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LEFT BI-QUASI IDEALS OF $\Gamma-$SEMIRINGS

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Abstract. In this paper we extend the notion of left bi-quasi-ideal in semiring to $\Gamma-$semiring and study the properties of left bi-quasi ideal. We characterize the left bi-quasi simple $\Gamma-$semiring and regular $\Gamma-$semiring using left bi-quasi ideals of $\Gamma-$semirings.
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1. Introduction

In 1995, Murali Krishna Rao [12.13,14] introduced the notion of a $\Gamma-$semiring as a generalization of $\Gamma-$ring, ring, ternary semiring and semiring. The notion of a semiring is an algebraic structure with two associative binary operations where one distributes over the other, was first introduced by Vandiver[22]in 1934 but semirings had appeared in earlier studies on the theory of ideals of rings. A universal algebra $(S, +, \cdot)$ is called a semiring if and only if $(S, +), (S, \cdot)$ are semigroups which are connected by distributive laws, $i.e., a(b + c) = ab + ac,\ (a + b)c = ac + bc,$ for all $a, b, c \in S$. In structure, semirings lie between semigroups and rings. The results which hold in rings but not in semigroups hold in semirings, since semiring is a generalization of ring. The study of rings shows that multiplicative structure of ring is an independent of additive structure whereas in semiring multiplicative structure of semiring is not an independent of additive structure of semiring. The additive and the multiplicative structure of a semiring play an important role in determining the structure of a semiring. The theory of rings and theory of semigroups have considerable impact on the development of theory of semirings. Semirings play an important role in studying matrices and determinants. Semirings are useful in the areas of theoretical computer science as well as in the solution of graph theory, optimization theory, in particular for studying automata,coding theory and formal languages. Semiring theory has many applications in other branches.Henriksen [3] and Shabir et al. [20] studied ideals in semirings. In 1952, the concept of bi-ideals was introduced by Good and Hughes [2] for semigroups. The notion of bi-ideals in rings and semirings were introduced by Lajos and Szasz [9, 10]. Steinfeld [21] first introduced the notion of quasi ideals for semigroups and then for rings. Iseki [4] introduced the concept of quasi ideal for a semiring. Quasi ideals in $\Gamma-$semirings studied by Jagtap and Pawar [8].We know that the notion of an one sided ideal of any algebraic structure is a generalization of notion of an ideal. The quasi ideals are generalization of left ideal and right ideal whereas the bi-ideals are generalization of quasi ideals. As a further generalization of ideals, Marapureddy Murali Krishna Rao[15,16] introduced the concept of bi-quasi-ideals which is a generalization of quasi ideal, bi-ideal and interior ideal and studied fuzzy bi-quasiideals of $\Gamma-$ semigroups and bi-quasi-ideals in semirings. As a generalization of ring, the notion of a $\Gamma-$ring was introduced by Nobusawa [18] in 1964. In 1981, Sen [19] introduced the notion of a $\Gamma-$semigroup as a generalization of semigroup. The notion of a ternary algebraic system was introduced by Lehmer [11] in 1932. Murali Krishna Rao and Venkateswarlu [17] studied regular $\Gamma-$incline and field $\Gamma-$semiring.

In this paper, we extend the notion of left (right) bi-quasi ideal of semiring which is a generalization of bi-ideal to $\Gamma-$semiring and study the properties of left bi-quasi ideal. We characterize the left bi-quasi simple $\Gamma-$semiring and regular $\Gamma-$semiring using left bi-quasi ideals of $\Gamma-$semirings.

2. Preliminaries

In this section we will recall some of the fundamental concepts and definitions, which are necessary for this paper.

Definition 2.1. Let $(M, +)$ and $(\Gamma, +)$ be commutative semigroups. Then we call M a $\Gamma-$semiring, if there exists a mapping $M \times \Gamma \times M \to M$ (images of (x, α, y) will be denoted by $x\alpha y, x, y \in M, \alpha \in \Gamma$) such that it satisfies the following axioms for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$

(i) $x\alpha(y + z) = x\alpha y + x\alpha z$ (ii) $(x + y)\alpha z = x\alpha z + y\alpha z$
(iii) $x(\alpha + \beta)y = x\alpha y + x\beta y$ (iv) $x\alpha(y\beta z) = (x\alpha y)\beta z$.

Every semiring R is a $\Gamma-$semiring with $\Gamma = R$ and ternary operation $x\gamma y$ defined as the usual semiring multiplication.

Definition 2.2. A $\Gamma-$semiring M is said to be commutative $\Gamma-$semiring if $x\alpha y = y\alpha x$, for all $x, y \in M$ and $\alpha \in \Gamma$.

Example 2.3. Let M be a set of all rational numbers and $\Gamma = M$. are commutative semigroups wIth respect to usual addition. Define the mapping $M \times \Gamma \times M \to M$ by $a\alpha b$ as usual multiplication for all $a, b \in M, \alpha \in \Gamma$. Then M is a $\Gamma-$semiring.

Definition 2.4. An element $1 \in M$ is said to be unity if for each $x \in M$ there exists $\alpha \in \Gamma$ such that $x\alpha 1 = 1\alpha x = x$.

Definition 2.5. A $\Gamma-$semiring M is said to have zero element if there exists an element $0 \in M$ such that $0 + x = x = x + 0$ and $0\alpha x = x\alpha 0 = 0$, for all $x \in M, \alpha \in \Gamma$.

Definition 2.6. Let M be a $\Gamma-$semiring. An element $a \in M$ is said to be idempotent of M if there exist $\alpha \in \Gamma$ such that $a = a\alpha a$ and a is also said to be α idempotent.

Definition 2.7. Let M be a $\Gamma-$semiring.If every element of M is an idempotent of M then M is said to be idempotent $\Gamma-$semiring M.

Definition 2.8. Let M be a $\Gamma-$semiring. An element $a \in M$ is said to be regular element of M if there exist $x \in M, \alpha, \beta \in \Gamma$ such that $a = a\alpha x\beta a$.

Definition 2.9. Let M be a $\Gamma-$semiring .If every element of M is a regular element of M then M is said to be regular $\Gamma-$semiring M.

Definition 2.10. A non-empty subset A of $\Gamma-$semiring M is called

(i) a $\Gamma-$subsemiring of M if $(A, +)$ is a subsemigroup of $(M, +)$ and $A\Gamma A \subseteq A$.
(ii) a quasi ideal of M if A is a $\Gamma-$subsemiring of M and $A\Gamma M \cap M\Gamma A \subseteq A$.
(iii) a bi-ideal of M if A is a $\Gamma-$subsemiring of M and $A\Gamma M\Gamma A \subseteq A$.
(iv) an interior ideal of M if A is a $\Gamma-$subsemiring of M and $M\Gamma A\Gamma M \subseteq A$.
(v) a left (right) ideal of M if A is a $\Gamma-$subsemiring of M and $M\Gamma A \subseteq A (A\Gamma M \subseteq A)$.
(vi) an ideal if A is a $\Gamma-$subsemiring of $M, A\Gamma M \subseteq A$ and $M\Gamma A \subseteq A$.
(vii) a $k-$ideal if A is a $\Gamma-$subsemiring of $M, A\Gamma M \subseteq A, M\Gamma A \subseteq A$ and $x \in M,\ x + y \in A, y \in A$ then $x \in A$.

3. Left bi-quasi ideals of $\Gamma-$semirings

In this section we introduce the notion of left bi-quasi ideal, right bi-quasi ideal and bi-quasi ideal. We study the properties of left bi-quasi ideals of $\Gamma-$semiring. Throughout this paper M is a $\Gamma-$semiring with unity element.

Definition 3.1. Let M be a $\Gamma-$semiring. A non-empty subset L of M is said to be left bi-quasi ideal (right bi-quasi ideal) of M if L is a subsemigroup of $(M, +)$ and $M\Gamma L \cap L\Gamma M\Gamma L \subseteq L$ $(L\Gamma M \cap L\Gamma M\Gamma L \subseteq L)$.

Definition 3.2. Let M be a $\Gamma-$semiring. L is said to be bi-quasi ideal of M if it is both a left bi-quasi and a right bi-quasi ideal of M.

Definition 3.3. A $\Gamma-$semiring M is called a left bi-quasi simple $\Gamma-$semiring if M has no left bi-quasi ideal other than M itself.

Examples 3.4.

(i) Let Q be the set of all rational numbers, $M = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid a, b, c, d \in Q \right\}$ be the additive semigroup of M matrices and $\Gamma = M$. Ternary operation $A\alpha B$ is defined as usual matrix multiplication of A, α, B, for all $A, \alpha, B \in M$.Then M is a $\Gamma-$semiring

(a) If $R = \left\{ \begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix} \mid 0 \neq a, 0 \neq b \in Q \right\}$ then R is a quasi ideal of $\Gamma-$semiring M and R is neither a left ideal nor a right ideal.

(b) If $S = \left\{ \begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix} \mid 0 \neq a \in Q \right\}$ then S is a bi-ideal of $\Gamma-$semiring M.

(ii) If $M = \left\{ \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} \mid a, b, c \in Q \right\}$ and $\Gamma = M$ then M is a $\Gamma-$semiring with respect to usual addition of matrices and ternary operation is defined as usual matrix multiplication and $A = \left\{ \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \mid 0 \neq a, 0 \neq b \in Q \right\}$. Then A is a left bi-quasi ideal and A is not a bi-ideal of $\Gamma-$semiring M.

Theorem 3.5. *If L is a left bi-quasi ideal of $\Gamma-$semiring M then $L\Gamma L \subseteq L$.*

Proof. Suppose L is a left bi-quasi ideal of $\Gamma-$semiring M. Then $M\Gamma L \cap L\Gamma M\Gamma L \subseteq L$.
We have $L\Gamma L \subseteq M\Gamma L$. Then $L\Gamma L \cap L\Gamma M\Gamma L \subseteq M\Gamma L \cap L\Gamma M\Gamma L \subseteq L$.
Let $x \in L\Gamma L$. Then $x = y\alpha z, y, z \in L, \alpha \in \Gamma$.
Since M has unity element, there exists $\beta \in \Gamma$ such that $x = y\alpha 1\beta z \in L\Gamma M\Gamma L$.
Therefore $x \in L\Gamma L \cap L\Gamma M\Gamma L \subseteq L$. Hence $L\Gamma L \subseteq L$. □

Theorem 3.6. *Every left ideal of $\Gamma-$semiring M is a left bi-quasi ideal of M.*

Proof. Let L be the left ideal of $\Gamma-$semiring M.
Then $M\Gamma L \subseteq L \Rightarrow M\Gamma L \cap L\Gamma M\Gamma L \subseteq M\Gamma L \subseteq L$.
Hence L is a left bi-quasi ideal of M. □

Theorem 3.7. *Every right ideal of $\Gamma-$semiring M is a left bi-quasi ideal of M.*

Proof. Let L be the right ideal of $\Gamma-$semiring M. Then $L\Gamma M \subseteq L$.
Then $M\Gamma L \cap L\Gamma M\Gamma L \subseteq L\Gamma M\Gamma L \subseteq L\Gamma L \subseteq L$.
Therefore $M\Gamma L \cap L\Gamma M\Gamma L \subseteq L$. Hence L is a left bi-quasi ideal of M. □

Corollary 3.8. *Every left ideal of* $\Gamma-$*semiring* M *is a right bi-quasi ideal of* M.

Theorem 3.9. *Every quasi ideal is a left bi-quasi ideal of* $\Gamma-$*semiring*

Proof. Let L be a quasi ideal of $\Gamma-$semiring M. Then $L\Gamma M \cap M\Gamma L \subseteq L$.
We have $L\Gamma M\Gamma L \subseteq L\Gamma M$, since $M\Gamma L \subseteq M$.
Therefore $M\Gamma L \cap L\Gamma M\Gamma L \subseteq M\Gamma L \cap L\Gamma M \subseteq L$.
Hence every quasi ideal is a left bi-quasi ideal of $\Gamma-$semiring M. □

Theorem 3.10. *Every bi-ideal of* $\Gamma-$*semiring* M *is a left bi-quasi ideal of* $\Gamma-$*semiring* M.

Proof. Let L be a bi-ideal of $\Gamma-$semiring M. Then $L\Gamma M\Gamma L \subseteq L$.
Therefore $M\Gamma L \cap L\Gamma M\Gamma L \subseteq L\Gamma M\Gamma L \subseteq L$.
Hence every bi-ideal of $\Gamma-$semiring M is a left bi-quasi ideal of M. □

Theorem 3.11. *Arbitrary intersection of left bi-quasi ideals of* $\Gamma-$*semiring* M *is either empty or a left bi-quasi ideal of* M.

Proof. Let M be a $\Gamma-$semiring and $L = \bigcap\limits_{i \in I} L_i$, where L_i is a left bi-quasi ideal of M.
Obviously L is a subsemigroup of $(M, +)$.
Therefore $M\Gamma \cap L_i \cap \cap L_i\Gamma M\Gamma \cap L_i \subseteq M\Gamma L_i \cap L_i\Gamma M\Gamma L_i \subseteq L_i$, for all $i \in I$.
Then $M\Gamma \cap L_i \cap \cap L_i\Gamma M\Gamma \cap L_i \subseteq \cap L_i$
Hence L is a left bi-quasi ideal of $\Gamma-$semiring M. □

Corollary 3.12. *If* L *is a left bi-quasi ideal and* R *is a right ideal of* $\Gamma-$*semiring* M *then* $L \cap R$ *is a left bi-quasi ideal of* $\Gamma-$*semiring* M.

Theorem 3.13. *Let* M *be a* $\Gamma-$*semiring and* T *be a non-empty subset of* M. *Every additive subsemigroup* D *of* M *containing* $M\Gamma T \cap T\Gamma M\Gamma T$ *and subset of* T, *is a left bi-quasi ideal of* M.

Proof. Let D be an additive subsemigroup of M containing $M\Gamma T \cap T\Gamma M\Gamma T$. and$D \subseteq T$.
We have $M\Gamma D \subseteq M\Gamma T$ and $D\Gamma M\Gamma D \subseteq T\Gamma M\Gamma T$, since $D \subseteq T$.
Therefore $M\Gamma D \cap D\Gamma M\Gamma D \subseteq M\Gamma T \cap T\Gamma M\Gamma T \subseteq D$.
Hence $\Gamma-$subsemigroup D is a left bi-quasi ideal of M. □

Theorem 3.14. *Let* M *be a* $\Gamma-$*semiring. If* $M = M\Gamma a$, *for all* $a \in M$. *Then every left bi-quasi ideal of* $\Gamma-$*semiring is a quasi ideal.*

Proof. Suppose M is a $\Gamma-$semiring with $M = M\Gamma a$, for all $a \in M$, L is a left bi-quasi ideal of $\Gamma-$semiring and $a \in L$. Then $M\Gamma L \cap L\Gamma M\Gamma L \subseteq L$, and

$$\begin{aligned} & M\Gamma a \subseteq M\Gamma L \\ \Rightarrow & M \subseteq M\Gamma L \subseteq M \\ \Rightarrow & M\Gamma L = M \\ \Rightarrow & L\Gamma M\Gamma L = L\Gamma M \\ \text{Therefore } \; & M\Gamma L \cap L\Gamma M \subseteq L. \end{aligned}$$

Hence the theorem. □

The proofs of the following theorems are similar to theorems in [8].

Theorem 3.15. *Let M be a regular $\Gamma-$semiring. Then every quasi ideal of $\Gamma-$semiring M is an ideal of $\Gamma-$semiring M.*

Theorem 3.16. *M is a regular $\Gamma-$semiring if and only $A\Gamma B = A \cap B$, for any right ideal A and left ideal B of $\Gamma-$semiring M.*

Theorem 3.17. *Let M be a regular $\Gamma-$semiring. Then every left bi-quasi ideal of $\Gamma-$semiring M is an ideal of $\Gamma-$semiring M.*

Proof. Let M be a regular $\Gamma-$semiring and L be a left bi-quasi ideal of $\Gamma-$semiring M. Then $M\Gamma L \cap L\Gamma M\Gamma L \subseteq L$. We know that $L\Gamma M$ and $M\Gamma L$ are right ideal and left ideal of $\Gamma-$semiring M.
By Theorem[3.16], we have $L\Gamma M\Gamma M\Gamma L = L\Gamma M \cap M\Gamma L$.
Therefore $L\Gamma M \cap M\Gamma L = L\Gamma M\Gamma M\Gamma L \subseteq M\Gamma L$
and $L\Gamma M \cap M\Gamma L = L\Gamma M\Gamma M\Gamma L \subseteq L\Gamma M\Gamma L$
Hence $L\Gamma M \cap M\Gamma L \subseteq M\Gamma L \cap L\Gamma M\Gamma L \subseteq L$.
Thus L is a quasi ideal of $\Gamma-$semiring M.
Therefore L is an ideal of $\Gamma-$semiring M. □

Theorem 3.18. *Let L be a left bi-quasi ideal of $\Gamma-$semiring M, $e\Gamma L \subseteq L$ and e be β-idempotent of $M, \beta \in \Gamma$. Then $e\Gamma L$ is a left bi-quasi ideal of M.*

Proof. Let L be a left bi- quasi ideal of $\Gamma-$semiring M and e be β-idempotent idempotent of M. i.e. $e\beta e = e$. Suppose $x \in L \cap e\Gamma M$. Then $x \in L$ and $x = e\alpha y, \alpha \in \Gamma, y \in M$.

$$\begin{aligned} x &= e\alpha y \\ &= e\beta e\alpha y \\ &= e\beta(e\alpha y) \\ &= e\beta x \in e\Gamma L \end{aligned}$$

$$\text{Therefore } L \cap e\Gamma M \subseteq e\Gamma L.$$

We have $e\Gamma L \subseteq L$ and $e\Gamma L \subseteq e\Gamma M$.
Therefore $e\Gamma L \subseteq L \cap e\Gamma M$. Hence $e\Gamma L = L \cap e\Gamma M$.
$e\Gamma L$ is the intersection of left bi-quasi ideal L and right ideal $e\Gamma M$.
By Corollary [3.12], $e\Gamma L$ is a bi-quasi ideal of M. □

Corollary 3.19. *Let L be a left bi-quasi ideal of $\Gamma-$semiring M, $L\Gamma e \subseteq L$ and e be the idempotent of M. Then $L\Gamma e$ is a left bi-quasi ideal of M.*

Theorem 3.20. *Let M be a $\Gamma-$semiring. Then the following statements are equivalent*

(i) *M is a left bi-quasi simple $\Gamma-$semiring M.*
(ii) *$M\Gamma a = M$, for all $a \in M$*
(iii) *$(a) = M$, for all $a \in M$*

Proof. Let M be a $\Gamma-$semiring with unity element.

(i) ⇒ (ii) Suppose that M is a left bi-quasi simple $\Gamma-$semiring , $a \in M$.
and $L = M\Gamma a$. Then L is a left ideal. Therefore by Theorem[3.6], L is a left bi-quasi ideal of $\Gamma-$semiring M.
Hence $M\Gamma a = M$, for all $a \in M$.

(ii) ⇒ (iii) Suppose that $M\Gamma a = M$, for all $a \in M$ and (a) is the smallest left bi-quasi ideal of M containing a.
Then $M\Gamma a \subseteq (a) \subseteq M$
$\Rightarrow M \subseteq (a) \subseteq M$.
Therefore $M = (a)$.

(iii)⇒ (i) Suppose (a) is the smallest bi-quasi ideal generated by $a, (a) = M$, for all $a \in M$ and A is a left bi-quasi ideal of M and $a \in A$.
Then $(a) \subseteq A \subseteq M$. Therefore $A = M$.
Hence M is a left bi-quasi simple $\Gamma-$semiring.

□

Theorem 3.21. *Let M be a $\Gamma-$semiring. Then M is a left bi-quasi simple $\Gamma-$semiring if and only if $M\Gamma a \cap a\Gamma M\Gamma a = M$, for all $a \in M$.*

Proof. Suppose M is a left bi-quasi simple $\Gamma-$semiring and $a \in M$.
By Theorem [3.12], $M\Gamma a \cap a\Gamma M\Gamma a$ is a left bi-quasi ideal of $\Gamma-$semiring M.
Therefore $M\Gamma a \cap a\Gamma M\Gamma a = M$, for all $a \in M$, since M is a left bi-quasi simple $\Gamma-$semiring .

Conversely suppose that $M\Gamma a \cap a\Gamma M\Gamma a = M$, for all $a \in M$.
Let T be a left bi-quasi ideal of $\Gamma-$semiring M and $a \in T$.

$$\begin{aligned} M &= M\Gamma a \cap a\Gamma M\Gamma a \\ &\subseteq M\Gamma T \cap T\Gamma M\Gamma T \subseteq T \subseteq M \\ \text{Therefore } \; M &= T. \end{aligned}$$

Hence M is a left bi-quasi simple $\Gamma-$semiring . □

Theorem 3.22. *Let M be a $\Gamma-$semiring. Then M is a left bi-quasi simple $\Gamma-$semiring if and only if $(a)_{bq} = M$, for all $a \in M$, where $(a)_{bq}$ is the left bi-quasi ideal generated by a.*

Proof. Let M be a $\Gamma-$semiring. Suppose that $(a)_{bq}$ is a left bi-quasi ideal generated by a and M is a left bi-quasi simple $\Gamma-$semiring. Then $(a)_{bq} = M$, for all $a \in M$.

Conversely suppose that B is a left bi-quasi ideal of $\Gamma-$semiring M and $(a)_{bq} = M$, for all $a \in M$. Let $b \in B$.
Then $(b)_{bq} \subseteq B \;\Rightarrow\; M = (b)_{bq} \subseteq B \subseteq M$.
Therefore M is a left bi-quasi simple $\Gamma-$semiring. □

Theorem 3.23. *Let M be a $\Gamma-$semiring. M is a regular $\Gamma-$semiring if and only if $B = M\Gamma B \cap B\Gamma M\Gamma B$ for every left bi-quasi ideal B of M.*

Proof. Suppose M is a regular $\Gamma-$semiring,B is a left bi-quasi ideal of M and $x \in B$.
We have $M\Gamma B \cap B\Gamma M\Gamma B \subseteq B$, since B is a left bi-quasi ideal of M.
As M is a regular, there exist $y \in M, \alpha, \beta \in \Gamma$ such that $x = x\alpha y\beta x$.
Then $x \in M\Gamma B$ and $B\Gamma M\Gamma B$. Therefore $B \subseteq B\Gamma M\Gamma B \cap M\Gamma B$.
Hence $B\Gamma M\Gamma B \cap M\Gamma B = B$.

Conversely suppose that $B = M\Gamma B \cap B\Gamma M\Gamma B$, for any left bi-quasi ideal B of M. Let R and L be right ideal and left ideal of Mrespectively. Then by Corollary [3.12] , $R \cap L$ is a bi-quasi ideal. Then

$$\begin{aligned} R \cap L &= M\Gamma(R \cap L) \cap (R \cap L)\Gamma M\Gamma(R \cap L) \\ &\subseteq (R \cap L)\Gamma M\Gamma(R \cap L) \\ &\subseteq R\Gamma M\Gamma L \\ &\subseteq R\Gamma L. \end{aligned}$$

We have $R\Gamma L \subseteq L$ and $R\Gamma L \subseteq R$. Therefore $R\Gamma L \subseteq R \cap L$.
Hence $R\Gamma L = R \cap L$. By Theorem [3.16] , M is a regular $\Gamma-$semiring. ☐

Theorem 3.24. *Let M be a $\Gamma-$semiring . Then every interior ideal of $\Gamma-$semiring M is a left bi-quasi ideal of M.*

Proof. Let B be an interior ideal of $\Gamma-$semiring M. Then

$$M\Gamma B\Gamma M \subseteq B$$

$$M\Gamma B \subseteq M\Gamma B\Gamma M \subseteq B.$$

$$M\Gamma B \cap B\Gamma M\Gamma M \subseteq M\Gamma B \subseteq B.$$

Hence every interior ideal is a left bi-quasi ideal of M. ☐

Corollary 3.25. *Let M be a $\Gamma-$semiring. Every interior ideal of $\Gamma-$semiring M is a right bi-quasi ideal of M.*

Corollary 3.26. *Let M be a $\Gamma-$semiring. Every interior ideal of $\Gamma-$semiring M is a bi-quasi ideal of M.*

Theorem 3.27. *Let B be a $\Gamma-$subsemiring of M. If $M\Gamma B\Gamma M \cap B\Gamma M\Gamma B \subseteq B$ then B is a bi-quasi ideal of M.*

Proof. Suppose B is a $\Gamma-$subsemiring of M and $M\Gamma B\Gamma M \cap B\Gamma M\Gamma B \subseteq B$.
$M\Gamma B \cap B\Gamma M\Gamma B \subseteq M\Gamma B\Gamma M \cap B\Gamma M\Gamma B \subseteq B$ and
$B\Gamma M \cap B\Gamma M\Gamma B \subseteq M\Gamma B\Gamma M \cap B\Gamma M\Gamma B \subseteq B$.
Hence B is a bi-quasi ideal of M. ☐

4. CONCLUSION

As a further generalization of ideals ,we introduce the notion of left (right) bi-quasi ideal of $\Gamma-$semiring which is a generalization of ideal,left ideal,right ideal, bi-ideal,quasi ideal and interior ideal of $\Gamma-$semiring and studied some of their properties. We introduced the notion of bi-quasi simple $\Gamma-$semiring and characterized the bi-quasi simple $\Gamma-$ semiring. In continuity of this paper, we study prime bi-quasi ideals, maximal and minimal bi-quasi ideals of $\Gamma-$ semiring.

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