{Y}^{\mathfrak{n}}, L^{\mathfrak{n}} \in \mathcal{T}$ such that $\mathfrak{m}^{\mathfrak{n}i} \in_i \mathfrak{Y}^{\mathfrak{n}}, q^{\mathfrak{n}i} \in_i L^{\mathfrak{n}}$ and $\mathfrak{Y}^{\mathfrak{n}} \cap_1 L^{\mathfrak{n}} = \emptyset_1^{\mathfrak{n}}$, $i = 1, 2, 4, 5$.

This implies that $\mathfrak{m}^{\mathfrak{n}i} \in_i \mathfrak{Y}^{\mathfrak{n}}$ but $q^{\mathfrak{n}i} \notin_i \mathfrak{Y}^{\mathfrak{n}}$ and $\mathfrak{m}^{\mathfrak{n}i} \notin_i L^{\mathfrak{n}}$ but $q^{\mathfrak{n}i} \in_i L^{\mathfrak{n}}$, by Theorem 2.17.

Hence, $(\mathfrak{X}, \mathcal{T})$ is a $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_1$– space at $\mathfrak{m}^{\mathfrak{n}i}$, $i = 1, 2, 4, 5$. □

Remark 3.27. For $i = 3$, Theorem 3.26 does not necessarily hold. For a counterexample, see Example 3.15, which valid with respect to $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_2$– space but not $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_1$– space.

Theorem 3.28. *Let $f : (\mathfrak{X}, \mathcal{T}_{\mathfrak{X}}) \to (\mathtt{Y}, \mathcal{T}_{\mathtt{Y}})$ be homeomorphic$_{(1,2)}$ neutrosophic crisp function. If $(\mathfrak{X}, \mathcal{T}_{\mathfrak{X}})$ is a $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_2$ – space at $\mathfrak{m}^{\mathfrak{n}i}$, then $(\mathtt{Y}, \mathcal{T}_{\mathtt{Y}})$ is $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_2$– space at $\mathfrak{m}^{\mathfrak{n}i}$.*

Proof. Let $q_1^{\mathfrak{n}i} \neq q_2^{\mathfrak{n}i}$ where $q_1^{\mathfrak{n}i}, q_2^{\mathfrak{n}i}$ are $\mathfrak{S}\mathbb{C}\mathfrak{ms}$ in $\mathtt{Y}$. Since f is a bijective there exist $\mathfrak{m}_1^{\mathfrak{n}i}, \mathfrak{m}_2^{\mathfrak{n}i}$ in $\mathfrak{X} \ni \mathfrak{m}_2^{\mathfrak{n}i} \neq \mathfrak{m}_1^{\mathfrak{n}i}$, $f\left(\mathfrak{m}_1^{\mathfrak{n}i}\right) = q_1^{\mathfrak{n}i}$, $f\left(\mathfrak{m}_2^{\mathfrak{n}i}\right) = q_2^{\mathfrak{n}i}$ (by Definition 2.12).

Since, $(\mathfrak{X}, \mathcal{T}_{\mathfrak{X}})$ is a $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_2$– space, there exist $\mathfrak{Y}^{\mathfrak{n}}, L^{\mathfrak{n}} \in \mathcal{T}_{\mathfrak{X}}$ such that $\mathfrak{m}^{\mathfrak{n}i} \in_i \mathfrak{Y}^{\mathfrak{n}}, q^{\mathfrak{n}i} \in_i L^{\mathfrak{n}}, \mathfrak{Y}^{\mathfrak{n}} \cap_1 L^{\mathfrak{n}} = \emptyset_1^{\mathfrak{n}}$, $i = 1, 2, 3, 4, 5$.

Because f is homeomorphic$_{(1,2)}$ it is an open$_{(1,2)}$ mapping (by Definition 2.14).

Hence, $f\left(\mathfrak{Y}^{\mathfrak{n}}\right), f\left(L^{\mathfrak{n}}\right) \in \mathcal{T}_{\mathtt{Y}}(by definition 2.13)$.

Also, $f\left(\mathfrak{m}_1^{\mathfrak{n}i}\right) = q_1^{\mathfrak{n}i} \in_i f\left(\mathfrak{Y}^{\mathfrak{n}}\right)$, $f\left(\mathfrak{m}_2^{\mathfrak{n}i}\right) = q_2^{\mathfrak{n}i} \in_i f\left(L^{\mathfrak{n}}\right)$ and $f(\mathfrak{Y}^{\mathfrak{n}}) \cap_1 f(L^{\mathfrak{n}}) = \emptyset_1^{\mathfrak{n}}$.

Hence, $(\mathtt{Y}, \mathcal{T}_{\mathtt{Y}})$ is a $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_2$– space at $\mathfrak{m}^{\mathfrak{n}i}$, $i = 1, 2, 3, 4, 5$. □

Theorem 3.29. *Let $(\mathfrak{U}^{\mathfrak{n}}, \mathcal{T}_{\mathfrak{U}^{\mathfrak{n}}})$ be a $\mathfrak{S}_s - \mathfrak{S}_{\mathfrak{p}}\mathbb{C}\mathcal{T}_{(1,2)}$- space, and let $(\mathfrak{X}, \mathcal{T}_{\mathfrak{X}})$ be a $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_2$– space at $\mathfrak{m}^{\mathfrak{n}i}$, $i = 1, 2, 5$. Then $(\mathfrak{U}^{\mathfrak{n}}, \mathcal{T}_{\mathfrak{U}^{\mathfrak{n}}})$ is also a $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_2$– space at $\mathfrak{m}^{\mathfrak{n}i}$, $i = 1, 2, 5$.*

Proof.

Case $i = 1$

Let $s_1^{\mathfrak{n}1} \neq s_2^{\mathfrak{n}1}$ where $s_1^{\mathfrak{n}1} =< \emptyset, \{s_1\}, \{s_1\}^c >, s_2^{\mathfrak{n}1} =< \emptyset, \{s_2\}, \{s_2\}^c > \in_1 \mathfrak{U}^{\mathfrak{n}}$.

Since $(\mathfrak{U}^{\mathfrak{n}}, \mathcal{T}_{\mathfrak{U}^{\mathfrak{n}}})$ is a subspace of $(\mathfrak{X}, \mathcal{T}_{\mathfrak{X}})$. it is follows from Remark 3.5 that these points are $\mathfrak{S}\mathbb{C}\mathfrak{ms}$ in $\mathfrak{X}$. Since, $(\mathfrak{X}, \mathcal{T}_{\mathfrak{X}})$ is a $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_2$ – space, there exist $\mathfrak{Y}^{\mathfrak{n}} < \mathfrak{Y}_1, \mathfrak{Y}_2, \mathfrak{Y}_3 >, L^{\mathfrak{n}} < L_1, L_2, L_3 > \in \mathcal{T}_{\mathfrak{X}}$ such that $s_1^{\mathfrak{n}1} \in_1 \mathfrak{Y}^{\mathfrak{n}}, s_2^{\mathfrak{n}1} \in_1 L^{\mathfrak{n}}$ and $\mathfrak{Y}^{\mathfrak{n}} \cap_1 L^{\mathfrak{n}} =< \mathfrak{Y}_1 \cap L_1, \mathfrak{Y}_2 \cap L_2, \mathfrak{Y}_3 \cup L_3 > = < \emptyset, \emptyset, \mathfrak{X} > = \emptyset_1^{\mathfrak{n}}$.

Consider

$\mathfrak{U}^{\mathfrak{n}} \cap_1 \mathfrak{Y}^{\mathfrak{n}} =< \mathfrak{U}_1 \cap \mathfrak{Y}_1, \emptyset \cap \mathfrak{Y}_2, \mathfrak{U}_3 \cup \mathfrak{Y}_3 > \in \mathcal{T}_{\mathfrak{U}^{\mathfrak{n}}}, \mathfrak{U}^{\mathfrak{n}} \cap_1 L^{\mathfrak{n}} =< \mathfrak{U}_1 \cap L_1, \emptyset \cap L_2, \mathfrak{U}_3 \cup L_3 > \in \mathcal{T}_{\mathfrak{U}^{\mathfrak{n}}}$ Since, $s_1^{\mathfrak{n}1} \in_1 \mathfrak{U}^{\mathfrak{n}}, s_1^{\mathfrak{n}1} \in_1 \mathfrak{Y}^{\mathfrak{n}}$, then $s_1 \notin \mathfrak{U}_3, s_1 \notin \mathfrak{Y}_3$. Hence, $s_1 \notin (\mathfrak{U}_3 \cup \mathfrak{Y}_3)$. So, $s_1^{\mathfrak{n}1} \in_1 (\mathfrak{U}^{\mathfrak{n}} \cap_1 \mathfrak{Y}^{\mathfrak{n}})$.

Since, $s_2^{\mathfrak{n}1} \in_1 \mathfrak{U}^{\mathfrak{n}}, s_2^{\mathfrak{n}1} \in_1 L^{\mathfrak{n}}$, then $s_2 \notin \mathfrak{U}_3, s_2 \notin L_3$.

Hence, $s_2 \notin (\mathfrak{U}_3 \cup \mathfrak{Y}_3)$. So, $s_2^{\mathfrak{n}1} \in_1 (\mathfrak{U}^{\mathfrak{n}} \cap_1 \mathfrak{Y}^{\mathfrak{n}})$. Hence, $(\mathfrak{U}^{\mathfrak{n}} \cap_1 \mathfrak{Y}^{\mathfrak{n}}) \cap_1 (\mathfrak{U}^{\mathfrak{n}} \cap_1 \mathfrak{Y}^{\mathfrak{n}}) =< (\mathfrak{U}_1 \cap \mathfrak{Y}_1) \cap (\mathfrak{U}_1 \cap \mathfrak{Y}_1), \emptyset, (\mathfrak{U}_3 \cup \mathfrak{Y}_3) \cup (\mathfrak{U}_3 \cup \mathfrak{Y}_3) > = < \emptyset, \emptyset, \mathfrak{X} > = \emptyset_1^{\mathfrak{n}}$.

Thus, the subspace is a $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_2$ – space at $\mathfrak{m}^{\mathfrak{n}1}$.

Case $i = 2$

$s_1^{\mathfrak{n}2} \neq s_2^{\mathfrak{n}2}$ where $s_1^{\mathfrak{n}2} =< \{s_1\}, \emptyset, \{s_1\}^c >, s_2^{\mathfrak{n}2} =< \{s_2\}, \emptyset, \{s_2\}^c > \in_2 \mathfrak{U}^{\mathfrak{n}}$.

Since $(\mathfrak{U}^{\mathfrak{n}}, \mathcal{T}_{\mathfrak{U}^{\mathfrak{n}}})$ is a subspace of $(\mathfrak{X}, \mathcal{T}_{\mathfrak{X}})$. it is follows from Remark 3.5 that these points are $\mathfrak{S}\mathbb{C}\mathfrak{ms}$ in $\mathfrak{X}$.

Since, $(\mathfrak{X}, \mathcal{T}_{\mathfrak{X}})$ is a $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_2$ – space, there exist $\mathfrak{Y}^{\mathfrak{n}} < \mathfrak{Y}_1, \mathfrak{Y}_2, \mathfrak{Y}_3 >, L^{\mathfrak{n}} < L_1, L_2, L_3 > \in \mathcal{T}_{\mathfrak{X}}$ such that $s_1^{\mathfrak{n}2} \in_2 \mathfrak{Y}^{\mathfrak{n}}, s_2^{\mathfrak{n}2} \in_2 L^{\mathfrak{n}}$ and $\mathfrak{Y}^{\mathfrak{n}} \cap_1 L^{\mathfrak{n}} =< \mathfrak{Y}_1 \cap L_1, \mathfrak{Y}_2 \cap L_2, \mathfrak{Y}_3 \cup L_3 > = < \emptyset, \emptyset, \mathfrak{X} > = \emptyset_1^{\mathfrak{n}}$.

Consider

$\mathfrak{U}^{\mathfrak{n}} \cap_1 \mathfrak{Y}^{\mathfrak{n}} =< \mathfrak{U}_1 \cap \mathfrak{Y}_1, \emptyset \cap \mathfrak{Y}_2, \mathfrak{U}_3 \cup \mathfrak{Y}_3 > \in \mathcal{T}_{\mathfrak{U}^{\mathfrak{n}}}, \mathfrak{U}^{\mathfrak{n}} \cap_1 L^{\mathfrak{n}} =< \mathfrak{U}_1 \cap L_1, \emptyset \cap L_2, \mathfrak{U}_3 \cup L_3 > \in \mathcal{T}_{\mathfrak{U}^{\mathfrak{n}}}$.

Since, $s_1^{\mathfrak{n}2} \in_2 \mathfrak{U}^{\mathfrak{n}}, s_1^{\mathfrak{n}2} \in_2 \mathfrak{Y}^{\mathfrak{n}}$, then $s_1 \in \mathfrak{U}_1, s_1 \in \mathfrak{Y}_1$. Hence, $s_1 \in (\mathfrak{U}_1 \cap \mathfrak{Y}_1)$, so $s_1^{\mathfrak{n}2} \in_2 (\mathfrak{U}^{\mathfrak{n}} \cap_1 \mathfrak{Y}^{\mathfrak{n}})$. Since, $s_2^{\mathfrak{n}2} \in_2 \mathfrak{U}^{\mathfrak{n}}, s_1^{\mathfrak{n}2} \in_2 \mathfrak{Y}^{\mathfrak{n}}$, then $s_2 \in \mathfrak{U}_1, s_2 \in \mathfrak{Y}_1$. Hence, $s_2 \in (\mathfrak{U}_1 \cap \mathfrak{Y}_1)$, so $s_2^{\mathfrak{n}2} \in_2 (\mathfrak{U}^{\mathfrak{n}} \cap_1 \mathfrak{Y}^{\mathfrak{n}})$. Hence, $(\mathfrak{U}^{\mathfrak{n}} \cap_1 \mathfrak{Y}^{\mathfrak{n}}) \cap_1 (\mathfrak{U}^{\mathfrak{n}} \cap_1 \mathfrak{Y}^{\mathfrak{n}}) =< (\mathfrak{U}_1 \cap \mathfrak{Y}_1) \cap (\mathfrak{U}_1 \cap \mathfrak{Y}_1), \emptyset, (\mathfrak{U}_3 \cup \mathfrak{Y}_3) \cup (\mathfrak{U}_3 \cup \mathfrak{Y}_3) > = < \emptyset, \emptyset, \mathfrak{X} > = \emptyset_1^{\mathfrak{n}}$.

Thus, the subspace is a $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_2$ – space at $\mathfrak{m}^{\mathfrak{n}2}$.

Case $i = 5$

Analogous to previous cases. □

Remark 3.30. The result of Theorem 3.29 does not necessarily hold for $i = 3, 4$.

Case $i = 3$ In Example 3.24, $(\mathfrak{X}, \mathcal{T}_{\mathfrak{X}})$ is $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}_2$ – space at $\mathfrak{m}^{\mathfrak{n}3}$, but $(\mathfrak{U}^{\mathfrak{n}}, \mathcal{T}_{\mathfrak{U}^{\mathfrak{n}}})$ is not $\mathfrak{S}_{\mathfrak{p}}\mathcal{T}3$ – space at $\mathfrak{m}^{\mathfrak{n}3}$. Indeed, $\mathcal{T}_{\mathfrak{U}^{\mathfrak{n}}} =< \emptyset_1^{\mathfrak{n}}, \mathfrak{U}^{\mathfrak{n}}, L^{\mathfrak{n}}, Q^{\mathfrak{n}} >, \mathfrak{U}^{\mathfrak{n}} =< \emptyset, \emptyset, \{a\} >, L^{\mathfrak{n}} =< \emptyset, \emptyset, \{a, c\} >, Q^{\mathfrak{n}} =< \emptyset, \emptyset, \{a, b\} >$.

Case $i = 4$

This follows from Remark 3.4.

4. Conclusion

In this study, we demonstrated that the structure of the neutrosophic crisp topological space heavily depends on the choice of algebraic operations, particularly the types of intersection, union, and complement. Fixing a first-type intersection alongside a second-type union and complement results in a topological framework whose properties differ significantly from both classical topology and the stable neutrosophic crisp topology.

One of the most important findings of this research is that the separation axioms in this context fundamentally depend on the types of neutrosophic crisp points and their associated membership relations. Unlike the classical case, where separation properties are uniformly defined, we found that many of these axioms do not hold for all five types of points; they may be valid in certain cases and fail in others.

Furthermore, we showed that some classical results cannot be generalized to this framework. For instance, the hereditary property does not hold for all types of points, and the closedness of sets does not necessarily imply that the space satisfies property $\mathcal{T}_1$ axiom. These results highlight the intrinsic differences between this space and both classical topology and the stable neutrosophic crisp topology, where property relations are generally more regular.

Overall, this work confirms that altering the fundamental operations leads to a substantial change in the behavior of topological properties within neutrosophic crisp spaces. This opens the door for future studies, particularly concerning higher separation axioms such as $\mathcal{T}_3$ and $\mathcal{T}_4$, as well as the concepts of regularity and normality within the same framework.

Conflicts of Interest. The authors declare that there are no conflicts of interest regarding the publication of this paper.

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