Asia Pacific Journal of Mathematics, Vol. 3, No. 2 (2016), 136-143 ISSN 2357-2205

COINCIDENCE POINT THEOREM FOR TWO PAIRS OF HYBRID MAPPINGS IN COMPLEX VALUED METRIC SPACES

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Abstract. In this paper using f is S-Weakly commuting we prove a coincidence point theorem for two pairs of hybrid mappings in a complex valued metric space.Our theorem is a generalization of Theorem 10 of Azam,Ahmad and Kumam [2].

2010 Mathematics Subject Classification. 54E40, 54E35, 54H25, 47H10.

Key words and phrases. coincidence point; S- weakly commuting; hybrid mappings.

1. Introduction

It is a well-known fact that the mathematical results regarding fixed points of contraction type mappings are very useful for determining the existence and uniqueness of solutions to various mathematical models.Over the last 40 years,the theory of fixed points has been developed regarding the results that are related to finding the fixed points of self and nonself nonlinear mappings in a metric space.

The study of fixed points for multi-valued contraction mappings was initiated by Nadler[18] and Markin[8].Several authors proved fixed point results in different types of generalized metric spaces[1, 3, 5, 7, 10, 11, 12, 13, 14, 15, 16, 17, 19].

Azam et al.[1] introduced the concept of a complex valued metric space and obtained sufficient conditions for the existence of common fixed points of a pair of mappings satisfying a contractive type condition.Subsequently,Rouzkard and Imdad [6] established some common fixed point theorems for maps satisfying certain rational expressions in complex valued metric spaces to generalize the results of [1].In the same way, Sintunavarat et al. [21, 22] obtained common fixed point results by replacing the constant of contractive condition to control functions.Recently ,Sitthikul and Saejung [9] and Klineam and Suanoom [4] established some fixed point results by generalizing the contractive conditions in the context of complex valued metric spaces.Very recently,Ahmad et al.[7] obtained some new fixed point results for multi-valued mappings in the setting of complex valued metric spaces.

Throughout this paper,N and C denote the set of all positive integers and the set of all complex numbers respectively.

A complex number $z \in C$ is an ordered pair of real numbers, whose first co-ordinate is called $Re(z)$ and second co-ordinate is called $Im(z)$. Let $z_1, z_2 \in C$. Define apartial order $\precsim$ on C as follows:
$z_1 \precsim z_2$ if and only if $Re(z_1) \leq Re(z_2), Im(z_1) \leq Im(z_2)$.
Thus $z_1 \precsim z_2$ if one of the following holds:
(1)$Re(z_1) = Re(z_2)$ and $Im(z_1) = Im(z_2)$,
(2)$Re(z_1) < Re(z_2)$ and $Im(z_1) = Im(z_2)$,
(3)$Re(z_1) = Re(z_2)$ and $Im(z_1) < Im(z_2)$,
(4)$Re(z_1) < Re(z_2)$ and $Im(z_1) < Im(z_2)$.
We will write $z_1 \precnsim z_2$ if $z_1 \neq z_2$ and one of (2), (3) and (4) is satisfed;also we will write $z_1 \prec z_2$ if only (4) is satisfed.

Definition 1.1. ([1]) Let X be a non empty set. A function $d : X \times X \to \mathbf{C}$ is called a complex valued metric on X if for all $x, y, z \in X$ the following conditions are satisfied:
(i) $0 \precsim d(x, y)$ and $d(x, y) = 0$ if and only if $x = y$;
(ii) $d(x, y) = d(y, x)$;
(iii) $d(x, y) \precsim d(x, z) + d(z, y)$.
The pair $(X, d)$is called a complex valued metric space.

Let $\{x_n\}$ be a sequence in X and $x \in X$.If for every $c \in C$ with $0 \preceq c$ there is $n_0 \in N$ such that for all $n > n_0, d(x_n, x) \prec c$,then $\{x_n\}$ is said to be convergent to x and x is called the limit point of $\{x_n\}$.We denote this by $lim_{n\to\infty} x_n = x$ or $x_n \to x$ as $n \to \infty$.If for every $c \in C$ with $0 \prec c$ there is $n_0 \in N$ such that for all $n > n_0, d(x_n, x_{n+m}) \prec c$, where $m \in N$,then $\{x_n\}$ is called Cauchy sequence in(X, d).If every Cauchy sequence is convergent in (X, d) then (X, d) is called a complete complexvalued metric space.
We require the follwing lemmas.
The following lemmas are very useful for further discussion.

Lemma 1.2. *([1]) Let (X, d) be a complex valued metric space and let $\{x_n\}$ be a sequence in X.Then $\{x_n\}$ converges to x if and only if $|d(x_n, x)| \to 0$ as $n \to \infty$.*

Lemma 1.3. *([1]) Let (X, d) be a complex valued metric space and let $\{x_n\}$ be a sequence in X.Then $\{x_n\}$ is a Cauchy sequence if and only if $|d(x_n, x_{n+m})| \to 0$ as $n, m \to \infty$.*

Now we follow the notations and definitions given in [7].

Let (X, d) be a complex valued metric space. We denote
$s(z_1) = \{z_2 \in C : z_1 \precsim z_2\}$ for $z_1 \in C$ and
$s(a, B) = \bigcup_{b \in B} s(d(a, b)) = \bigcup_{b \in B} \{z \in C : d(a, b) \precsim z\}$ for $a \in X$ and $B \in C(X)$.
For $A, B \in C(X)$, we denote
$s(A, B) = \Big(\bigcap_{a \in A} s(a, B)\Big) \cap \Big(\bigcap_{b \in B} s(b, A)\Big)$.

Remark 1.4. ([7]) Let (X, d) be a complex valued metric space and let CB(X) be a collection of nonempty closed subsets of X. Let $T : X \to CB(X)$ be a multivaluedmap.For $x \in X$ and $A \in CB(X)$,
define $W_x(A) = \{d(x, a) : a \in A\}$.
Thus , for $x, y \in X$. $W_x(Ty) = \{d(x, u) : u \in Ty\}$.

Definition 1.5. ([7]) Let (X, d) be a complex valued metric space.A nonempty subset A of X is called bounded from below if there exists some $z \in C$ such that $z \precsim a$ for all $a \in A$.

Definition 1.6. ([7]) Let (X, d) be a complex valued metric space. A multivalued mapping $F : X \to 2^C$ is called bounded from below if for each $x \in X$ there exists $z_x \in C$ such that $z_x \precsim u$ for all $u \in Fx$.

Definition 1.7. ([7]) Let (X, d) be a complex valued metric space.The multivalued mapping $T : X \to CB(X)$ is said to have the lower bound proerty (l.b.Property) on (X, d) if the for any $x \in X$, the multi-valued mapping $F_x : X \to 2^C$ defined by $F_x(y) = W_x(Ty)$ is bounded from below.That is for $x, y \in X$,there exists an element $l_x(Ty) \in C$ such that $l_x(Ty) \precsim u$, for all $u \in W_x(Ty)$, where $l_x(Ty)$ is called a lower bound of T associated with (x, y).

Definition 1.8. ([7]) Let (X, d) be a complex valued metric space.The multivalued mapping $T : X \to CB(X)$ is said to have the gretest lower bound proerty
(g.l.b.Property) on (X, d) if the gretest lower bound of $W_x(Ty)$ exists in C for all $x, y \in X$. We denote $d(x, Ty)$ by the g.l.b.Property of $W_x(Ty)$. That is $d(x, Ty) = inf\{d(x, u) : u \in Ty\}$.

Definition 1.9. ([20]) Let $f : X \to X, S : X \to CB(X)$.$f$ is said to be S-weakly commuting at $x \in X$ if $f^2x \in Sfx$.

2. Main Results

Theorem 2.1. *Let (X,d) be a complex valued metric space.*
Let $S,T:X\to CB(X)$ be multi valued mappings $f,g:X\to X$ satisfying

$(2.1.1) Sx\subseteq g(X)$, $Tx\subseteq f(X)$, $\forall x\in X$

$(2.1.2) ad(fx,Ty)+bd(gy,Sx)+\frac{cd(fx,Ty)d(gy,Sx)}{1+d(fx,gy)}\in s(Sx,Ty)$

for all $x,y\in X$ and a, b, c are non negative reals such that $2a+2b<1$,

$(2.1.3) f$ *is S weakly commuting and g is T weakly commuting,*

$(2.1.4) f(X)$ *is complete.*

Then (f,S) and (g,T) have the same coincidence point.

Proof. Let x_1 be an arbitrary point in X. Write $y_1=fx_1$.Since $Sx_1\subseteq g(X)$, there exists $x_2\in X$ such that $y_2=gx_2\in Sx_1$.
From (2.1.2) ,we have

$ad(fx_1,Tx_2)+bd(gx_2,Sx_1)+\frac{cd(fx_1,Tx_2)d(gx_2,Sx_1)}{1+d(fx_1,gx_2)}\in s(Sx_1,Tx_2)$.

$ad(fx_1,Tx_2)+bd(gx_2,Sx_1)+\frac{cd(fx_1,Tx_2)d(gx_2,Sx_1)}{1+d(fx_1,gx_2)}\in\left(\bigcap_{x\in Sx_1}s(x,Tx_2)\right)$.

$ad(fx_1,Tx_2)+bd(gx_2,Sx_1)+\frac{cd(fx_1,Tx_2)d(gx_2,Sx_1)}{1+d(fx_1,gx_2)}\in s(x,Tx_2),\forall x\in Sx_1$.

$ad(fx_1,Tx_2)+bd(gx_2,Sx_1)+\frac{cd(fx_1,Tx_2)d(gx_2,Sx_1)}{1+d(fx_1,gx_2)}\in s(gx_2,Tx_2)$.

$ad(fx_1,Tx_2)+bd(gx_2,Sx_1)+\frac{cd(fx_1,Tx_2)d(gx_2,Sx_1)}{1+d(fx_1,gx_2)}\in\bigcup_{x\in Tx_2}s\left(d(gx_2,x)\right)$.

Since $Tx_2\subseteq f(X)$,there exists some $x_3\in X$ with $y_3=fx_3\in Tx_2$ suchthat $ad(fx_1,Tx_2)+bd(gx_2,Sx_1)+$
$\in s\left(d(gx_2,fx_3)\right)$.

Hence

$d(gx_2,fx_3)\precsim ad(fx_1,Tx_2)+bd(gx_2,Sx_1)+\frac{cd(fx_1,Tx_2)d(gx_2,Sx_1)}{1+d(fx_1,gx_2)}$.

$d(y_2,y_3)\precsim ad(y_1,y_3)+bd(y_2,y_2)+\frac{cd(y_1,y_3)d(y_2,y_2)}{1+d(y_1,y_2)}$.

$|d(y_2,y_3)|\le a\,|d(y_1,y_2)|+a\,|d(y_2,y_3)|$.

$|d(y_2,y_3)|\le\frac{a}{1-a}\,|d(y_1,y_2)|$. ‘.....(1)

Now,

$ad(fx_3,Tx_2)+bd(gx_2,Sx_3)+\frac{cd(fx_3,Tx_2)d(gx_2,Sx_3)}{1+d(fx_3,gx_2)}\in s(Sx_3,Tx_2)$.

$ad(fx_3,Tx_2)+bd(gx_2,Sx_3)+\frac{cd(fx_3,Tx_2)d(gx_2,Sx_3)}{1+d(fx_3,gx_2)}.\in\left(\bigcap_{y\in Tx_2}s(Sx_3,y)\right)$.

$ad(fx_3, Tx_2) + bd(gx_2, Sx_3) + \frac{cd(fx_3,Tx_2)d(gx_2,Sx_3)}{1+d(fx_3,gx_2)}. \in s(Sx_3, y), \forall y \in Tx_2$

$ad(fx_3, Tx_2) + bd(gx_2, Sx_3) + \frac{cd(fx_3,Tx_2)d(gx_2,Sx_3)}{1+d(fx_3,gx_2)} \in s(Sx_3, fx_3).$

$ad(fx_3, Tx_2) + bd(gx_2, Sx_3) + \frac{cd(fx_3,Tx_2)d(gx_2,Sx_3)}{1+d(fx_3,gx_2)} \in \bigcup_{y \in Sx_3} s\left(d(y, fx_3)\right).$

Since $Sx_3 \subseteq g(X)$,there exists some $x_4 \in X$ with $y_4 = gx_4 \in Sx_3$ such that $ad(fx_3, Tx_2) + bd(gx_2, Sx_3)+$

$\in s\left(d(gx_4, fx_3)\right).$

Hence
$d(gx_4, fx_3) \precsim ad(fx_3, Tx_2) + bd(gx_2, Sx_3) + \frac{cd(fx_3,Tx_2)d(gx_2,Sx_3)}{1+d(fx_3,gx_2)}.$

$d(y_3, y_4) \precsim ad(y_3, y_3) + bd(y_2, y_4) + \frac{cd(y_3,y_3)d(y_2,y_4)}{1+d(y_3,y_2)}.$

$|d(y_3, y_4)| \leq b\,|d(y_2, y_3)| + b\,|d(y_3, y_4)|$

$\left|d(y_{3,}y_4)\right| \leq \frac{b}{1-b}\left|d(y_{2,}y_3)\right|.$ ......(2)
putting $h = max\left\{\frac{a}{1-a}, \frac{b}{1-b}\right\}$ and we continuing in this way, we get

$$\begin{aligned}
\left|d(y_n, y_{n+1})\right| &\leq h\left|d(y_{n-1}, y_n)\right| \\
&\leq h^2\left|d(y_{n-2}, y_{n-1})\right| \\
&. \\
&. \\
&. \\
&\leq h^{n-1}\left|d(y_1, y_2)\right|
\end{aligned}$$

Now for $m > n$ consider

$$\begin{aligned}
\left|d(y_{n,}y_m)\right| &\leq \left|d(y_{n,}y_{n+1}) + d(y_{n+1,}y_{n+2}) + \ldots\ldots + d(y_{m-1,}y_m)\right| \\
&\leq h^{n-1} + h^n + \ldots. + h^{m-2}\left|d(y_{1,}y_2)\right| \\
&\leq \left[\frac{h^{n-1}}{1-h}\right] \rightarrow\ 0\ as\ m, n \rightarrow \infty.
\end{aligned}$$

Thus $\{y_n\}$ is a Cauchy sequence in X.
Since $f(X)$ is complete , $\{y_{2n+1}\} = \{fx_{2n+1}\}$ is Cauchy,it follows that $\{y_{2n+1}\}$ converges to $u \in f(X)$.Hence there exists $v \in X$ such that $u = fv$.
Since $\{y_n\}$ is a Cauchy sequence and $\{y_{2n+1}\} \rightarrow u$ it follow that $\{y_{2n}\} \rightarrow u$.

$ad(fv, Tx_{2n}) + bd(gx_{2n}, Sv) + \frac{cd(fv,Tx_{2n})d(gx_{2n},Sv)}{1+d(fv,gx_{2n})} \in s(Sv, Tx_{2n}).$

$ad(fv, Tx_{2n}) + bd(gx_{2n}, Sv) + \frac{cd(fv,Tx_{2n})d(gx_{2n},Sv)}{1+d(fv,gx_{2n})} \in \left(\bigcap_{y \in Tx_{2n}} s(Sv, y)\right).$

$ad(fv, Tx_{2n}) + bd(gx_{2n}, Sv) + \frac{cd(fv,Tx_{2n})d(gx_{2n},Sv)}{1+d(fv,gx_{2n})} \in s(Sv, y), \forall y \in Tx_{2n}.$

$ad(fv, Tx_{2n}) + bd(gx_{2n}, Sv) + \frac{cd(fv,Tx_{2n})d(gx_{2n},Sv)}{1+d(fv,gx_{2n})} \in s(Sv, y_{2n+1}).$

$ad(fv,Tx_{2n})+bd(gx_{2n},Sv)+\frac{cd(fv,Tx_{2n})d(gx_{2n},Sv)}{1+d(fv,gx_{2n})}\in\bigcup_{u^1\in Sv}s\left(d(u^1,y_{2n+1})\right).$

There exists $v_n\in Sv$ such that

$ad(fv,Tx_{2n})+bd(gx_{2n},Sv)+\frac{cd(fv,Tx_{2n})d(gx_{2n},Sv)}{1+d(fv,gx_{2n})}\in s\left(d(v_n,y_{2n+1})\right).$

Therefore $d(v_n,y_{2n+1})\precsim ad(fv,Tx_{2n})+bd(gx_{2n},Sv)+\frac{cd(fv,Tx_{2n})d(gx_{2n},Sv)}{1+d(fv,gx_{2n})}.$

Using g.l.b.property, we get

$d(v_n,y_{2n+1})\preceq ad(fv,y_{2n+1})+bd(y_{2n},v_n)+\frac{cd(fv,y_{2n+1})d(y_{2n},v_n)}{1+d(fv,y_{2n})}.$

Using triangular inequality, we obtain

$d(v_n,y_{2n+1})\precsim ad(fv,y_{2n+1})+bd(y_{2n},y_{2n+1})+bd(y_{2n+1},v_n)+\frac{cd(fv,y_{2n+1})d(y_{2n},v_n)}{1+d(fv,y_{2n})}.$

$d(v_n,y_{2n+1})\precsim\frac{a}{1-b}d(fv,y_{2n+1})+\frac{b}{1-b}d(y_{2n},y_{2n+1})+\frac{c}{1-b}\frac{d(fv,y_{2n+1})d(y_{2n},v_n)}{1+d(fv,y_{2n})}.$

Now consider

$d(fv,v_n)\precsim d(fv,y_{2n+1})+d(y_{2n+1},v_n).$

$\precsim d(fv,y_{2n+1})+\frac{a}{1-b}d(fv,y_{2n+1})+\frac{b}{1-b}d(y_{2n},y_{2n+1})+\frac{c}{1-b}\frac{d(fv,y_{2n+1})d(y_{2n},v_n)}{1+d(fv,y_{2n})}$

$|d(fv,v_n)|\leq|d(fv,y_{2n+1})|+\frac{a}{1-b}|d(fv,y_{2n+1})|+\frac{b}{1-b}|d(y_{2n},y_{2n+1})|$

$+\frac{c}{1-b}\frac{|d(fv,y_{2n+1})|\left|d(y_{2n},v_n)\right|}{\left|1+d(fv,y_{2n})\right|}.$ Letting $n\to\infty$,

we obtain

$|d(fv,v_n)|\to 0$ as $n\to\infty$.By Lemma 1.2,we have $v_n\to fv$ as $n\to\infty$.

Since Sv is closed and $\{v_n\}\subseteq Sv$, it follows that $fv\in Sv$.

Now $u=fv\in Sv$ and $Sv\subseteq g(X)$ it follows that $u=fv=gw$ for some $w\in X$.

$ad(fx_{2n-1},Tw)+bd(gw,Sx_{2n-1})+\frac{cd(fx_{2n-1},Tw)d(gw,Sx_{2n-1})}{1+d(fx_{2n-1},gw)}\in s(Sx_{2n-1},Tw).$

$ad(fx_{2n-1},Tw)+bd(gw,Sx_{2n-1})+\frac{cd(fx_{2n-1},Tw)d(gw,Sx_{2n-1})}{1+d(fx_{2n-1},gw)}.$

$\in\left(\bigcap_{y^1\in Sx_{2n-1}}s(y^1,Tw)\right).$

$ad(fx_{2n-1},Tw)+bd(gw,Sx_{2n-1})+\frac{cd(fx_{2n-1},Tw)d(gw,Sx_{2n-1})}{1+d(fx_{2n-1},gw)}.$

$\in s(y^1,Tw),\forall y^1\in Sx_{2n-1}.$

$ad(fx_{2n-1},Tw)+bd(gw,Sx_{2n-1})+\frac{cd(fx_{2n-1},Tw)d(gw,Sx_{2n-1})}{1+d(fx_{2n-1},gw)}\in s(y_{2n},Tw).$

$ad(fx_{2n-1},Tw)+bd(gw,Sx_{2n-1})+\frac{cd(fx_{2n-1},Tw)d(gw,Sx_{2n-1})}{1+d(fx_{2n-1},gw)}\in\bigcup_{u^1\in Tw}s\left(d(y_{2n},u^1)\right).$

There exists some $w_n\in Tw$ such that

$ad(fx_{2n-1},Tw)+bd(gw,Sx_{2n-1})+\frac{cd(fx_{2n-1},Tw)d(gw,Sx_{2n-1})}{1+d(fx_{2n-1},gw)}\in s(d(y_{2n},w_n)).$

$d(y_{2n}, w_n) \precsim ad(fx_{2n-1}, Tw) + bd(gw, Sx_{2n-1}) + \frac{cd(fx_{2n-1},Tw)d(gw,Sx_{2n-1})}{1+d(fx_{2n-1},gw)}$.

Using g.l.b.property, we obtain

$d(y_{2n}, w_n) \precsim ad(y_{2n-1}, w_n) + bd(gw, y_{2n}) + \frac{cd(y_{2n-1},w_n)d(gw,y_{2n})}{1+d(y_{2n-1},gw)}$.

Using triangular inequality, we have

$d(y_{2n}, w_n) \precsim ad(y_{2n-1}, y_{2n}) + ad(y_{2n}, w_n) + bd(gw, y_{2n}) + \frac{cd(y_{2n-1},w_n)d(gw,y_{2n})}{1+d(y_{2n-1},gw)}$.

$$d(y_{2n}, w_n) \precsim \frac{a}{1-a} d(y_{2n-1}, y_{2n}) + \frac{b}{1-a} d(gw, y_{2n}) + \frac{c}{1-a} \frac{d(y_{2n-1},w_n)d(gw,y_{2n})}{1+d(y_{2n-1},gw)} \ .$$

Now consider $d(gw, w_n) \precsim d(gw, y_{2n}) + d(y_{2n}, w_n)$.

$$\precsim d(gw, y_{2n}) + \frac{a}{1-a} d(y_{2n-1}, y_{2n}) + \frac{b}{1-a} d(gw, y_{2n}) + \frac{c}{1-a} \frac{d(y_{2n-1},w_n)d(gw,y_{2n})}{1+d(y_{2n-1},gw)}.$$

$$|d(gw, w_n)| \leq |d(gw, y_{2n})| + \frac{a}{1-a} |d(y_{2n-1}, y_{2n})| + \frac{b}{1-a} |d(gw, y_{2n})| + \frac{c}{1-a} \frac{|d(y_{2n-1},w_n)||d(gw,y_{2n})|}{|1+d(y_{2n-1},gw)|}.$$

Letting $n \to \infty$ we get

$|d(gw, w_n)| \to 0$ as $n \to \infty$.By Lemma 1.2, we have $w_n \to gw$ as $n \to \infty$.

Since Tw is closed and $\{w_n\} \subseteq Tw$, it follows that $gw \in Tw$.

We have $u = fv = gw \in Tw$.

Since f is S-weakly commuting and g is T-weakly commuting we have

$f^2v \in Sfv \Rightarrow fu \in Su$ and $g^2w \in Tgw \Rightarrow gu \in Tu$.

Thus the pairs (f, S) and (g, T) have the same coincident point. □

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